Applied Mechanics and Materials
ISSN: 1662-7482, Vol. 885, pp 276-289
doi:10.4028/www.scientific.net/AMM.885.276

Submitted: 2018-04-30
Revised: 2018-07-26
Accepted: 2018-08-06
Online: 2018-11-20

Cross-Domain Tolerance Analysis for Directional Control Valves Based on Imperfect Information

TAUTENHAHN, Ralf[1,a], WEBER, Jürgen[1,b]

[1]Chair of fluid-mechatronic systems, Technische Universität Dresden, Dresden, Germany
[a]ralf.tautenhahn@tu-dresden.de, [b]fluidtronik@mailbox.tu-dresden.de

Keywords:manufacturing tolerances, analysis, multi-physics, uncertainty, imprecision

Abstract. The task of tolerance analysis usually addresses the question of the mechanical mountability of an assembly. When talking about directional control valves this viewpoint has to be extended to a cross-domain tolerance analysis; an analysis whose task is to determine the possible variation in the key product characteristics induced by a specific tolerance concept. As the available information about the noise factors to be toleranced is almost always imperfect, generalised methods for their representation and the propagation of their impact on the key product characteristics are required. In this study the capabilities and potentials of belief and plausibility measures as well as fuzzy random variables are compared to traditional worst-case and statistical tolerance analysis.

Introduction

Directional control valves are mechatronic systems whose working principle places high demands on the manufacturing processes. This is not only to ensure the mechanical mountability of the whole assembly but is also a matter of the required accuracy for the key product characteristics such as leakage flow, flow gain or response dynamics. To determine the scope of this analysis the parameter diagram, first introduced by the Japanese quality engineer Genichi Taguchi, can help to distinguish between input, output, control and noise factors like shown in figure 1. The outputs of the investigated product vary more or less due to the existence of noise factors but have to match the specified functional limits in order to gain customer satisfaction. Therefore, tolerances need to be established for all those noise factors that occur during production, e.g. variations of geometric features or material properties, taking into consideration the noise factors arising from the environment, e.g. temperature.

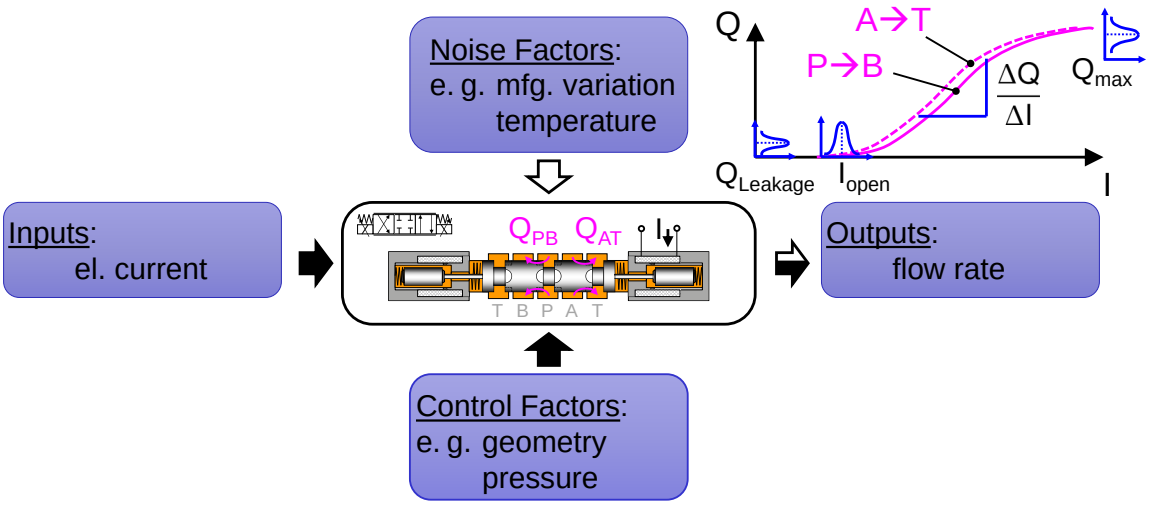


Fig. 1: System context for tolerancing illustrated by a parameter diagram following Taguchi's quality engineering method

During product development the resulting variations of the outputs are usually not sufficiently determinable. This is because the information available about the prospective noise factors is almost

always imperfect, for example due to the existence of natural variability and incomplete knowledge of the system, its environment, or the manufacturing processes.

According to [16] perfect information is characterized by precision, certainty and consistency. While precision and consistency are characteristics of the information itself, certainty expresses an opinion on the relationship between information and reality. Imprecision exists when several characteristic attributes meet the information. In the case of uncertain information, it can not be decided for sure whether the statement contained in the information corresponds to a characteristic attribute of reality. Inconsistency occurs when reality can not match the information. So both tolerance analysis and tolerance synthesis require the treatment of imperfect information, with the consistency of the information is considered as a matter of fact in this context.

Handling of Imperfect Information

While imprecision can be modelled for example within the set theory like it is done in traditional worst case tolerance analyses, uncertainty is typically expressed by measures like probability, possibility or plausibility. Probability approaches, e.g. statistical tolerance analysis, are most often used in literature as they allow wider tolerance ranges for the individual features to meet the acceptable tolerance ranges of the key product characteristics. This is due to the fact that they do not overestimate the coincidence of extreme values. On the other hand, the knowledge of the actual probability distribution functions is often quite limited; something that calls the trustworthiness of the results in question. During early design phases hardly anything is known about the prospective probability distribution functions. Furthermore, the available information about the noise factors may change during product development. In this case neither the worst case nor the probabilistic approach allow for a realistic evaluation of a valve's operational capability resulting from a chosen tolerance concept.

The latter requires a mathematical modelling for the noise factors, which can deal with both imprecise and uncertain as well as simultaneously imprecise and uncertain information. In [6] two directions for generalization of probability theory are presented within the so called Generalized Information Theory that fulfil this requirement. The first direction leads within the field of measure theory to a whole series of so-called imprecise probabilities by utilising further monotonic measures. A generalization in the field of set theory represents the second direction of generalization and leads, for example by the consideration of fuzzy numbers as events of a random experiment, to fuzzy probabilities.

These generalized model approaches can be seen on the one hand as complementary mathematical tools that are more or less capable to represent the existing information depending on the application [3]. On the other hand, [17] describes an inclusive view of the different forms of imprecise probabilities that can be derived from a most generalized form.

[2 ,3] claim that the readability of the results decreases with more generalization, and at the same time the required numerical effort increases disproportionately, especially for higher-dimensional problems. As a result, two of the simplest possible variants in each of the mentioned directions of generalization are discussed below.

