Darmstadt
Discussion Papers
in ECONOMICS

TECHNISCHE UNIVERSITÄT DARMSTADT

The Population Question in a Neoclassical Growth Model

A Brief Theory of Production per Capita

Tim Lüger

Nr. 235

Arbeitspapiere der Volkswirtschaftlichen Fachgebiete der TU Darmstadt

ISSN: 1438-2733

ISSN 1438-2733

Printed in Germany
Technische Universität Darmstadt
Department of Law and Economics
D – 64289 Darmstadt
Germany
www.wi.tu-darmstadt.de

The Population Question in a Neoclassical Growth Model

A Brief Theory of Production per Capita

Tim Lueger*

March 19, 2019

Abstract

This work seeks to answer the "population question," i.e. the effect of population growth on production per capita. This question has lingered in economic thought for centuries and to this day two general lines of thought can be identified, which might be marked as the "optimist" and the "pessimist" view. While the optimists claim that an increase in population will – chiefly owed to concomitant specialization and technological progress – raise average production per capita, the pessimists maintain that the latter would decline as a result of resources becoming relatively more scarce. Integrating both approaches and using a neoclassical framework, this work intends to show that sustainably increasing productivity is predominantly the result of reducing too high fertility toward a lower level such that diminishing returns are outweighed by the benefits from labor division. The paper argues that the historical reduction of fertility can almost completely explain long–run development.

JEL classification: B12, B22, J1, O47, N01, N3

Keywords: Population Question, Division of Labor, Diminishing Returns, Demographic Transition, Economic Development, Classical Growth Theory, Neoclassical Growth Theory, Unified Growth Theory, History of Economic Thought

*Preliminary version. Author affiliation: Darmstadt University of Technology, Faculty of Law and Economics, Department of Economic Theory (Email: lueger@vwl.tu–darmstadt.de; phone no. +49 6151 16–57255).

1 Introduction: The Population Question

This work seeks to answer the "population question," i.e. the effect of population growth on production per capita. This question has lingered in economic thought for centuries and to this day two general lines of thought can be identified, which might be marked as the "optimist" and the "pessimist" view.[1] While the optimists claim that an increase in population will – chiefly owed to concomitant specialization and technological progress – raise average production per capita, the pessimists maintain that the latter would decline as a result of resources becoming relatively more scarce. Integrating both approaches and using a neoclassical framework, this work intends to show that sustainably increasing productivity[2] is predominantly the result of reducing too high fertility toward a lower level such that diminishing returns are outweighed by the benefits from labor division. This result is broadly in line with Galor (2011), who suspects that a substitution of child quality for child quantity initiated the take–off toward a sustained path of economic development. In particular, Ashraf et al. (2013) find a negative effect of fertility on GDP per capita that can account for about 10% of long-run growth. The paper argues that the historical reduction of fertility can almost completely explain long–run development. Being a quite popular objection (e.g. Becker (1991)), it will certainly be replied that the observed negative effect of fertility on productivity is merely another illustration of a statistical correlation being misread as a causal effect. This author intends to sufficiently clarify that the modeled relationship reflects a very deep insight from classical as well as neoclassical growth theory.[3]

To arrive at the above result, the paper proceeds as follows. In the next section it will be explained that, building on the assumption of an efficient division of labor, the original neoclassical production function is already an exhaustive instrument in determining productivity, even without explicitly accounting for "technology."[4] Following Smith (1776) and the classical economists, the extension of the (static) production function approach toward a neoclassical (dynamic) growth model is justified by the as-

[1] See for example Bloom et al. (2003). For an optimist view, see for example Becker (1988) or Kremer (1992). Examples of a pessimist view are Hardin (1968) or Ehrlich (1968).

[2] For simplicity, production per capita will be abbreviated as *productivity*. We will also, following Lucas (1988), refer to an increase in productivity (intensive growth) as *economic development*, whereas the expression *economic growth* will be used to characterize an increase in gross production (extensive growth). These terms have often been confused in the past, in particular in that literature which is concerned with "growth in economic development."

[3] This paper does not attempt to model a theory of population. The perhaps most recent evaluation of this paper's argument is provided by Chatterjee and Vogl (2018).

[4] As is for example done by Romer (1986).

sumption that additional labor is responsible for generating the benefits from a division of labor and is therefore "the source of all value." The third section clarifies that what was commonly meant by the variable "labor" should be substituted by the variable "unskilled labor," which is well approximated by population size. On this basis, a new model of population as source of all value is presented, where regular innovations are supposed to be embodied by population growth through a division of labor and subsequent specialization and structural equations are derived. The latter are evaluated in section four and briefly linked to unified growth theory in section five. Section six concludes and suggests possible extensions of the work.

2 A Labor Theory of Production and Development

2.1 Static Theory

> *The annual labour of every nation is the fund which originally supplies it with all the necessaries and conveniences of life which it annually consumes [...]* [5]

The one decisive regular cause by which population growth is classically assumed to enhance production is the *division of labor*. Smith (1776) suggested that, in an environment favoring the security of property and income, the inherent tendency of individuals to exchange their products would result in a division of labor.[6] What Smith meant by "division of labor" can be understood as the efficient cooperation of all productive individuals of the economy to maximize total production. This efficient level of cooperation would be achieved if all production processes were perfectly subdivided between those individuals. Such a perfect division implied that every new individual entering the economy would tend to induce a new subdivision of production into smaller, more easily conductible, efficient production processes and therefore raise production.[7]

Nonetheless, in spite of the generally observed tendency toward an efficient division of labor in a free market economy, the classics had already noted large regional differences in individual productivity. Obviously, these differences were owed to the relative abundance or scarcity of some other production factor. Senior (1836) extended the

[5] Smith (1776), Introduction.

[6] Throughout this paper we will presume the existence of such institutional conditions.

[7] We may even relax the above presumption by stating that efficient institutions are one result of such a perfect division of labor.

doctrine of labor division by stating that what Smith had really meant was the efficient subdivision of production processes through an efficient combination of *several* production factors.[8] He argued that this concept ought in fact be termed the "division of production" instead of the "division of labor" and could be formulated as a relationship between production and an efficient use of all these production factors – a production function.[9]

Perpetuating this production function approach, Clark (1899) argued that, since production factors are in reality dynamically interconnected, a separate analysis of the effect of every single factor required the consideration of an abstract static state of an economy where production factors were assumed to be independent. To Clark, it seemed obvious that the economist had to start with the easier task of modeling a static production function first, where all except the production factor of interest were held constant (*ceteris paribus*) such that no causal relationships between production factors interfered.[10] Roughly at the same time, the static model was mathematically advanced by Wicksteed (1894), adding to the above considerations the perhaps most powerful proposition for employing a valid aggregate macroeconomic production function: the replication argument. It states that under static conditions, a replication of an exhaustive list of production factors must universally generate a replication of production. Correspondingly, an aggregate production function is to be defined as a static production function fulfilling the replication argument, which was later formulated as the doctrine of constant returns to scale.[11]

> *Now it must of course be admitted that if the physical conditions under which a certain amount of wheat, or anything else, is produced were exactly repeated the result would be exactly repeated also, and a proportional increase of the one would yield a proportional increase of the other. The crude division of the factors of production into land, capital and labour must indeed be abandoned [...]. We must regard every kind and quality of labour that*

[8] A production factor being defined as an input resource that positively contributes to production.

[9] "[...] division of production would have been a more convenient expression than division of labour; but Adam Smith's authority has given such currency to the term division of labour, that we shall continue to employ it, using it, however, in the extended sense in which it appears to have been used by Adam Smith." Senior (1836), p. 159.

[10] "Why, then, do we wish to know the laws of an imaginary static state? Because the forces that act in such a state continue to act in a dynamic one. [...] In dealing with the complex problems of an advancing economy, the key of success is the separate study of the static forces that constantly act within it." Clark (1899), p. 60.

[11] See Hicks (1936).

can be distinguished from other kinds and qualities as a separate factor; and in the same way every kind of land will be taken as a separate factor. [...] Each of these may be scheduled in its own unit, and when this has been done the enumeration of the factors of production may be regarded as complete. On this understanding it is of course obvious, that a proportional increase of all the factors of production, will secure a proportional increase of the product.[12]

Notwithstanding the requirement of an exhaustive list of factors, most classical economists seem to have agreed on the usage of merely two factors required for production Y, labor L and capital K.[13] Wicksteed declared the just use of this simplification as long as capital was viewed to serve as an approximate residual "catch–all–variable" to incorporate an exhaustive list of all hitherto omitted production factors required for total production, measured in a complex unit, for example in exchange value.[14] As a result, capital could be defined as all *things of value required for production*, whereas the explicit use of other production factors would have unnecessarily complicated the theoretical and empirical analysis – a simplification that was later used by Keynes (1936), Robinson (1954), Solow (1957) and many others.[15,16] Obviously, when following this definition, a potential factor specifically accounting for "technology" would become obsolete.

Alongside to this generalized capital, labor remained the main factor of interest as long as labor productivity $\frac{Y}{L}$ seemed to best measure individual productivity.[17]

[12] Wicksteed (1894), p. 33.

[13] Often, the additional factor land has been added: "[...] it has been usual to take each of the great factors of production such as Land, Capital and Labour, severally, to enquire into the special circumstances under which that factor co–operates in production [...]." Wicksteed (1894), p. 7.

[14] "All the constituents of this generalised 'capital' are regarded as reduced to their expression in money." Wicksteed (1894), p. 13.

[15] "The capital in existence at any moment may be treated simply as 'part of the environment on which labour works.'" Keynes (1936) in Robinson (1954), p. 214.

[16] "Were the data available, it would be better to apply the analysis to some precisely defined production function with many precisely defined inputs. One can at least hope that the aggregate analysis gives some notion of the way a detailed analysis would lead." Solow (1957), p. 312, footnote.

[17] We follow here McCulloch's notion of productive labor: "So long as an individual employs himself in any way not detrimental to others, and accomplishes the object he has in view, his labour is obviously productive; while, if he do not accomplish it, or obtain some sort of equivalent advantage from the exertion of his labour, it is as obviously unproductive. This definition seems sufficiently clear, and leads to no perplexities; [...] it is not possible to adopt any other without being involved in endless difficulties and contradictions." McCulloch (1864), part I, chapter I, section II.

> *Here, then, is the simple and decisive test by which we are to judge of the expediency of all measures affecting the wealth of the country, and of the value of all innovations. If they make labour more productive, [...] they must be advantageous; [...] Considered in this point of view, that great branch of the science which treats of the production of wealth will be found to be abundantly simple, and easily understood.* [18]

To this end, Wicksteed suggested that labor had to be separated out of the infinite number of production factors.

> *What we really want is to separate out labour and dose it with land–plus–capital, if possible to satiety.*[19] *[. . .] It is perfectly legitimate to start with a unit of [labour], assume that the command of the other factors of production is so exercised as to secure the maximum productive result, and then treat the product as a function of [labour] and pounds sterling. [. . .] and we may, if we choose, select any one factor to measure in its proper unit while measuring all the rest in a common unit.* [20]

Moreover, Wicksteed concluded that such an aggregate production function implied *diminishing returns* to each production factor, i.e. that an incremental static use of any separate factor would yield an increasingly diminishing marginal product as well as diminishing productivity of that factor.[21] Hence, the idea of diminishing returns became a universal law for the accumulation of any production factor, which is one of the conclusive statements of the static theory of production.[22] Cobb and Douglas (1928) built on Wicksteed's approach and suggested a specific form of an aggregate production function that would account for the above conditions.[23] Their aggregate production function $Y = F(K, L) = K^{\alpha}L^{1-\alpha}$ *with* $0 < \alpha < 1$, where α reflects the constant production elasticity of capital, is still a commonly taught instrument of the

[18] McCulloch (1864), part I, chapter I, section II.
[19] Wicksteed (1894), p. 14, footnote.
[20] Wicksteed (1894), p. 39.
[21] Wicksteed (1894), p. 14, footnote: "Then if [labour] remains constant and capital–plus–[land] increases, we shall have increasing returns per unit of [labour] and decreasing returns per unit of capital. But if capital is constant and [labour] increases, we shall have increasing returns per unit of the former and decreasing returns per unit of the latter."
[22] See for example Humphrey (1997).
[23] "The theory referred to (due to J.B. Clark, Wicksteed et al.) states that production, labor and capital are so related that [...] production is a first degree homogeneous function of labor and capital." Cobb and Douglas (1928), p. 151.

neoclassical growth school, although it does not necessarily display the "true" form of the aggregate production function.

In summary, we may conclude that Smith's concept of labor division has survived the marginal revolution in the form of an aggregate production function centering on the production factor labor. With regard to the population question, the static theory of production supports the idea that each additional amount of labor entering an efficient division of labor causes production to rise and labor productivity to shrink due to diminishing returns.

2.2 Dynamic Theory

By assuming that all production factors are efficiently employed, so far nothing has been said about the potential dynamic effects, or gains, derived from a division of labor. On that account, Smith emphasized that

> *[T]he greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is anywhere directed, or applied, seem to have been the* effects *of the division of labour.*[24]

Extending the static theory of production toward a dynamic theory of growth, Solow (1956) and Swan (1956) integrated the Cobb–Douglas production function into Harrod's (1939) and Domar's (1946) concepts of intertemporal capital accumulation by using as main dynamic equation $K_{t+1} = sY_t + (1-\delta)K_t$, or in units per labor

$$\frac{K_{t+1}}{L} = s\left(\frac{K_t}{L}\right)^{\alpha} + (1-\delta)\frac{K_t}{L} \tag{1}$$

with time index t for the corresponding year, annual savings rate s and annual capital depreciation rate δ. Considering the direction of causality between the production factors, this framework assumes that the amount of labor is exogenously supplied, while the amount of capital is allowed to adapt over time. The level of labor is therefore assumed to be unaffected by capital changes, whereas changes in the amount of labor may generally cause changes in the amount of capital. Again, the rationale behind the exogenous use of labor as compared to capital can be traced back to classical economics and in particular to Locke (1689) and McCulloch (1864), who considered labor as the only source of value, without which capital would be not worth anything.

[24] Smith (1776), book I, chapter I.

> *'Tis labour, then which puts the greatest part of value upon land, without which it would scarcely be worth of any thing. Tis to that we owe the greatest part of all its useful products;' [...] Locke has here all but established the fundamental principle on which the science [of economics] rests. Had he carried his analysis a little farther, he could not have failed to perceive that neither water, leaves, skins, nor any one of the spontaneous productions of nature, has any value, except what it derives from the labour required for its appropriation. The utility of such products makes them be demanded; but it does not give them value. This is a quality which can be communicated only through the agency of voluntary labour of some sort or other. [...] It is to labour, therefore, and to it only, that man owes every thing possessed of value.*[25]

On these grounds, dynamic changes in labor productivity can be modeled as a response to an exogenous labor shock as follows. As a starting point, neoclassical economists reasonably assume a static equilibrium in which capital accumulation is equal to zero and capital depreciation equals saving, i.e. for $K_{t+1} = K_t = K$ we have

$$\delta \frac{K}{L} = s \left(\frac{K}{L} \right)^{\alpha}. \tag{2}$$

The resulting "steady state" equilibrium of capital per labor $\left(\frac{K}{L}\right)^{*} = \frac{K_0}{L_0}$ is marked on the x–axis of figure 2.1 (ignoring g_{L_2} at this instance). In this situation, a positive labor shock would statically reduce capital per labor toward $\frac{K_0}{L_1}$, where savings are higher than capital depreciation. Subsequently, additional capital will be accumulated and capital per labor eventually reconverges to its original steady state such that $\frac{K_0}{L_0} = \frac{K_1}{L_1}$ with $K_1 > K_0$ and $L_1 > L_0$. As a result, although diminishing returns have reduced labor productivity in the short run, a growing labor force seems to be capable of accumulating and maintaining a larger amount of capital in the long run, reflecting the abstract "gains" from a division of labor, which will be analyzed in more detail at a later point. The same mechanism applies reversely if labor shrinks. In that case, relative labor scarcity increases labor productivity in the short run without being able to maintain the old amount of capital in the long run. As a general result, it might be deducted that every change in the variable labor is in the long run followed by a proportional change in the variable capital such that we may write in terms of growth rates

[25] Locke (1689) "Of Civil Government" book ii §§ 42, 43 in McCulloch (1864), part I, chapter I, section II.

$$g_Y = \alpha g_K + (1-\alpha) g_L = g_L, \tag{3}$$

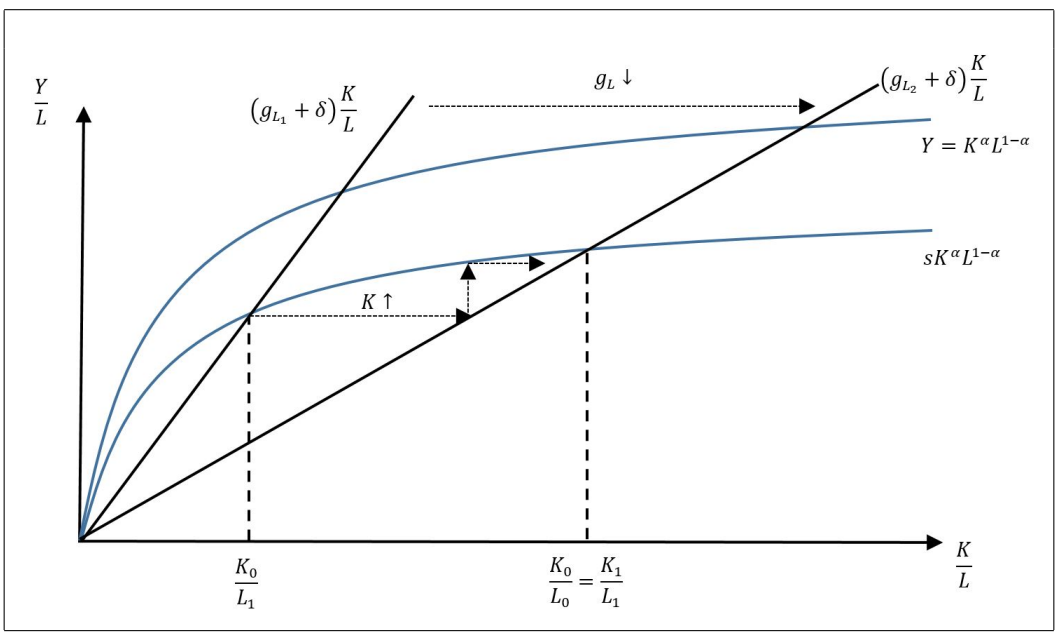
where g_Y, g_K and g_L denote the growth rates of production, capital and labor respectively.[26] Consequently, labor productivity $\frac{Y}{L}$ would in this framework ultimately remain constant after a labor shock, since we have $g_{Y/L} = g_Y - g_L = 0$.