Evidence Theory (a.k.a. Dampster-Shafer-Theory). The following section only sums up the basic ideas of evidence theory. A more detailed introduction can be found for example in [1, 10, 11]. This mathematical framework utilizes a measure defined by eq. (1), the so-called Basic Probability Assignment (BPA) m_X to express the evidences that certain elements from a subset U of the universal set X (representing all possible states of the system under consideration) may occur. The subsets U_j with $m_X \;\; U_j > 0$ are called focal elements.

The peculiarity of this representation is that all available information regarding the occurrence of realizations is distributed on the power set 2^X without any further assignment to the elements of a subset $U \subseteq X$ would be possible.

$$m_X : 2^X = \{U \mid U \subseteq X\} \mapsto [0;1]\,, \sum_{j=1}^{n_U} m_X\left(U_j\right) = 1, m_X(\emptyset) = 0 \tag{1}$$

As shown in figure 2a the definition of m_X allows specification of evidences on overlapping subsets, which can be used to formalize linguistically imprecise expressions (e.g. small, medium, large values). In addition, there are a number of computational rules that can be used to combine the evidences of different sources [12].

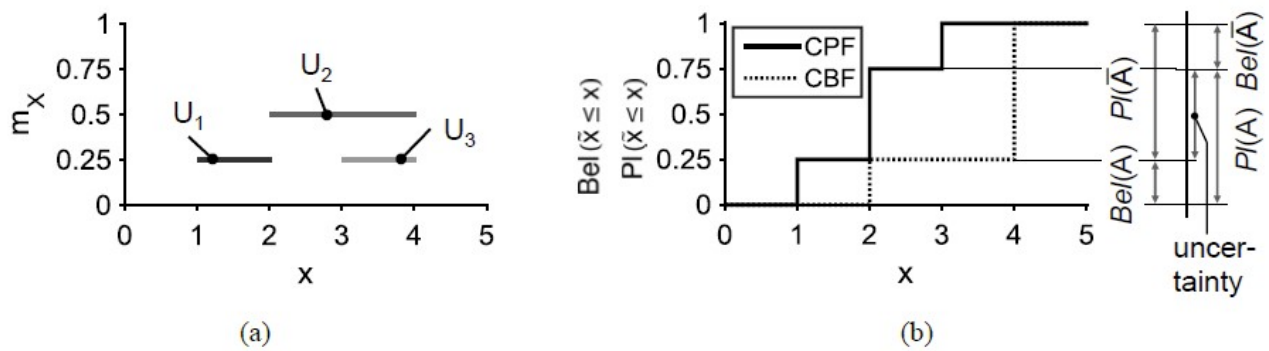


Fig. 2: Example illustrating the definitions of a) the Basic Probability Assignment (BPA) m_X; b) the associated cumulative belief function CBF and cumulative plausibility function CPF with the relation between Bel and Pl describing the uncertainty

Due to the uncertainty, only a lower and a upper bound can be defined for the conviction of an event A to occur:

- *belief*: sum of all evidences that argue in favour of the event A

$$Bel : 2^X \mapsto [0;1]\,, Bel(A) = \sum_{U_j \subseteq A} m_X(U_j) \tag{2}$$

- *plausibility*: sum of all evidences that do not argue against the event A

$$Pl : 2^X \mapsto [0;1]\,, Pl(A) = \sum_{U_j \cap A \neq \emptyset} m_X(U_j) \tag{3}$$

For both Bel and Pl, the following dependencies exist with respect to the event A and the complementary event $\bar{A}$:

$$Bel\,(A) + Bel(\bar{A}) \neq 1 \tag{4}$$
$$Pl\,(A) + Bel(\bar{A}) = 1 \tag{5}$$

By analogy with the cumulative distribution function in probability theory a cumulative belief function CBF and a cumulative plausibility function CPF are defined to characterise the evidence space (compare figure 2):

$$CBF = \{[x, Bel\,(v_x)] : x \in \Omega\}\,, \qquad v_x = \{\tilde{x} : \tilde{x} \in \Omega \text{ and } \tilde{x} \leq x\} \tag{6}$$
$$CPF = \{[x, Pl\,(v_x)] : x \in \Omega\}\,, \qquad v_x = \{\tilde{x} : \tilde{x} \in \Omega \text{ and } \tilde{x} \leq x\} \tag{7}$$

Due to the uncertainties in $\mathbf{x}$, the functional relationship $y = f(\mathbf{x})$ results in an uncertainty in y. The determination of the focal elements of y is difficult with real technical questions because the number of focal elements for $\mathbf{x}$ increases by $\prod_{i=1}^{n_X} n_{U,i}$ very fast with the number of input variables n_X and the number of focal elements $n_{U,i}$ per input variable x_i. For each of these elements, the characteristic functions $\chi_{U_Y}(y)$ which define the focal elements of y must then be determined by interval arithmetic, numerical optimization, or Monte-Carlo Simulation. The associated high number of model evaluations usually leads to an impractical computational effort. Therefore, [5] proposes a sample-based determination of the cumulative distribution functions for the measures Bel_Y and Pl_Y. In addition, to further reduce the computational effort, it's recommended to use meta-models and to progressively include only evidences for those input variables x_i, against which y has the highest sensitivities. The main advantage of this calculation approach is that you only have to know the focal elements of the input variable $\mathbf{x}$ and must assign the elements of a random sample and the corresponding outputs y to the same.

Fuzzy random variables. Normalized fuzzy sets are defined by a continuous membership function $\forall x \in X : \mu_A(x) \in [0;1]$ in contrast to the characteristic function $\forall x \in X : \chi_A(x) \in \{0;1\}$ in classical set theory. This allows to express a gradual assignment of an element of X to a set A.

In recent decades, several definitions for fuzzy random variables have been coined by various researchers [13]. In the following, the representation of [7, 14] will be taken up, because it's one of the most comprehensive views and its applicability in the area of engineering sciences has been proven. Fuzzy random variables are considered as an extension of the traditional probability space (Ω, Σ, p). They are the result of the mapping from Ω to the set $\mathbb{F}(\mathbb{R})$ of all the fuzzy quantities on $\mathbb{R}$. In contrast to a classical, crisp random variable $\mathcal{X}$, the realizations of a fuzzy random variable $\tilde{\mathcal{X}}(\omega)$ are not singletons $x(\omega)$ but fuzzy quantities $\tilde{x}(\omega)$ for each elementary event $\omega \in \Omega$. This concept is depicted in figure 3a and includes the representation of the traditional probability space as a special case.