It has been argued that this model would be incomplete as it does not account for the seemingly historically observed increases in labor productivity.[27] This claim often overlooks the effect stemming from a potential decrease in labor growth. Supposing that labor would increase every period at the same constant rate $g_L = \frac{L_{t+1}}{L_t} - 1$ yields the following modified dynamic law and steady–state value for labor productivity:

$$(1+g_L)\frac{K_{t+1}}{L_{t+1}} = (1-\delta)\frac{K_t}{L_t} + s\left(\frac{K_t}{L_t}\right)^{\alpha} \Rightarrow \left(\frac{Y}{L}\right)^{*} = \left(\frac{s}{\delta + g_L}\right)^{\frac{\alpha}{1-\alpha}}. \tag{4}$$

Since higher labor growth reduces the steady–state value of labor productivity, we find that an exogenous decrease of labor growth toward g_{L_2} is well–qualified for causing labor productivity growth during the transition between steady states (see figure 2.1).

Figure 2.1: A labor growth slowdown in the Solow model.



[26] As Solow emphasized, "[T]he common rate of growth is just the exogenously given growth rate of the labor force." Solow (2000), p. 357.

[27] As a result, the additional factor "technology" (Solow (1956)) or "measure of our ignorance" (Abramovitz (1956)) was introduced , embodied in the so–called "Solow–Residual."

3 A Population Theory of Growth and Development

3.1 Static Theory

While empirical long–run estimates for production are generally readily obtainable and generalized capital can only be measured as a residual, we require empirical values for labor as well as those of production elasticity of labor to test the validity of the neoclassical model. To this end, the crucial questions arise what labor is supposed to mean in theory and in what units it thus ought to be measured empirically.[28]

Theoretically, we will again follow the classical view of Senior (1836), who defined labor as "the voluntary exertion of bodily or mental faculties for the purpose of production."[29] Such a definition comprises the quality and the quantity of labor, or to use a slightly different modern wording, skilled as well as unskilled labor. Empirically, the first assessment of economic growth based on the above aggregate production function was conducted by Cobb and Douglas (1928). Problematically, they used Wicksteed's production exhaustion theorem[30] to interpret the empirical share of labor income on total income as production elasticity of *labor* $(1-\alpha)$, whereas they measured the production factor L in units of *laborers* with the following reservation:

> *Such an index [L] of course makes no allowance for possible changes in the quality of the laborers or in the intensity of their work. [...] When they can be measured, then they should be included.*[31]

Notwithstanding this qualification, conventional empirical and theoretical studies still seem to erroneously follow Cobb and Douglas' provisional model and continue to confuse the production elasticity of labor with that of the number of laborers without adjusting

[28] Unfortunately, while the Cambridge capital controversy has questioned the correct measurement of the production factor capital, no such debate can be found on the empirical use of the production factor labor.

[29] Senior (1836), p. 152.

[30] Since $F(K,L)$ is homogeneous of degree one, the Euler theorem can be applied as follows, where r represents the marginal product of capital and w the marginal product of labor:

$$Y = F(K,L) = K^{\alpha}L^{1-\alpha} = \frac{\partial Y}{\partial K}K + \frac{\partial Y}{\partial L}L = \alpha Y + (1-\alpha)Y = rK + wL \Leftrightarrow 1-\alpha = \frac{wL}{Y}.$$

"[...] under ordinary conditions of competitive industry, it is sensibly or approximately true that if every factor of production draws a remuneration determined by its marginal efficiency or significance, the whole product will be exactly distributed." Wicksteed (1894), p. 38.

[31] Cobb and Douglas (1928), p. 149.

for the quality of the laborers.[32] To make up for this defect we may first attempt to assess whether a labor measurement exists that can account for the quality as well as the quantity of labor. However, once finding that it is still not possible to measure a unit of labor quality without making fantastic assumptions, it will – in pursuing the population question and in explaining economic development – appear much more promising to focus on a theoretical conception that separates out the quantity of labor. We will therefore now discuss the necessary adjustments for a neoclassical model based on unskilled labor as central variable.

Firstly, we define an unskilled individual I as an individual who has just entered the labor market and has therefore become productive for the first time. We may then define a unit of unskilled labor n as the amount of labor provided by such an unskilled individual. Eventually, the aggregate amount of unskilled labor L_u is defined as one unit of unskilled labor multiplied by the number W of all productive individuals, i.e. $L_u = I{\cdot}n{\cdot}W$. Since we are interested in the *annual* amount of unskilled labor of the *average* individual, these values can be standardized using $I = n = 1$.[33] In contrast to Cobb and Douglas, who employed the number of laborers W to measure the overall amount of unskilled labor, this work uses the size of the population for the following reasons. Firstly, from a theoretical point of view, unskilled labor moves – although with a maturity lag – almost proportionally with population. Empirically, population data generating per capita values are amongst the most objective and transparent measurements, relatively easy to collect, and arguably possess the longest historical time series. Finally, we are mainly interested in production per capita, i.e. (population) productivity instead of productivity per laborer.

To exhaustively subdivide labor into independent components, we will then refer to the production factor displaying the quantity of labor as population(N), while terming the remaining factor human capital (H), incorporating the residual quality of labor. The latter is supposed to comprise every acquired productive skill in addition to unskilled labor. Statically, like every other production factor, population and human capital must

[32] For example, Barro and Sala–i–Martin (2003) use units of *labor* instead of units of *laborers, workers* or *population* in order to calculate output per worker and output per capita, pp. 27–28.

[33] "The dimension of time enters negatively into all the quantities we are discussing. 'Land' is use of land per unit of time. Labour is hours of work per unit of time, etc. But the universality of this condition enables us to dispense with any special consideration of it." Wicksteed (1894), p. 20, footnote.

necessarily exhibit diminishing returns. Following Mankiw et al.'s (1992) extension of the Solow model[34], this subdivision suggests the use of the production function

$$Y = K^{\alpha}L^{1-\alpha} = K^{\alpha}H^{\beta}N^{1-\alpha-\beta} \; with \; 0 < \alpha, \beta, (1-\alpha-\beta) < 1. \tag{5}$$

3.2 Dynamic Theory

Given our new static division of production, we will now again inquire into the relationships between the production factors. In section 2 we assumed that the decisive causal relation runs from labor, the source of all value, to capital accumulation. We will see that this causal effect can be renewed inasmuch as population growth may be considered as source of all human capital as well as capital accumulation – a result that can be derived if we take a more detailed look at Smith's dynamic effects from the division of labor. According to Smith, all men are born equal and every worker acquires in the same way both productive skills and productive capital over his lifetime to optimize individual production. Senior remarked about Smiths main idea:

> *The advantages derived from the division of labour are attributed by Smith to three different circumstances. 'First, to the increase of dexterity in every particular workman; secondly, to the saving of the time which is commonly lost in passing from one species of work to another; and lastly, to the invention of a great number of machines which facilitate and abridge labour, and enable one man to do the work of many.'*[35]

These effects are all reducible to gains from specialization and can account for an exhaustive dynamic theory of growth. Firstly, "the saving of time" is owed to specialization across a given territory. Obviously, as soon as new individuals are added to the division of labor, the economy becomes more densely populated and the efficient geographical distribution reduces any kind of transport costs between production processes. This advantage simply reflects the static use of the factor N including diminishing returns through the relative abundance of this factor. Secondly, "the increase in dexterity" is owed to specialization over time. If the same production process is performed frequently on tighter geographical constraints, individuals will successively tend to improve their

[34] This paper makes two important deviations from the Mankiw et al. (1992) model: Population is used instead of labor and total factor productivity is assumed to be non–existent, following the Wicksteed approach.

[35] Smith (1776) in Senior (1836), p. 159.

productive skills by way of learning and subsequently use their experience to become a specialist in their field.[36] Since skilled labor H can only be accumulated by repeatedly employing unskilled labor N, population growth can be rightfully viewed as the only source of human capital accumulation. Thirdly, "the invention of machines" refers to a regular tendency toward automation of specialized processes. Whenever production processes have been subdivided into such small steps that their repetition can be easily conducted through some non–human agency, capital K tends to be substituted for labor H and N. Accordingly, we may state that population, "the starting and ending point of all economic activity"[37], is really the source of all value and the most regular trigger of economic growth. "Hence the peopling process is essential"[38] and we shall start every inquiry on economic growth by examining the effects stemming from any foregoing population changes. As Young (1928) put it,

> *Senior's positive doctrine is well known, and there were others who made note of the circumstance that with the growth of population and of markets, new opportunities for the division of labour appear and new advantages attach to it. In this way, and in this way only, were the generally commonplace things which they said about 'improvements' [...].*[39]

Based on Smith's gains from labor division, we can derive additional static and dynamic interpretations of the neoclassical growth model. Since capital and human capital are in the same way frequently and proportionally accumulated after new productive individuals have entered the economy, we should reasonably assume that human capital is subject to the same law of accumulation as capital ($\delta_H = \delta_K \equiv \delta,\ s_H = s_K \equiv s$) and can be measured – like any other production factor – in the same complex unit. Thus, we can make use of a model, where population N is separated out of the inifinite number of production factors and where human capital and physical capital are aggregated into a complex production factor generalized capital, or *broad capital* C, as follows.[40]

[36] "Men are much more likely to discover easier and readier methods of attaining any object, when the whole attention of their minds is directed towards that single object." Smith (1776), book I, chapter I.

[37] Bairoch (1988), p. 127.

[38] Lange (2012), p. 21.

[39] Young (1928), p. 529.

[40] Intuitively, the argumentation follows Smith: "The improved dexterity of a workman may be considered in the same light as a machine or instrument of trade which facilitates and abridges labour, and which, though it costs a certain expense, repays that expense with a profit." Smith (1776), book I, chapter I.

$$Y = K^{\alpha} H^{\beta} N^{1-\alpha-\beta} \equiv C^{\gamma} N^{1-\gamma} \; with \; 0 < \gamma < 1 \tag{6}$$

$$(1 + g_N) \frac{C_{t+1}}{N_{t+1}} = s \frac{Y_t}{N_t} + (1 - \delta) \frac{C_t}{N_t} \tag{7}$$

With regard to the causal relation between production factors and in analogy to the old labor model of section 2, we assume that exogenous changes in population are the source of all value and that broad capital would in the long run react proportionally to population. Consequently, constant population growth would again cause constant capital growth, constant economic growth and constant production per capita. The corresponding stable steady state with $\frac{C_{t+1}}{N_{t+1}} = \frac{C_t}{N_t} = \frac{C}{N}$ is given by

$$\left(\frac{Y}{N}\right)^{*} = y^{*} = \left(\frac{s}{\delta + g_N}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{s}{\delta + b - d}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{s}{b}\right)^{\frac{\gamma}{1-\gamma}} \tag{8}$$

where we have defined the constant (natural) population growth rate as $g_N = b - d =$ *birth rate – death rate* and made the assumption that $\delta = d$, since the skills of a population as well as the (unskilled) population itself depreciate with the death rate ($\delta = \delta_H = \delta_N = d$).[41]

However, allowing for a varying birth rate, the productivity ratio of two subsequent steady states providing the following "unit–free" measurement of the growth factor would still depend on the birth rate:[42]

$$\frac{y_t^*}{y_{t-j}^*} = \left(\frac{b_{t-j}}{b_t}\right)^{\frac{\gamma}{1-\gamma}} \; with \; 0 < \gamma < 1. \tag{9}$$

From this formula it becomes apparent that the steady–state growth rate of productivity is determined neither by the level of population nor by population growth itself. Instead it is governed by the (inverted) *growth of population growth.* What is more, this intertemporal representation of productivity allows us to distinguish the essentially conflicting effects of population growth. While the numerator of the right hand side of equation (9) has a delayed positive effect on productivity growth, representing the gains from labor division, the denominator affects productivity immediately negatively, representing the losses from diminishing returns. Ultimately, the population question

[41] Obviously, if the death rate is high and population is written off, H is increasingly lost and K cannot be maintained and depreciates correspondingly. If the death rate is zero (and the birth rate positive), population grows infinitely together with human capital and physical capital. Depreciation rate as well as death rate normally lie within the range $[0.01, 0.04]$ (see also Mankiw et al. (1992), p. 410).

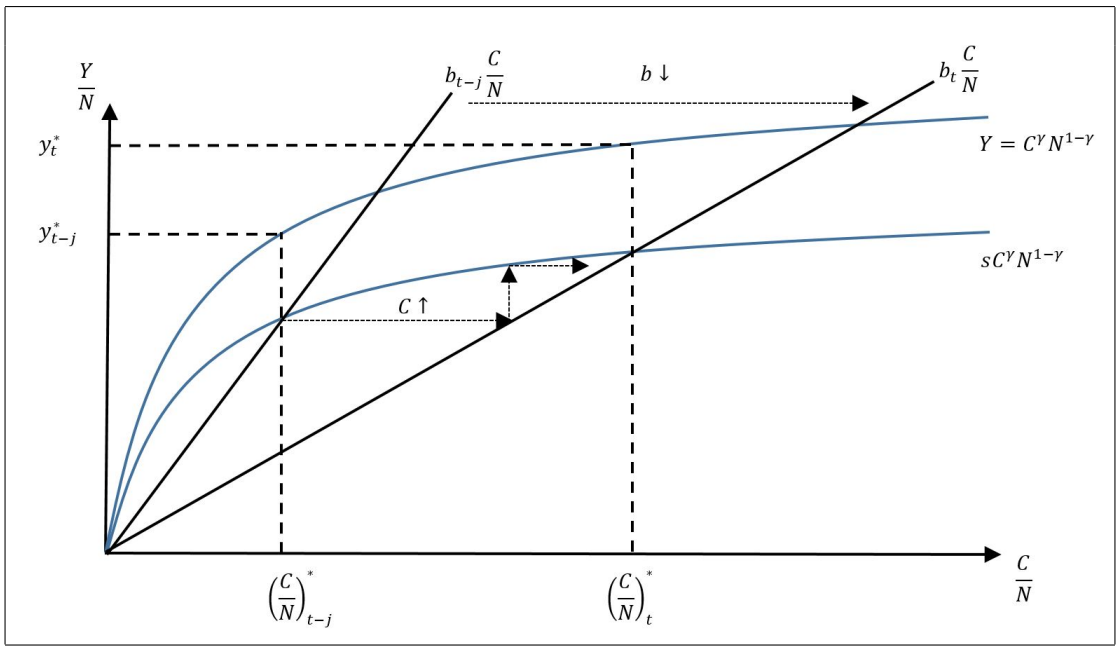
[42] The author is well aware of the vast literature on endogenized savings. However, this issue should be disentangled from attempting to answer the population question.

boils down to these two opposing forces describing the essential conflict by which we have to judge population growth economically.

> *The 'population' question [...] will then be found ultimately to turn on a balance between the significance to each man of other free men regarded as appliances and the significance to him of the space those other men occupy. Is their room [effects from diminishing returns] or their company [effects from labor division] the more important?* [43]

More explicitly, as is depicted in figure 3.1, a steadily decreasing birth rate is expected to yield continuous productivity growth, as in this case the right hand side of equation (9) will be larger than one.[44] Hence, this theory suggests that the causes for the prevalence

Figure 3.1: A population slowdown in the Solow model.



of sustainable development in any economy may be reduced to the beneficial effects from labor division outweighing the detrimental effects of diminishing returns as long as the birth rate is decreasing. Whenever, in contrast, the birth rate increases, the losses from diminishing returns tend to outweigh the gains from labor division and to create a situation of economic regress. This subject of overpopulation due to diminishing returns seems equally obvious as scientifically neglected or avoided.

[43] Wicksteed (1894), p. 22, footnote.

[44] A result that has been confirmed by a number of studies on economic development including Sachs and Malaney (2002).

4 Empirical Evaluation of the Steady–State Equation

4.1 Methodology

To support the above model, the subsequent empirical exercise will focus on the estimation of steady–state equation (9) and proceed as follows. To begin with, production per capita data, measured by the ratio of annual real GDP to population, and birth rate data, measured by the ratio of annual births to population, are readily available over a certain time span. To employ an appropriate $j-$value, we will in the following theoretically justify a presumed time span between steady states. To find evidence of a causal relationship, we will recover the remaining parameter γ by estimating the expression $\beta = \frac{\gamma}{1-\gamma}$. Based on the assumption of constant returns to scale and indicating a negative effect of a change in the birth rate on productivity, the estimated value of the parameter γ is expected to lie within the range $(0, 1)$. As a reference point, Cobb and Douglas (1928) estimated the production elasticity of capital of the labor model to be $\alpha \approx 0.25$. More recently, Mankiw et al.'s (1992) estimations suggested a parameter value $\alpha \approx 0.33$ and most conventional calibrations assume a production elasticity of capital within the range $(0.25, 0.33)$. Nonetheless, when withdrawing the variable human capital from the variable labor and adding it to the variable physical capital, Mankiw et al.'s estimator rises to $\alpha \equiv \gamma \approx 0.66$. Such an estimator would suggest a much higher exponent $\frac{\gamma}{1-\gamma}$ in equation (9) and correspondingly a larger leverage effect of changes in the birth rate on GDP per capita. In fact, if the following estimation confirms the conjecture that $\gamma \approx 0.66$, population growth may have a much larger impact on economic development than is usually suspected.