For numerical treatment a convex fuzzy quantity can be discretized according to different levels α of the membership function $\mu_A(x)$. In this case crisp subsets of elements with a value of the membership function $\mu_A(x)$ greater than the specified α-level are treated progressively. These crisp subsets have random boundaries $\boldsymbol{\mathcal{X}}_{\boldsymbol{\alpha}} = [\mathcal{X}_{\alpha,l}; \mathcal{X}_{\alpha,r}]$.

The occurrence of $\tilde{\mathcal{X}}(\omega) = \tilde{x} \in A$ for any crisp subset $A \in \mathbb{R}$ can be considered an event of a random experiment. Since the realizations $\tilde{x}$ are fuzzy quantities a partial overlapping of the realization $\tilde{x}$ with the set A may occur in addition to the two crisp defined cases: the realization $\tilde{x}$ is contained completely or not at all in the set A. As a result, not only one probability P exists for the considered event, but several $\mathbf{P}_\alpha$.

$$\tilde{P}(A) = \{(\mathbf{P}_\alpha(A); \mu(\mathbf{P}_\alpha(A))) \mid \mu(P_\alpha(A)) = \alpha; \alpha \in [0;1]\} \tag{8}$$

$$\mathbf{P}_\alpha(A) = [P_{\alpha,l}(A); P_{\alpha,r}(A)] \mid \alpha \in [0;1] \tag{9}$$

$$P_{\alpha,l}(A) = P(\boldsymbol{\mathcal{X}}_\alpha \subseteq A) \tag{10}$$

$$P_{\alpha,r}(A) = P(\boldsymbol{\mathcal{X}}_\alpha \cap A \neq \emptyset) \tag{11}$$

The fuzzy probability distribution function $\tilde{F}_{\tilde{\mathcal{X}}}(x)$ of a fuzzy random variable $\tilde{\mathcal{X}}$ is defined analogously to the traditional probability distribution function $F_{\mathcal{X}}(x)$ of a random variable $\mathcal{X}$.

$$\tilde{F}_{\tilde{\mathcal{X}}}(x) = \tilde{P}(A_i) \mid A_i = \{t \mid t < x; x, t \in \mathbb{R}\} \tag{12}$$

The boundaries of this fuzzy probability $\tilde{P}(A_i)$ can be deduced from the α-level sets $\boldsymbol{\mathcal{X}}_\alpha$ as shown in figure 3b and expressed by eqs. (15) and (16).

$$\tilde{F}_{\tilde{\mathcal{X}}}(x) = \{(\mathbf{F}_\alpha(x); \mu(\mathbf{F}_\alpha(x))) \mid \mu(F_\alpha(x)) = \alpha; \alpha \in [0;1]\} \tag{13}$$
$$\mathbf{F}_\alpha(x) = [F_{\alpha,l}(x); F_{\alpha,r}(x)] \mid \alpha \in [0;1] \tag{14}$$
$$F_{\alpha,l}(x) = P(\boldsymbol{\mathcal{X}}_\alpha \subseteq A_i) = P(\mathcal{X}_{\alpha,r} < x \mid x \in \mathbb{R}) \tag{15}$$
$$F_{\alpha,r}(x) = P(\boldsymbol{\mathcal{X}}_\alpha \cap A_i \neq \emptyset) = P(\mathcal{X}_{\alpha,l} < x \mid x \in \mathbb{R}) \tag{16}$$

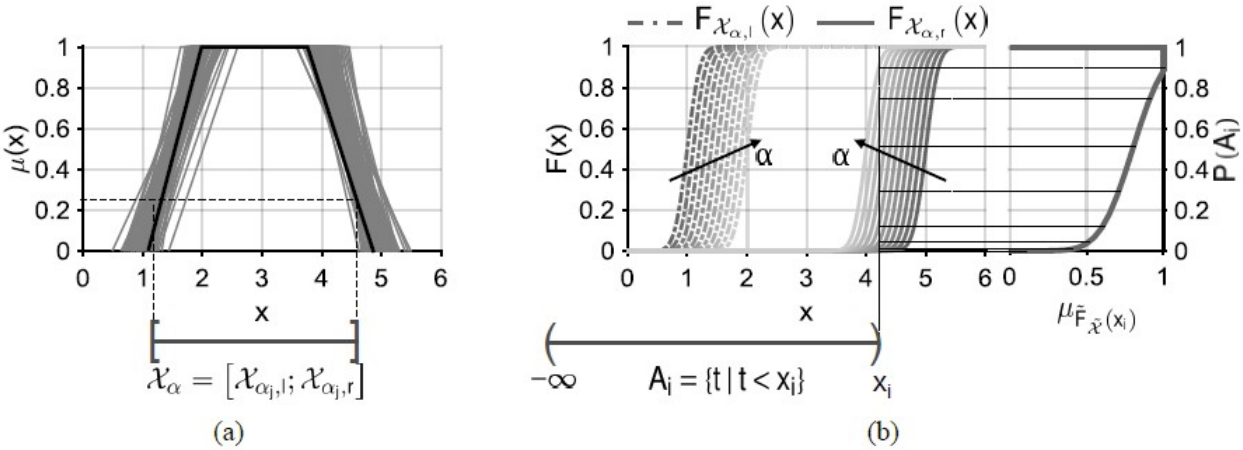


Fig. 3: Example illustrating the definitions of a) fuzzy random variables and b) fuzzy probability distributions function

Furthermore, with eq. (17) a fuzzy random variable $\tilde{\mathcal{X}}$ can be interpreted as the fuzzy set of all possible originals $\mathcal{X}_j$. An original $\mathcal{X}_j$ is hereby defined as a random variable $\mathcal{X}$ that is completely contained in a fuzzy random variable $\tilde{\mathcal{X}}$.

$$\tilde{\mathcal{X}} = \{\mathcal{X}_j = \mathcal{X}(\mathrm{s}_j) \mid \mu(\mathcal{X}_j) = \mu(\mathrm{s}_j) \forall \mathrm{s}_j \in \tilde{\mathrm{s}}\} \tag{17}$$
$$\tilde{\mathcal{X}} = \mathcal{X}(\tilde{\mathrm{s}}) \tag{18}$$

With the definition of a fuzzy bunch parameter vector $\tilde{\mathrm{s}} = \{(\mathrm{s}_j, \mu(\mathrm{s}_j))\}$ any original $\mathcal{X}_j = \mathcal{X}(\mathrm{s}_j)$ of a fuzzy random variable $\tilde{\mathcal{X}}$ can be uniquely identified by a specific bunch parameter $\mathrm{s}_j \in \tilde{\mathrm{s}}$. As a result, the fuzzy set of all originals $\mathcal{X}_j$ is defined by the membership value $\mu(\mathrm{s}_j)$. Through the so-called bunch parameter representation in eq. (18) the fuzziness is transferred to the bunch parameters. Each original has a probability distribution function $F_{\mathcal{X}}(\mathrm{s}_j, x)$. The set of distribution functions $F_{\mathcal{X}}(\mathrm{s}_j, x)$ of all originals $\mathcal{X}_j$ from one fuzzy random variable $\tilde{\mathcal{X}}$ can also be expressed by the fuzzy bunch parameter representation of its fuzzy distribution function: $\tilde{F}_{\tilde{\mathcal{X}}}(x) = F_{\mathcal{X}}(\tilde{\mathrm{s}}, x)$.