Ideally, once we have estimated a consistent parameter value, we can confirm or reject the time frame presumption for j. To estimate equation (9), we will employ the usual ordinary bivariate least squares (OLS) method. Due to the fact that the OLS estimator is the best *linear* unbiased estimator (BLUE), equation (9) will be linearized by taking logs of both ratios, yielding the approximate growth of GDP per capita as explained variable and the approximate growth of birth rate as explanatory variable (see equation (10)).[45] OLS estimation is in this case a valid approach, since the variable birth rate is viewed as the (independent) source of all value in GDP per capita.[46] This

[45] For an alternative methodology including a measurement of convergence see Mankiw et al. (1992).

[46] Employing a vector autoregression generates insignificant results due to the large number of required lags and, therefore, parameters.

also means that the additional use of an intercept to account for unobserved effects is not necessary.

$$ln\left(\frac{y_t^*}{y_{t-j}^*}\right) = \beta\ ln\left(\frac{b_t}{b_{t-j}}\right)\ with\ \beta = -\frac{\gamma}{1-\gamma} < 0. \tag{10}$$

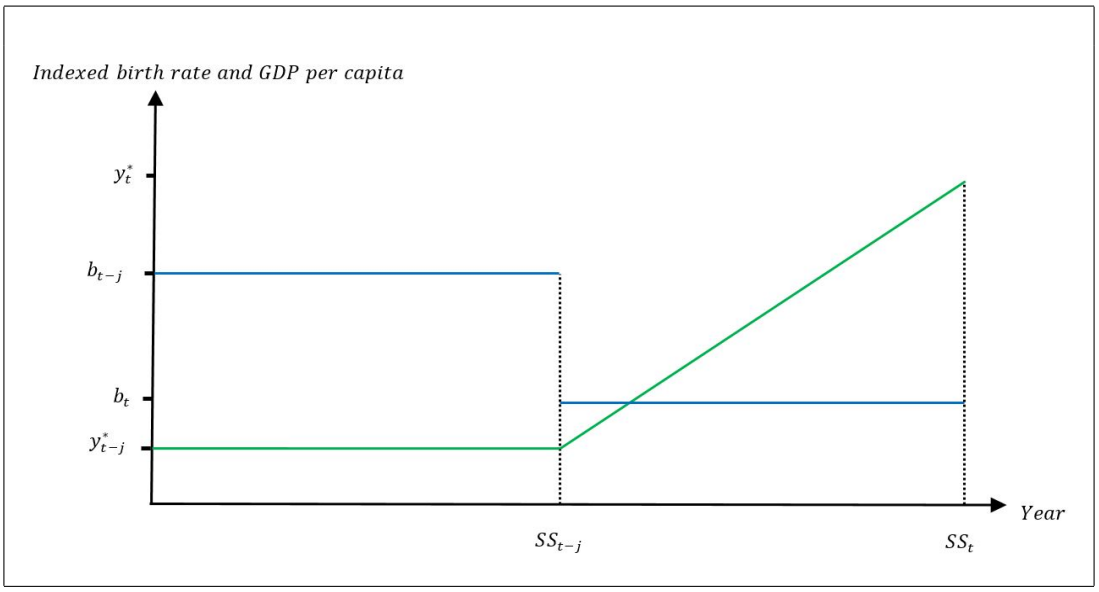
As can be seen, an ideal determination of β would only be realized under a comparison of two steady states y_t^*, y_{t-j}^*. Since the transition from one steady state to another might take a certain number of years after a shock in the variable birth rate, we have to account for this transitional period by using an appropriate value for j. Although the birth rate is expected to affect productivity immediately negatively through b_t, its positive effect of labor division is realized with a delay through b_{t-j}. To account for the latter effect, it has been stated that a newborn cohort will raise productivity only after it has generated the ability of unskilled labor with a maturity lag of one "generation" of ϕ years, and it seems plausible to assume a period of at least15 years. In addition, broad capital accumulation by way of "dexterity" and the "invention of machines" has been assumed to take place over the whole working life of a cohort, i.e. we add a maximum amount of $\psi \approx 50$ years.[47] Thus, since the above combined gains are probably fully achieved after $j = \phi + \psi$ years, we must assume a maximum accumulation period of $j \approx 65$ years to account for the transition between steady states. Consequently, ideal results from an OLS estimation can only be expected if we could employ time series of GDP per capita and birth rate over a time horizon of $2j$ years where the birth rate stays constant for the first j years, changing abruptly to another level in $j+1$ (treatment) and remaining constant on the new level for j years, as is exemplified in figure 4.1. After the birth rate has changed, GDP per capita is predicted to react over the latter period (treatment effect).

4.2 Estimation of γ and j

Firstly, to get an idea about the empirical relationship between birth rate and GDP per capita, available aggregate global data series provided by the World Bank are displayed in figure 7.1 in appendix 7.2. Here, we observe a relatively steady decline of the birth rate as well as a parallel rise in GDP per capita over the period 1960–2014. The corresponding calculation of the aggregate parameter yields $\beta^g = -2.0$ (see column (1)

[47] For a more extensive lag model, see e.g. Becker and Murphy (1992) or Liso et al. (2001).

Figure 4.1: Illustration of two time series required for an ideal estimation of equation (11).



of table 4.1) and we conclude that an average birth rate reduction by 1% is connected with an average rise of GDP per capita by about 2%.

Secondly, using country-level data,[48] we find that the birth rate decreased during the period 1960–2014 in each of the 104 available series.[49] Consequently, we would expect a rise in GDP per capita in every country at least until the year 2014, which we will thus use as reference year for the terminal steady state. To find evidence for the expected relationship, we plot the dependent variable (approximate productivity growth) against the independent variable (approximate growth of birth rate) over the complete 55–year period for all 104 countries with available data (see figure 7.3) and estimate the OLS coefficient of equation (11) (column (2) of table 4.1).

$$ln\left(\frac{y_{i,2014}}{y_{i,1960}}\right) = \beta\ ln\left(\frac{b_{i,2014}}{b_{i,1960}}\right)\ \ for\ i = country\ 1,\ ..,\ country\ 104 \tag{11}$$

The R–squared of 0.698 indicates that the greater part of the variation in GDP per capita is explained by variation in the birth rate. The magnitude of the coefficient is somewhat smaller than the aggregate coefficient, which is probably owed to the fact

[48] Since population is a relatively immobile factor of production, an estimation on the country level is expected to yield significant results.

[49] See table 7.1 in appendix 7.3 for a list of countries studied.

that the OLS approach does not weigh countries according to their population size.[50] Nonetheless, since the coefficient is highly significant, we have found some evidence of the true parameter β lying approximately within the range $[-2.02, -1.56]$ over the observed time span 1960–2014. However, since the above theory states that a change of the birth rate affects GDP per capita over the following $j \approx 65$ years, a calculation employing a time horizon of merely 55 years probably underestimates the magnitude of the aggregate coefficient, suggesting a somewhat higher true value.

Thirdly, in order to extend the maximum time span for j toward 65, we turn to the historical Mitchell (2013) database and estimate the corresponding coefficient for all 36 countries providing data on GDP per capita and birth rate over the period 1949–2014 (column (3) of table 4.1).[51] The greater magnitude of this "long-run" coefficient as well as the higher R-squared seem to confirm our expectation.

Table 4.1: Calculated and estimated coefficients.

	$ln\left(\frac{b_{i,2014}}{b_{i,t}}\right)$ (1)	$ln\left(\frac{b_{i,2014}}{b_{i,t}}\right)$ (2)	$ln\left(\frac{b_{i,2014}}{b_{i,t}}\right)$ (3)	$ln\left(\frac{b_{i,2014}}{b_{i,t}}\right)$ (4)
$ln\left(\frac{y_{i,2014}}{y_{i,t}}\right)$	-2.02	-1.56***	-2.19***	-2.16***
		(.100)	(.134)	(0.178)
R^2		0.698	0.789	0.94
t	1960	1960	1949	1901
$\# \ i$	1^a	104^a	36^b	10^b

Sources: a=World Bank (2018), b=Mitchell (2013)

*** indicates significance at 1% level. Standard errors are reported in parentheses.

To show that the true parameter j actually centers around 65 years, we will employ the World Bank as well as the Mitchell data series and display the evolution of the coefficient β for increasing j. As is shown in figure 4.2, the coefficient remains significant for all j–values. As expected, by increasing the transitional time span j, the coefficient tends to increase as well until settling at a value of approximately –2.0 after 60–65 years.

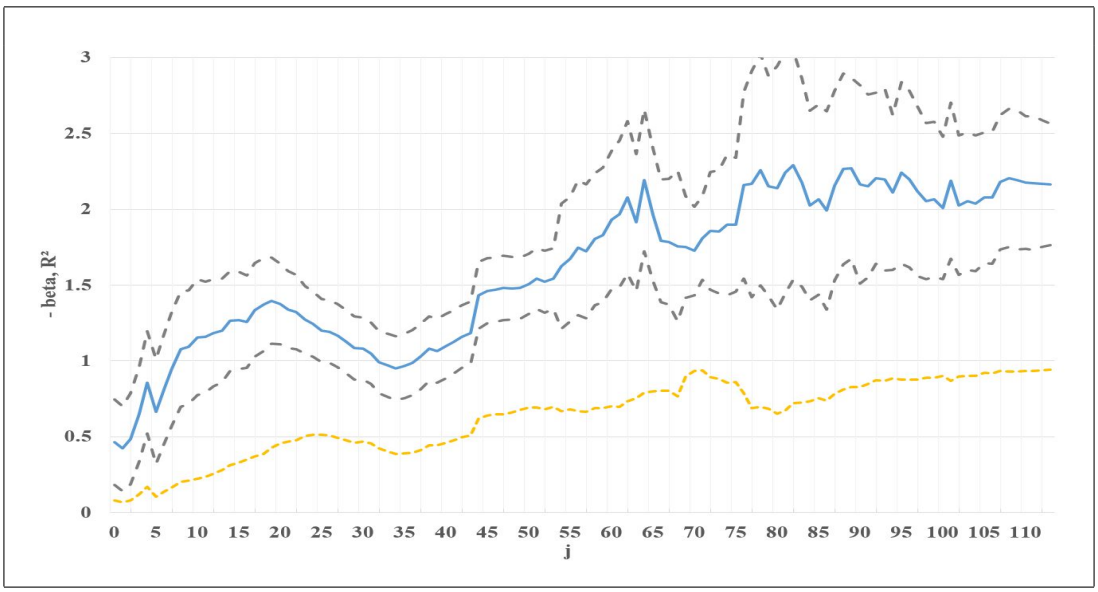
[50] If for example China and India were assigned a weight according to their population size, the value would be larger.

[51] 1945 has been left out for obvious reasons.

Thereafter, the coefficient remains relatively constant[52] at an average value of −2.0 and the 95% confidence interval roughly within the boundaries $[-3.0, -1.5]$ for $j > 60$.

On the one hand, these observations confirm the predicted strong impact of birth rate changes on GDP per capita. Since the displayed R–squared tends to steadily increas over time and displays a value of 0.94 for $j = 112$ (column (4) of table 4.1), it appears that changes in the birth rate are capable of explaining over 90% of the subsequent GDP per capita growth. On the other hand, they provide evidence of the idea that the full effect of changes in the birth rate is achieved after approximately 65 years.

Figure 4.2: Magnitude of OLS-estimator β (blue) with 95% confidence intervals (gray) and R^2 (yellow) with increasing time span j.



As a result, using the average coefficient value for $j > 60$ as a benchmark, the theory suggests that a 1% decline in the birth rate causes on average a 2% increase in GDP per capita over the subsequent 65–70 years.[53] These values imply that a birth rate reduction from 4% to 1% – as is observed in developed countries – raises production per capita by the factor 16.[54] Finally, the production elasticity of broad capital $\gamma = \frac{\beta}{1+\beta}$ can be

[52] Constancy is defined as a linear trend with slope parameter $< |0.002|$ over the corresponding time span.

[53] The possibility of reverse causality is dealt with in appendix 7.1.

[54] As an empirical comparison, we employ the (longest available) British time series during the period 1802–2014, where the birth rate decreases from a maximum average value of 4.1% between 1802

calculated to lie approximately within the 95% confidence interval $[0.6, 0.7]$, confirming Mankiw et al.'s (1992) estimation results.

5 Unified Growth Theory

The economic growth and development model put forward in this paper is for the most part a variation of the neoclassical model suggested by Mankiw et al. (1992). However, since this neoclassical model has been commonly accused of being incapable of explaining economic development endogenously,[55] the new branch "unified growth theory" has emerged more recently, trying to explain long–run economic development from a rather demographic perspective and at a very high level of abstraction. Unified growth theory was particularly advanced by Galor (2011) and builds on the following observed stylized facts of economic development.

1. Every economy was at some point over the past three centuries caught in a *regime of stagnation*, where productivity remained at a low level and birth rates at a high level.[56]

2. Today, almost all economies have left the regime of stagnation in favor of a *growth regime*, where productivity increases from a low level to a high level.

3. Roughly at the same time as these economies left the regime of stagnation, a *fertility transition* set in, by which the birth rate declined from a high level to a low level.[57]

Unified growth theory tries to make sense out of these stylized facts by initially stating that a high birth rate was causal for keeping productivity down during the regime of stagnation. Obviously, our neoclassical model fits perfectly into this framework, as it provides a sophisticated and well-established theory by which a decreasing birth rate was equally causal in allowing productivity to increase. This means that the mono-causal negative relationship between birth rate and productivity continues to operate

and 1826 to a minimum average value of 1.3% between 1974 and 2014 (see figure 7.2 in appendix 7.3). Since the historical GDP per capita data series by Clark (2009) has raised some debate, the Broadberry et al. (2015) data over the period 1802–1870 are analogously employed. The Clark data suggest an increase in GDP per capita by the factor 20 ($\beta^C = -2.54$), the Broadberry et al. data suggest an increase by the factor 26 ($\beta^B = -2.78$). Here, the Clark data appear to be more plausible as they are broadly in line with the implications of the World Bank data ($\beta^g = -2.02$) and the Mitchell data ($\beta^M = -2.12$).

[55] A view which is, as this paper has shown, probably not correct.

[56] See also Clark (2007).

[57] See Thompson (1930), who observed the fertility transition as part of the demographic transition.

over the whole time span under consideration and figure 4.1 provides the missing link between the second and the third stylized facts, stating that the fertility transition is rather a cause for and less a consequence of the escape from stagnation. Furthermore, this extended unified growth theory suggests a fourth stylized fact which remains to be evaluated, namely that

4. approximately 60 years after the fertility transition has been completed, an economy will enter a *new regime of stagnation*, where productivity stabilizes on a high level as long as birth rates remain on a low level.

Finally, a paper on the population question is bound to mention the Malthusian principle of population as a building stone of unified growth theory, according to which fertility is positively affected by economic development. For simplicity, this principle has been completely avoided in this paper by assuming exogenous population growth. Nonetheless, a unified growth theory should ideally include a theory of population that can explain changes in the birth rate endogenously. [58]Here, it is promising to return to Malthus' (1826) "preventive check", which he claimed to be capable of reducing the birth rate within manageable limits.[59]

6 Conclusion

In this brief exercise on the population question, the conventional Solow model has been modified by renewing classical assumptions. It has been shown that a Solow model where the production factor labor is replaced with population and which expressly excludes exogenous "technology" can account for the largest part of the historically experienced sustainable rise in production per capita. Instead of modeling "technology" endogenously, the following assumptions have been used to account for innovation. Firstly, the Cobb–Douglas production function is theoretically based on Smith's assumption of an efficient division of labor. Secondly, the accumulation of production factors is theoretically based on Smith's gains from the division of labor, which are made up of specialization across space and time as well as automation of repetitive laboring processes. This means that every progress that is commonly very

[58] The principle of population can be easily modeled by using a law of population accumulation of the form $N_{t+1} = bY_t - dN_t + N_t$.

[59] "[T]he preventive check is perhaps best measured by the smallness of the proportion of yearly births to the whole population." Malthus (1826), book II, ch.XI.

loosely termed "measure of our ignorance" is included in the production factor "broad capital."

With regard to the population question, the size of population has been found to yield constant long-run returns and therefore to have a positive effect on economic growth (measured by GDP) and a neutral effect on economic development (measured by GDP per capita). Although constant returns contradict permanent development, they do not contradict sustainable development. The corresponding steady-state equation states that, if positive population growth is reduced from a high level to a lower level, population pressure from diminishing returns relaxes while the formerly established division of labor derived from a larger population continues to have a positive effect. Accordingly, this work suggests that a population growth rate change (in this work measured in terms of birth rate) is the best predictor for economic development. To advance economic theory, the above conclusions recommend a further move away from exogenous and endogenous growth theories toward unified growth theories to explain long–run economic development in one framework with the demographic transition.

Evaluating annual data on 104 countries over a period of 55 years and 24 countries over a period of 114 years, our estimators imply that a birth rate reduction from 4% to 1% – as it is observed in developed countries – raises production per capita by the factor 16. If these results are correct, the historically observed decline in fertility can be suspected of being the cause of most of the observed economic development. Moreover, our estimations suggest that the production elasticity of broad capital lies in the range $(0.60, 0.75)$, suggesting a production elasticity of population in the range $(0.25, 0.40)$ which has often erroneously been calculated to lie in the interval $(0.66, 0.75)$. Although employing a quite different approach, the results are roughly in line with those of Mankiw et al. (1992) and Ashraf et al. (2013), providing supportive evidence of the neoclassical growth model. Further research will be required to confirm the idea that an appropriate form of the aggregate production function might be approximated by $Y = K^{1/3}H^{1/3}N^{1/3}$.