In order to solve engineering problems the fuzzy mapping $\tilde{f} : \tilde{\mathcal{X}} \tilde{\mapsto} \tilde{\mathcal{Y}}$ of some inputs $\mathbf{x}$ to the outputs $\mathbf{y}$ of a system under consideration has to be carried out. One way to do this is based on the bunch parameter representation of fuzzy random variables, and is referred to as fuzzy stochastic analysis type I in [9, 8], or as fuzzy stochastic sampling in [14, 4]. The bunch parameter representation of the fuzzy random variables $\tilde{\mathcal{X}} = \mathcal{X}(\tilde{\mathrm{s}})$ and $\tilde{\mathcal{Y}} = \mathcal{Y}(\tilde{\boldsymbol{\sigma}})$ and the functional relationship $\tilde{f}(x) = f(\tilde{s}, x)$ allows for a separate numerical treatment of fuzziness and uncertainty. As part of an outer loop, the α-level optimization is done for all bunch parameters. Since each specific element $\mathrm{s}_j \in \tilde{\mathrm{s}}$ of a α-level set uniquely specifies the distribution functions of the considered uncertain information, a probabilistic analysis can be conducted in an inner loop for each iteration of the outer loop. As already discussed for the case of Evidence Theory hereby the number of model evaluations increases strongly compared to worst-case or traditional stochastic tolerance analysis.

Application in tolerance analysis. As mentioned before the design engineer has to deal with imprecise and/or uncertain information when developing a tolerance concept for a directional control valve. Beside worst-case tolerance analysis engineers tend to statistical tolerancing because it is said to be more economically efficient even when small amount of incomplete interchangeability occurs. This might look legitimately at the first glance because the randomness from a high number of different manufacturing steps for one feature superimposes to normal distribution according to the central limit theorem and our empirical knowledge suggests a concentration at the mean value with falling incidence for realisations further away. However, in probability theory the uniform distribution has to express a lack of knowledge but this already implies a specific probability density function which is most unlikely to occur in reality. Furthermore, the specification limits are typically located in the marginal areas of the resulting probability density functions of the system's outputs. These marginal areas are particularly sensitive to small errors induced by the estimation of the probability density functions' parameters without a statistical reliable data base.

To avoid unnecessary small tolerances which usually arise when worst-case tolerancing is applied and to avoid making unrealistic assumptions about probability distribution functions in statistical tolerance analysis both fuzzy random variables and belief or plausibility measures are considered as possible compromise solutions. In the context of tolerance analysis we have to associate the imprecise and/or uncertain information about the noise factors with the allowed tolerance for each feature. In figure 4 two different possibilities are illustrated for each of the following propagation methods: fuzzy analysis (FA), stochastic analysis (SA), evidence analysis (EA) and fuzzy stochastic analysis (FSA). For each case a way to express some preference for the centre of the tolerance zone (solid line) or a complete lack of knowledge (dotted line) is exemplified by scaling of the corresponding information to the width of the tolerance zone. In the case of fuzzy analysis the support of the membership function may refer to the tolerance zone and the interval with $\mu = 1$ may express values that are for sure within the set of outcomes. As already discussed for stochastic analysis a normal distribution with $T/2 = 3 \cdot \sigma$ and $\mu = 0$ and a uniform distribution with $f(x) = 1/T \quad \forall x \in [-T/2; T/2]$ are typical assumptions. The focal elements used in evidence analysis may be different subsets of the tolerance zone with higher BPA m_X in the center. When there is no evidence for any subset of the tolerance zone only one focal element is used which holds the BPA $m_X = 1$. Fuzzy stochastic analysis allows for an imprecise definition of any probability distribution function parameter by using bunch parameter representation. For example an interval may be used to express the imprecision about the true mean value of a normal distribution. This results in upper and lower boundaries for the probability density function depending on the α-level considered.

Beside these two examples it's also possible to describe some kind of skewness of the supposed probability density function (e.g. when approaching the nominal value from on side), or multimodal distributions (e.g. in the case of multiple machines producing the same part) which are typical results from manufacturing process like turning or milling.

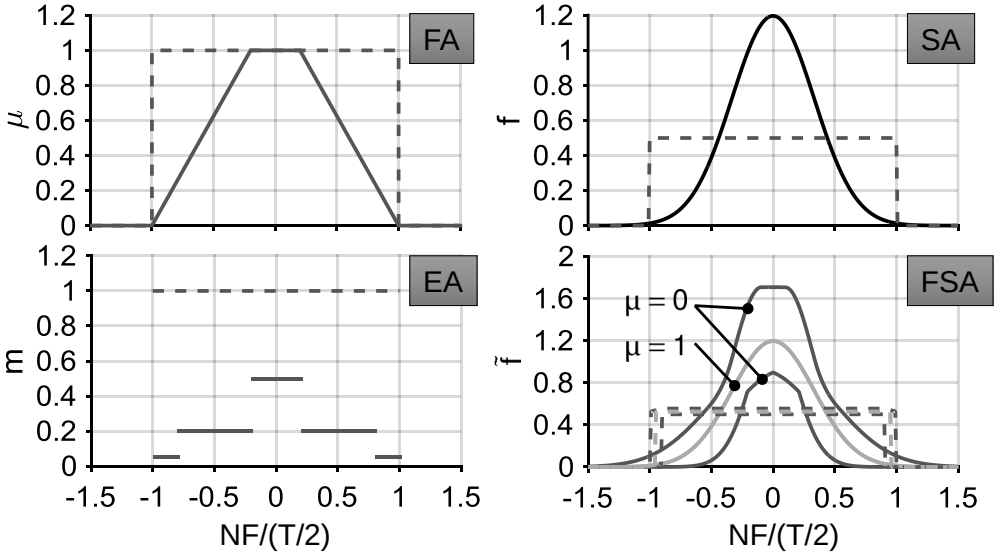


Fig. 4: Different mathematical representations for the imperfect information about noise factors associated to the tolerance zone