While this paper provides a relatively simple approach on productivity, adhering to Cobb and Douglas' (1928) "method of attack," the model should be extended by accounting for a fourth constant production factor – land – that is not subject to accumulation and depreciation. This will probably imply that even the density of the population is relevant in determining development and that population growth exhibits diminishing returns in the long run.

Under the assumption of perfect competition, a calculation of γ may alternatively be conducted by using the production exhaustion theorem to determine the income share of population. To this end, we would have to employ average unskilled labor wages or minimum wages with regard to the whole population (which may in fact be termed "geographical wages" or "population wages") and to compute their income share $(1-\gamma)$ on total GDP. This share should be found to lie in the interval $[0.25, 0.40]$.

Finally, since physical and human capital accumulation of a country are sometimes strongly encouraged by foreign investments, future research on the topic may allow for a varying national savings rate. These external adjustments toward an efficient international division of labor may account for the remaining unobserved variation in our regressions.

References

[1] Abramovitz, M.: 1956, "Resource and Output Trends in the United States since 1870". *American Economic Review*. 46, 5–23.

[2] Ashraf, Q., Weil, M., Wilde, J.: 2013 "The effect of fertility reduction on economic growth". *Pop Dev Rev*. 39, 97–130.

[3] Bairoch, P.: 1988, *Cities and Economic Development: From the Dawn of History to the Present*. Chicago: University of Chicago Press.

[4] Becker, G. S.: 1988, "Family Economics and Macro Behavior". *American Economic Review*. 78, 1–13.

[5] Becker, G. S.: 1991, *A Treatise on the Family*, Harvard University Press, Cambridge.

[6] Becker, G. S., Murphy, K. M.: 1992, "The Division of Labor, Coordination Costs, and Knowledge". *The Quarterly Journal of Economics*. 107(4), 1137–1160.

[7] Bloom, D. E., Canning, D. and Sevilla, J.: 2003, *The Demographic Dividend – A New Perspective on the Economic Consequences of Population Change*. RAND, Pittsburgh.

[8] Broadberry, S., Campbell, B. M. S., Klein, A., Overton, M., Van Leeuwen, B.: 2015, *British Economic Growth 1270-1870*. Cambridge: Cambridge Univ. Press.

[9] Chatterjee, S. and Vogl, T.: 2018, "Escaping Malthus: Economic growth and fertility change in the developing world". *American Economic Review*, 108(6), 1440–1467.

[10] Clark, J.B.: 1899, *The Distribution of Wealth*. New York.

[11] Clark, G.: 2007, *A farewell to alms: a brief economic history of the world*. Princeton: Princeton Univ. Press.

[12] Clark, G.: 2009, *The macroeconomic aggregates for England, 1209-2008*. Davis, Calif.: Dep. of Economics, Univ. of California.

[13] Cobb, C. W. and P. H. Douglas: 1928, "A Theory of Production". *American Economic Review*. 18(Supplement), 139–165.

[14] Domar, E.: 1946, "Capital expansion, rate of growth, and employment". *Econometrica.* 14, 137–147.

[15] Ehrlich, P. R.: 1968, *The population bomb: Population control or race to oblivion?* New York (USA), Sierra Club-Ballantine.

[16] Galor, O.: 2011, *Unified Growth Theory.* Princeton: Princeton University Press.

[17] Hardin, G.: 1968, "The Tragedy of the Commons". *Science.* 162(3859), 1243–1248.

[18] Harrod, R.F.: 1939, "An essay in dynamic theory". *Economic Journal.* 49, 14–33.

[19] Hicks, J. R.: 1936, "Distribution and Economic Progress: A Revised Version". *The Review of Economic Studies.* 4(1), 1–12.

[20] Humphrey, T. M.: 1997, "Algebraic Production Functions and Their Uses Before Cobb-Douglas". *Federal Reserve Bank of Richmond Economic Quarterly*, 83(1), 51–83.

[21] Keynes, J. M.: 1936, *The general theory of employment, interest and money.* London: Macmillan.

[22] Kremer, M.: 1993, "Population-Growth and Technological-Change - One Million BC to 1990". *Quarterly Journal of Economics* 108(3), 681–716.

[23] Lange, J.: 2012, "Allyn Young's Inquiry into the Nature and Cause of the Growth of Markets". *Working Paper Université Paris Descartes*, first draft.

[24] De Liso, N., Filatrella, G., Weaver, N.: 2001, "On endogenous growth and increasing returns: Modeling learning-by-doing and the division of labor". *Journal of Economic Behavior and Organization.* 46(1), 39-55.

[25] Lucas, R. E.: 1988, "On the Mechanics of Economic-Development". *Journal of Monetary Economics.* 22(1), 3–42.

[26] Malthus, T. R.: 1826, *An essay on the principle of population or, A view of its past and present effects on human happiness.* London: J. Murray, 6th edition.

[27] Mankiw, N., Romer, D. and Weil, D.: 1992, "A Contribution to the Empirics of Economic Growth". *Quarterly Journal of Economics.* 107, 407–37.

[28] Marx, K.: 1867, Das Kapital, Erster Band. Hamburg. Verlag von Otto Meissner.

[29] McCulloch, J. R.: 1863, *A treatise on the principles and practical influence of taxation and the funding system.* Edinburgh: A. and C. Black.

[30] Mitchell, B.: 2010, *International Historical Statistics, 1750–2010.*

[31] Robinson, J.: 1954, "The Production Function and the Theory of Capital". *Review of Economic Studies.* 2(1), 2.

[32] Romer, P.: 1986, "Increasing Returns and Long-Run Growth". *Journal of Political Economy.* 94(5).

[33] Sachs, J., Malaney, P.: 2002, "The economic and social burden of malaria". *Nature.* 415, 680–685.

[34] Senior, N. W.: 1836, *An outline of the science of political economy.* London: Clowes.

[35] Smith, A.: 1776, *An inquiry into the nature and causes of the wealth of nations.* Dublin: Printed for Messrs. Whitestone and 19 others.

[36] Solow, R. M.: 1956, "A Contribution to the Theory of Economic Growth". *Quarterly Journal of Economics.* 70, 65–94.

[37] Solow, R. M.: 1957, "Technical Change and the Aggregate Production Function". *Review of Economics and Statistics.* 39, 312–20.

[38] Swan, T. W.: 1956, "Economic Growth and Capital Accumulation". *Economic Record.* 32(2)

[39] Thompson, W. S.: 1930, *Danger Spots in World Population.* New York: Alfred A. Knopf.

[40] Wicksteed, P. H.: 1894, *An Essay on the Co-ordination of the Laws of Distribution.* 1932 edition, Reprint No. 12, London: London School of Economics.

[41] The World Bank: 2018, "World Development Indicators". URL: http://data.worldbank.org/data-catalog/world-development-indicators

[42] Wrigley. E. A. and R. S. Schofield: 1981, *The Population History of England, 1541-1871: A Reconstruction.* London: Edward Arnold.

[43] Young, A. A.: 1928, "Increasing Returns and Economic Progress". *The Economic Journal*. 38(152).

7 Appendix

7.1 Reverse Causality: A Population Objection

The observed correlation between birth rate and GDP per capita has prompted large academic circles to believe that rising productivity generally induces individuals to lower their fertility, since running a regression for the (inverted) equation

$$ln\left(\frac{b_t}{b_{t-j}}\right) = \alpha\ ln\left(\frac{y_t}{y_{t-j}}\right) \tag{12}$$

would naturally yield an inverted significant coefficient $\alpha = \frac{1}{\beta}$.[60] However, this hypothesis must be rejected for many reasons of which we may state merely two. Firstly, due to a fertility decision lag and a pregnancy lag, a contemporaneous effect of y_t on b_t can barely exist.