Computational Models

In addition to determining a suitable mathematical representation for the noise factors each of the associated propagation methods requires the numerical calculation of the system's output quantities. For hydraulic proportional valves, the flow rate/signal characteristic curve is one of the most quality-relevant features. It is characterized by the following values which have been already exemplified in figure 1:

- Leakage flow rate Q_{Leakage} in initial position ($I = 0\ A$)
- Opening current I_{on}
- Characteristic slope $\mathrm{d}Q/\mathrm{d}I$
- Maximum flow rate Q_{max}

In order to determine the flow rate/signal characteristic curve the steady-state values of the volumetric flow rate are measured at different coil currents while ports A and B are short-circuited and a constant pressure is supplied on port P. The system's functional behaviour can be split up into three sub-functions corresponding to the different physical domains involved:

1. Convert $E_{el.} \rightarrow E_{mech.}$ Solenoids are the most commonly used control elements of fluidic valves. The electric current through the coil generates a magnetic field. Due to the ferromagnetic material properties of the yoke comprising the coil, the tube, the armature, the armature counterpart and the washer, the magnetic flux is concentrated on these components. Since the armature and armature counterpart are separated by an air gap, a force effect (reluctance force) arises as a result of the abrupt change in the permeabilities. Because of their special design, proportional solenoids for hydraulic valves exhibit a force characteristic that is almost independent of stroke and increases approximately linearly with the coil current in their operational range. The magnetic material properties and the geometry of the connection between the tube and the armature counterpart determine the stroke/force characteristics most.
2. Adjust shift element The kinetics of the spool as closing element can be modelled as a translational rigid body movement with one degree of freedom as a first approximation. The corresponding motion differential equation $m \cdot \ddot{x} = \sum F_{ext.}$ describes the influence of the external forces on the motion. These

are the actuating forces of the control elements, frictional forces, the forces of the return springs and unbalanced pressure forces. The force generated by the solenoid $F_{M,a/b}$ is passed via the spool on the opposite return spring. According to the applied force a proportional change in length of the return spring allows the continuous positioning of the valve's spool.

3. Change flow resistance Most hydraulic proportional valves exhibit a cylindrical shift element with several grooves referred to as spool. It is longitudinally slidable in a bore of the valve housing or a socket inserted therein. Depending on the relative position between the spool's grooves and the cavities in the housing the connections between the ports are sealed or opened. For the mobility of the spool a clearance gap between the spool and the housing is required. As a result, typically the flow regime changes from an laminar gap flow (leakage) to a turbulent flow through an iris-shaped orifice with the spool's movement.

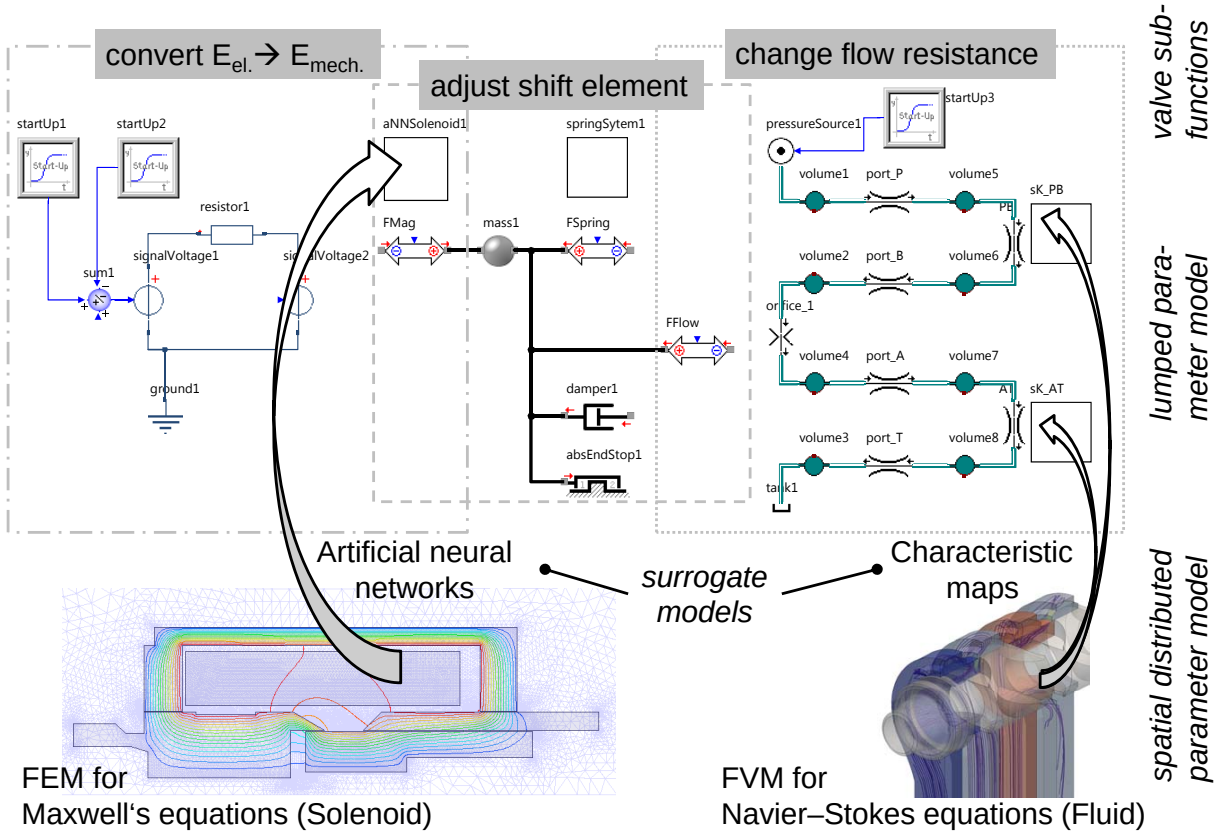


Fig. 5: Calculation of flow rate/signal characteristics for hydraulic proportional valves by lumped parameter model: representation of identified sub-functions and application of surrogate models

Obviously, directional control valves show strong interactions between the electrical, magnetic, mechanical and fluidic domains as well as non-linear transfer functions. So no closed analytical equation for the calculation of the key product characteristics is available and simulation models are needed to determine the system's response. The known propagation methods for imperfect information usually require numerous model evaluations. In this case the computational costs are critical and simulations with spatial distributed parameters, e. g. FEM for magnetic fields, CFD for fluid flows, are not or only to a limited extent permissible. As an alternative, simulations with lumped parameters allow for the consideration of cross-domain interactions and provide a sufficiently fast model computation. Therefore, these kind of models is preferable for use in tolerance analysis of hydraulic proportional valves.

In order to compare different tolerance concepts all identified noise factors need to be related to the key product characteristics within these simulation models. Unfortunately, some of the underlying physics cannot be modelled with respect to manufacturing variations or other noise factors by

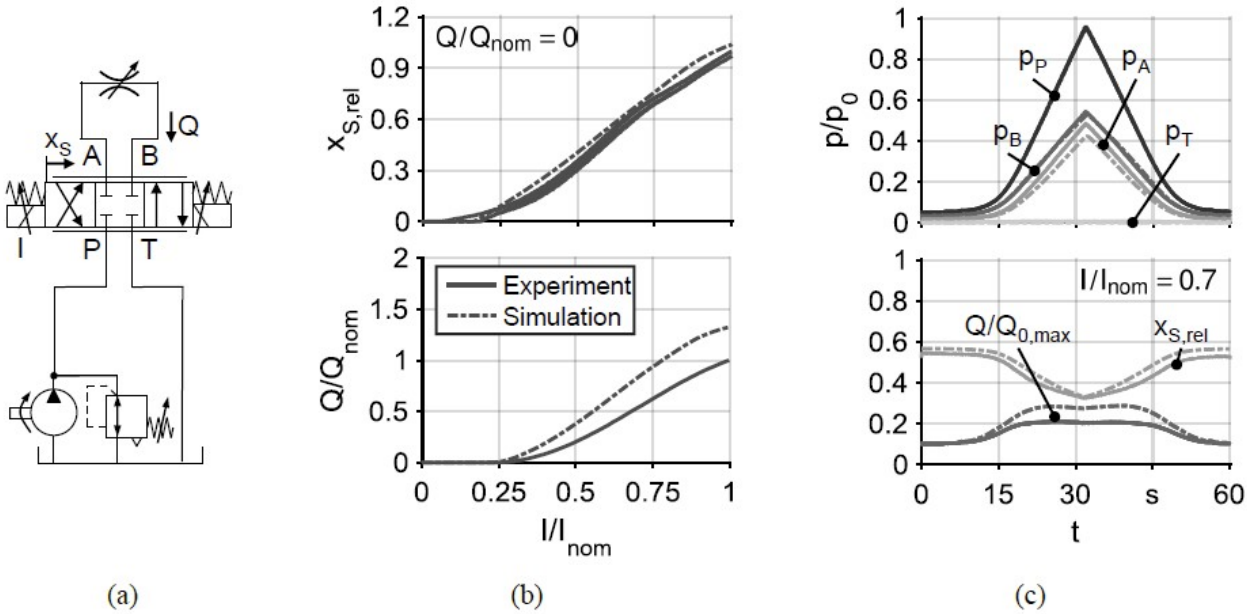
differential-algebraic equations as used in lumped parameter models. In such cases it is possible to describe these effects phenomenologically with different kinds of surrogate models. These take into account the noise factors directly or at least as multipliers which were estimated from limiting samples. As illustrated in figure 5 artificial neural networks can be used advantageously to model the influence of noise factors to the solenoid's force/stroke/current characteristic map. For that purpose they are trained with a sample data set resulting from FEM calculations conducted for random geometry and material parameters. Due to the required calculation time this is only reasonable for two-dimensional FEM models. In the case of solenoids without rotational symmetry reluctance networks can be an alternative choice. While leakage flow may be calculated based on Hagen–Poiseuille equation with respect to noise factors like clearance and eccentricity, the hydraulic resistance of the orifice, emerging at large spool strokes, is dominated by the resulting cross sectional area. That's why the characteristic maps for the relationship between flow rate or flow force,stroke and pressure difference are sufficiently determinable by a series of CFD simulations.Table 1 summarises the chosen implementation of the identified noise factors for the directional control valve under consideration.

Table 1: Consideration of noise factors for the analysed hydraulic proportional valve

sub-function	**geometry based implementation**	**phenomenological implementation**
convert $E_{\text{el.}} \rightarrow E_{\text{mech.}}$	• iron core • coil (temp.-dependent)	• magnetic permeability
adjust shift element	• stroke range • spring pre-stress • mass of armature/spool	• spring rate (temp.-dependent) • damping
change flow resistance	• leakage flow	• flow rate $= f(x_S, \Delta p)$ • flow force $= f(x_S, \Delta p)$

The simulation results from the system's model and the corresponding measurements are in good alignment as depicted in figure 6. Since the spool position matches for both simulation and measurements, the difference in the resulting flow/signal characteristic curve probably arises from simplifications made within the CFD models, e.g. neglected wall-roughness for the cast iron housing.

In most cases the computational cost for simulations with lumped parameters are still too high for a fuzzy stochastic analysis or the calculation of belief and plausibility measures. A common approach is to use surrogate models for estimating the whole system's behaviour with less computational effort. According to Simpson et al.[15] artificial neural networks are especially suitable for systems with much more than ten parameters. It is important to check the quality of the surrogate models with respect to the resulting error and against overfitting. The latter is done by using an additional test set, one that is not used for the generation of the surrogate model. When there is only a small error for these additional samples (as can be seen in figure 7, the model is said to have a good generalisation: something that is essential for utilization in tolerance analysis.



(a) (b) (c)

Fig. 6: Comparison of measurment and simulation results for the hydraulic proportional valve under consideration: a) test circuit; b) variation of input current; c) variation of pressure supply

Results

The aforementioned surrogate models were used to determine the system's responses during tolerance analysis based on different methods for representing the noise factors within the specified tolerance zone. Altogether 93 noise factors and two exemplary outputs are included in this study. For the sake of simplicity the noise factors are considered to prefer the centre of their respective tolerance zone as illustrated by the solid lines in figure 4. The results are depicted in figure 8 for comparison to each other. As can be seen easily all four methods predict some preference of the outcomes shifted by a small amount to the left/right from the centre of all possible values. The latter is gained from the support of the fuzzy interval and corresponds to the results from the worst-case analysis as shown on the topmost diagram.