Secondly, since we stated that birth rate has a delayed effect on GDP per capita, we may also test for a delayed effect (l) of GDP per capita on birth rate of the form

$$ln\left(\frac{b_t}{b_{t-j}}\right) = \alpha\ ln\left(\frac{y_{t-l}}{y_{t-l-j}}\right). \tag{13}$$

However, although in developed countries GDP per capita has steadily increased over the 20th century, we observe – beginning in the 1970s – a constant birth rate in those countries.[61] If GDP per capita would indeed have a negative impact on birth rate, we should instead observe a further declining birth rate after 1970, which is not the case.[62]

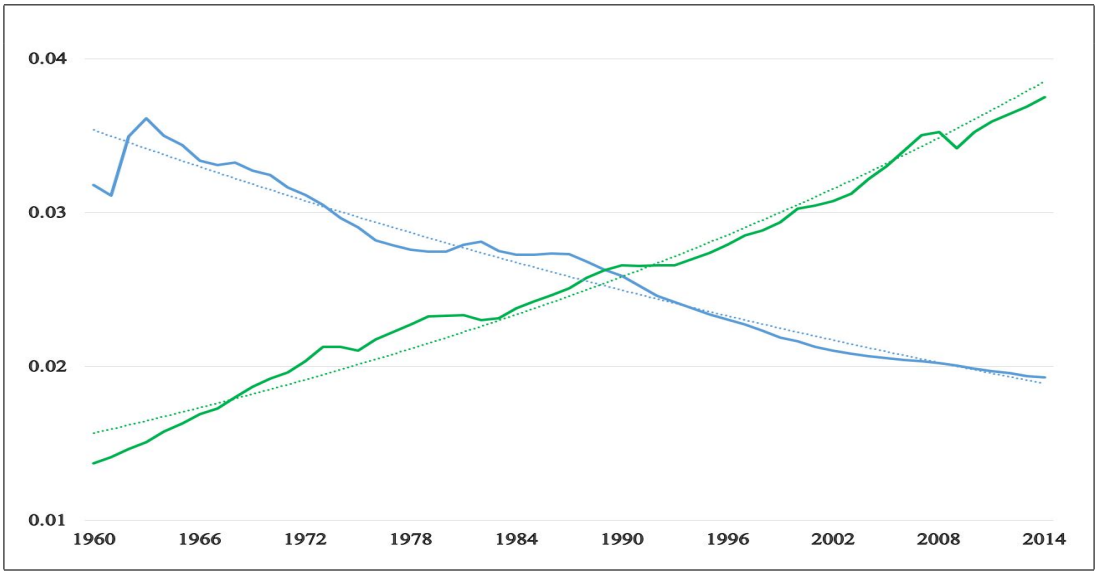
[60] See for example Becker and Lewis (1973). This currently quite popular idea is widely known as the "demographic economic paradox".

[61] See footnote 49 for our empirical definition of constancy. The longest series displaying a constant birth rate is given by the UK-data (40 years).

[62] Further reading on the rejection of the Becker–Hypothesis is provided by Galor (2011).

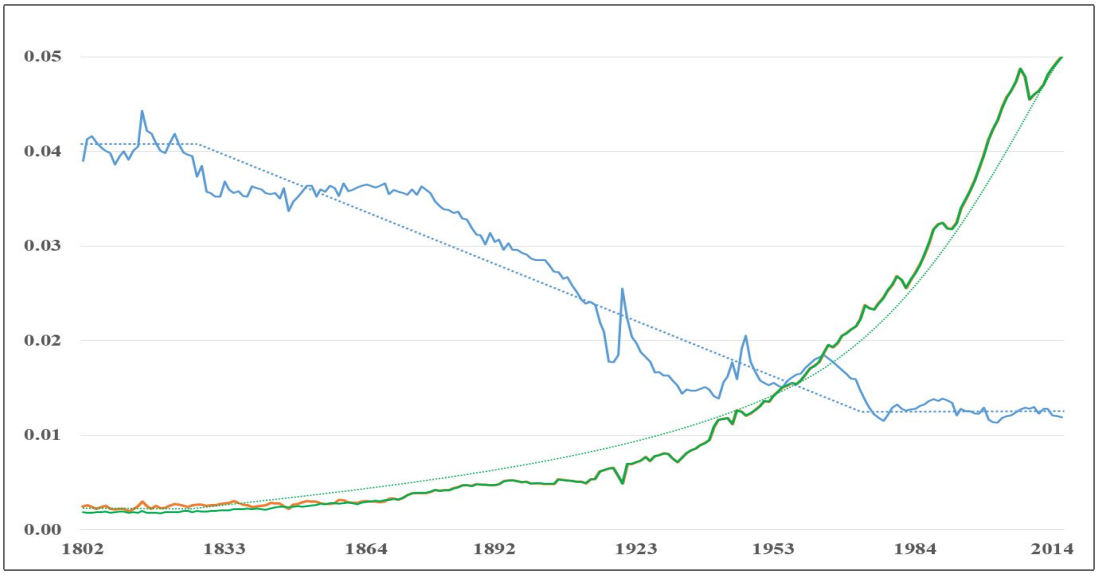
7.2 Historical Development of Birth Rate and GDP per Capita

Figure 7.1: Birth rate (blue) and indexed GDP per capita (green), aggregate global data, 1960–2014.



Source: World Bank (2018).

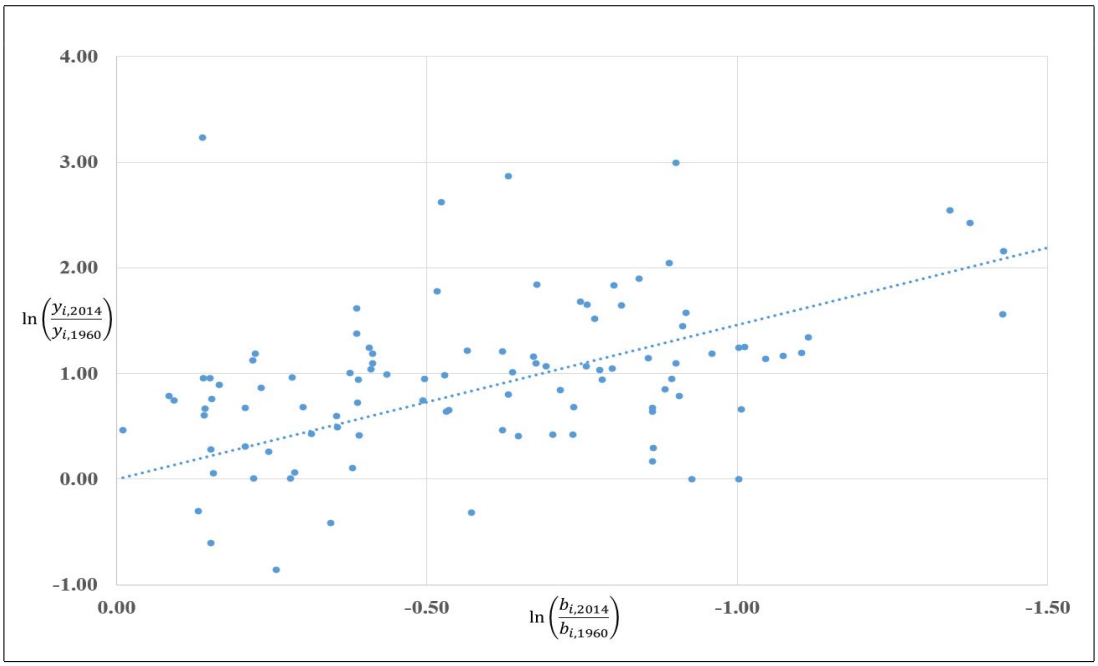
Figure 7.2: Birth rate (blue) and indexed GDP per capita with exponential trend (green, Clark data orange), Britain, 1802–2014.



Source: Wrigley and Schofield (1981), Broadberry et al. (2015), Clark (2009), Mitchell (2010), World Bank (2018)

7.3 Cross-Sectional Relationship between (Negative) Growth of Birth Rate and Growth of GDP per Capita

Figure 7.3: Scatterplot of 104 countries comparing (negative) growth of birth rates (x–axis) and growth of GDP per capita (y–axis) between 1960 and 2014.



Source: World Bank (2018).

7.4 List of Countries

Table 7.1: List of 104 countries studied. 36 countries with long-run data are starred.

Algeria	Germany*	Nicaragua
Argentina*	Ghana	Niger
Australia*	Greece*	Nigeria
Austria*	Guatemala	Norway*
Bangladesh	Guinea	Pakistan
Belgium*	Guinea–Bissau	Panama*
Benin	Haiti	Paraguay
Bolivia	Honduras*	Peru
Botswana	Hong Kong	Philippines*
Brazil	India*	Portugal*
Burkina Faso	Indonesia	Romania
Burundi	Iran, Islamic Rep.	Rwanda
Cameroon	Ireland*	Senegal
Canada*	Israel*	Sierra Leone
Central African Republic	Italy*	Singapore
Chad	Jamaica*	South Africa
Chile*	Japan*	Spain*
China	Jordan	Sri Lanka*
Colombia*	Kenya	Sweden*
Congo, Dem. Rep.	Korea, Rep.	Switzerland*
Congo, Rep.	Lesotho	Syrian Arab Republic
Costa Rica	Madagascar	Tanzania
Cote d'Ivoire	Malawi	Thailand
Cyprus*	Malaysia	Togo
Denmark*	Mali	Trinidad and Tobago
Dominican Republic	Mauritania	Tunisia
Ecuador	Mauritius	Turkey
Egypt, Arab Rep.*	Mexico*	Uganda
El Salvador*	Morocco	United Kingdom*
Equatorial Guinea	Mozambique	United States
Ethiopia	Myanmar	Uruguay*
Finland*	Namibia	Venezuela, RB*
France	Nepal	Zambia
Gabon	Netherlands*	Zimbabwe
Gambia, The	New Zealand*	(Cuba)*

Arbeitspapiere der Volkswirtschaftlichen Fachgebiete der TU Darmstadt

ISSN: 1438-2733