The resulting variation has to be compared to the customer needs/ product specification. As done in statistical tolerancing a small amount of incomplete interchangeability may be allowed. For the presented study the interval containing 90 % of the outcomes was identified and is illustrated in figure 9. The determined range of variation is by far too wide for a hydraulic proportional valve. This is the result of using tolerance ranges from the tolerance class "middle" according to the DIN ISO 2768-1:1991 norm for all features as a theoretical basis. Nevertheless, the main advantage of evidence analysis and fuzzy stochastic analysis becomes clear. Compared to statistical tolerance analysis these methods need no unjustified assumptions concerning the true probability density functions but predict a smaller variation interval for the outputs than worst-case analysis does. When comparing evidence theory to worst-case tolerance analysis only a little more information is required to get this much more meaningful result. In this case, five focal elements were used to define a preference for occurrence at the centre of the tolerance zone for the six noise factors with the highest sensitivity indices. For the remaining 87 noise factors the same amount of information like in worst-case analysis was used: all values within the tolerance range are possible. On the other hand the fuzzy stochastic analysis also covers the two special cases of fuzzy analysis and stochastic analysis. This is quite useful to handle the ongoing acquisition of knowledge about the noise factors during progress of product development. Depending on the available information the individual noise factors are included within a common framework as fuzzy intervals, fuzzy random variables, or crisp probability distribution functions. Fur-

thermore the predicted range of variation is always less than in worst-case analysis. So it is possible to adjust the chosen tolerance concept for example based on sensitivities and tolerance/cost-functions and thereby increase economical efficiency without any measurable fall off in quality.

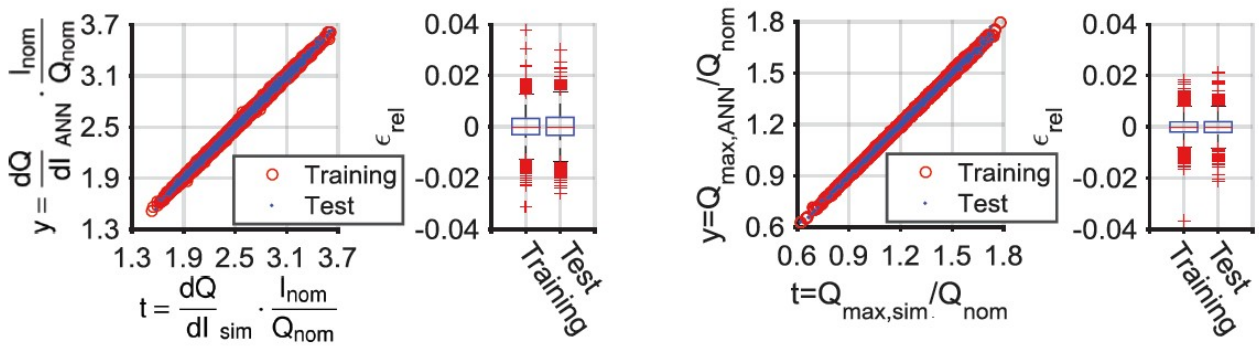


Fig. 7: Investigation of the surrogate models' generalisation and the error for: a) slope of flow rate/signal characteristics and b) maximum flow rate

Summary

This paper explains a method for computational tolerance analysis by the example of a directional control valve. A comparison of different approaches to describe the noise factors, their mathematical representation in relation to tolerances, and a way to propagate their impact on the key product characteristics through simulation models, were all addressed. Further application to other valve types (or even to complex fluid power systems) and their application in optimization based tolerance synthesis could be the focus of future research.

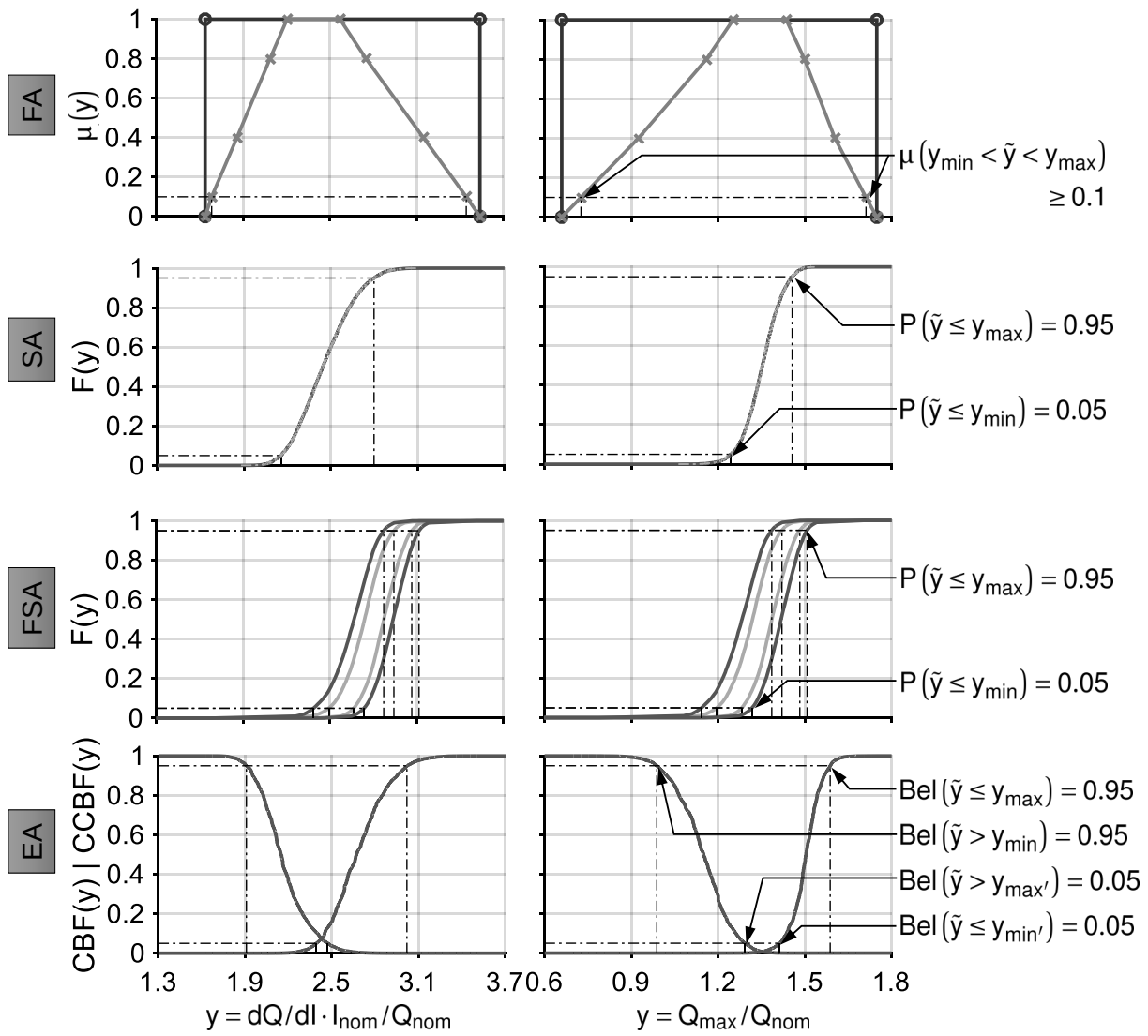


Fig. 8: Comparison of the results from different methods used to deal with imperfect information about noise factors

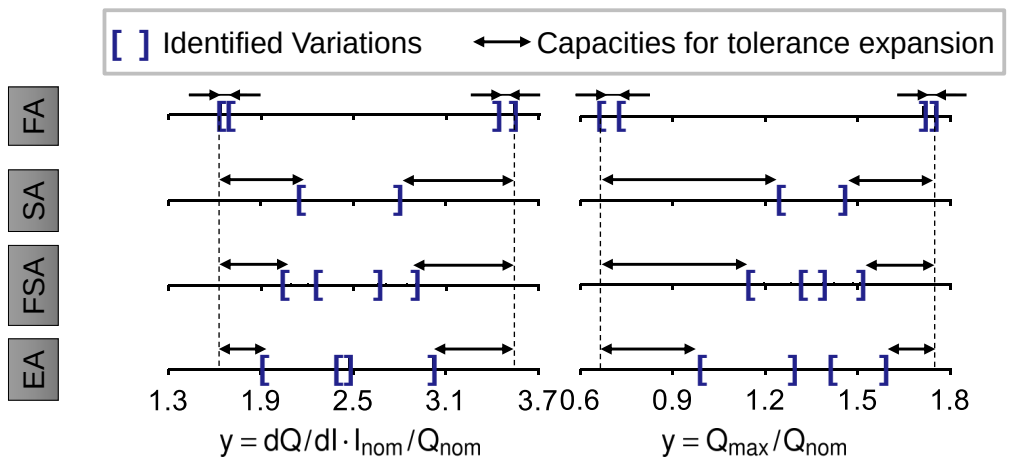


Fig. 9: Capacities for tolerance expansion resulting from the different concepts handling imperfect information when accepting 90 % of the outputs exceeding the specification limits

References

[1] Dj.M. Maric, P.F. Meier and S.K. Estreicher: Mater. Sci. Forum Vol. 83-87 (1992), p. 119

[2] H. Agarwal et al. "Uncertainty Quantification Using Evidence Theory in Multidisciplinary Design Optimization". In: Reliability Engineering and System Safety 85.1-3 (July 2004), pp. 281–294. ISSN: 09518320. DOI: 10.1016/j.ress.2004.03.017.

[3] T. Augustin et al., eds. Introduction to Imprecise Probabilities. Wiley Series in Probability and Statistics. Chichester, UK: John Wiley and Sons, Ltd, May 9, 2014. ISSN: 978-1-118-76311-7.

[4] M. Fuchs. "Clouds, p-Boxes, Fuzzy Sets, and Other Uncertainty Representations in Higher Dimensions". In: Acta Cybernetica 19.1 (2009), pp. 61–92. ISSN: 0324-721X. DOI: 10.14232/actacyb.19.1.2009.5.

[5] W. Graf, J.-U. Sickert, and F. Steinigen. "Numerical Simulation of Structures Using Generalized Models for Data Uncertainty". In: WIT Transactions on Modelling and Simulation. Vol. 48. May 20, 2009, pp. 511–521. DOI: 10.2495/CMEM090461.

[6] J. Helton et al. "A Sampling-Based Computational Strategy for the Representation of Epistemic Uncertainty in Model Predictions with Evidence Theory". In: Computer Methods in Applied Mechanics and Engineering 196.37-40 (Aug. 2007), pp. 3980–3998. ISSN: 00457825. DOI: 10.1016/j.cma.2006.10.049.

[7] G. J. Klir. "Generalized Information Theory: Aims, Results, and Open Problems". In: Reliability Engineering and System Safety 85.1-3 (July 2004), pp. 21–38. ISSN: 09518320. DOI: 10.1016/j.ress.2004.03.003.

[8] B. Möller and M. Beer. Fuzzy Randomness. Berlin, Heidelberg: Springer Berlin Heidelberg, 2004. ISSN: 978-3-642-07312-0.

[9] B. Möller and U. Reuter. Uncertainty Forecasting in Engineering: With 15 Tables. OCLC: 255434772. Berlin: Springer, 2007. 202 pp. ISSN: 978-3-540-37173-1.

[10] B. Möller et al. "Numerical Simulation Based on Fuzzy Stochastic Analysis". In: Mathematical and Computer Modelling of Dynamical Systems 13.4 (Aug. 2007), pp. 349–364. ISSN: 1387-3954, 1744-5051. DOI: 10.1080/13873950600994514.

[11] W. L. Oberkampf. "Uncertainty Quantification Using Evidence Theory". In: Proceedings from the Advanced Simulation and Computing Workshop. 2005.

[12] G. Presser. DEMPSTER-SHAFER Evidenztheorie – Versuch einer anschaulichen Einführung. Forschungsbericht 729. Universität Dortmund, May 18, 2000, pp. 19–24. DOI: 10 . 17877 / DE290R-14881.

[13] K. Sentz and S. Ferson. Combination of Evidence in Dempster-Shafer Theory. SAND2002-0835. Albuquerque, NM: Sandia, 2002.

[14] A. F. Shapiro. "A Fuzzy Random Variable Primer". In: Actuarial Research Clearing House. Itasca, Ill.: Committee on Research of the Society of Actuaries, 2008.

[15] J.-U. Sickert. "Fuzzy-Zufallsfunktionen Und Ihre Anwendung Bei Der Tragwerksanalyse Und Sicherheitsbeurteilung". TU Dresden, 2005.

[16] T. Simpson et al. “Metamodels for Computer-Based Engineering Design: Survey and Recommendations”. In: Engineering with Computers 17.2 (July 2001), pp. 129–150. DOI: 10.1007/PL00007198.

[17] P. Smets. “Imperfect Information: Imprecision and Uncertainty”. In: Uncertainty Management in Information Systems. Ed. by A. Motro and P. Smets. Boston, MA: Springer US, 1997, pp. 225–254. ISSN: 978-1-4613-7865-5.

[18] P. Walley. “Towards a Unified Theory of Imprecise Probability”. In: International Journal of Approximate Reasoning 24.2-3 (May 2000), pp. 125–148. ISSN: 0888613X. DOI: 10 . 1016 / S0888-613X(00)00031-1.