Predicting Short-Term HR Response to Varying Training Loads Using Exponential Equations

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Abstract

Aim of this study was to test whether a monoexponential formula is appropriate to analyze and predict individual responses to the change of load bouts online during training. Therefore, 234 heart rate (HR) data sets obtained from extensive interval protocols of four participants during a twelve-week training intervention on a bike ergometer were analyzed. First, HR for each interval was approximated using a monoexponential formula. HR at onset of exercise (HR_{start}), HR induced by load (HR_{steady}) and the slope of HR (c) were analyzed. Furthermore, a calculation routine incrementally predicted HR_{steady} using measured HR data after onset of exercise. Validity of original and approximated data sets were very high (r^2 =0.962, *SD* =0.025; *Max* =0.991, *Min* =0.702). HR_{start} was significantly different between all participants (one exception). HR_{steady} was similar in all participants. Parameter c was independent of the duration of intervention and intervals regarding one training session but was significantly different in all participants (one exception). Final HR was correctly predicted on average after 58.8 s *(SD* = 34.77, *Max* =150 s, *Min* =30 s) based on a difference criteria of less than 5 bpm. In 3 participants, HR_{steady} was predicted correctly in 142 out of 175 courses (81.1%).

KEYWORDS: ADAPTATION, HR RESPONSES, MONOEXPONENTIAL EQUATION

Introduction

In the physical training process, optimal training adaptations require individually optimized strain on the human body. However, it is very challenging to apply individually optimal stress that induces the required strain without any prior knowledge of the factors influencing this individual response (e.g., training condition, fatigue caused by prior training load). Whereas an overestimated load might result in overstraining and potential risks for the athletes, insufficient load might result in ineffective training and minor or no adaptation to training. A fast and reliable prediction of this individual strain is therefore essential for an optimal training regarding training effectiveness and efficiency as well as risk minimization, and time-optimization.

To identify the individual strain, the individual response to the change of load can be measured in various systems of the human body, for example, the cardiovascular system (e.g., heart rate, blood pressure), the cardiorespiratory system (e.g., oxygen uptake), the metabolic system (e.g., lactate, ammonia), the hormonal system (e.g., cortisol, IGF-I), the immune system (e.g., leucocytes), or the autonomous system (e.g., adrenaline). Especially in endurance training, the individual heart rate (HR in beats per minute, bpm) response has become a very important indicator to measure and determine responses representing individual strain of the human organism.

As identical load can induce completely different responses in different individuals, the prediction of individual strain is very challenging. Even in the same individual, a varying response can be observed depending on a variety of influencing factors, e.g., different environmental conditions, the current psychophysical state, muscle temperature or exhaustion of the cardio respiratory system (Hoffmann, Wiemeyer, & Hardy, 2015).

Another difficulty arises as the HR is not a fixed signal but is modulated by numerous influencing factors, e.g. the autonomous nervous system, arterial and cardiopulmonary baroreflexes or humoral mechanisms. Therefore, the HR shows considerable variability due to these influences (Sykrs, 1973).

Additionally, the short-term response of HR to the change of load bouts is delayed and seems to follow an exponential curve. When applying a submaximal exercise intensity, two physiologically different HR dynamics can be observed: the HR increases at the onset of exercise (HR_{start}) and the HR plateau corresponding to the applied load (HR_{steady}) representing the zone of steady state. If the load exceeds the submaximal range an additional upward drift of the HR can be observed (Åstrand & Rodahl, 1970).

In the literature, different formulas and procedures describing and modeling this individual HR response to the change of load bouts can be found (Ludwig, Sunduram, Füller, Asteroth, & Prassler, 2015). Besides analytical models such as exponential models (e.g. Bunc, Heller, & Leso, 1988), systems of linear equations (e.g. Le, Jaitner, Tobias, & Litz, 2008; Baig, 2014) and systems of non-linear equation (e.g. Cheng et al., 2008; Stirling, Zakynthinaki, Refoyo, & Sampredo, 2008), machine learning approaches such as artificial neuronal networks (e.g. Xiaro, Chen, Yuchi, Ding, & Jo, 2010; Sumida, Mizumoto, & Yasomoto, 2013) have been used for approximation.

The problem of most approaches is that calculation of steady state HR corresponding to the change of load requires the estimation of additional variables. These parameters mostly need to be estimated in additional tests prior to the training. Le et al. (2008), for example, presume that the individual anaerobic threshold is known; Stirling et al. (2008) require the individual maximal HR for calculation. Additionally, several procedures such as artificial neuronal networks (Xiaro et al., 2010) or machine learning approaches require a data set for learning the

individual adaptation parameters.

To enable a calculation of HR without any additional knowledge, the Bunc formula was used as a straightforward method taking into account only the resting HR value and HR data obtained while training. This data was used for calculating the individual steady state HR and the individual slope of the adaptation course of HR. This formula describes the course of the HR response by the following equation:

$$HR_{current} = a - b \cdot e^{-c \cdot t} \quad \text{(Bunc et al., 1988, p.41)} \qquad (1)$$

Legend:

a - steady state HR level elicited by the change of load (HR_{steady} in Figure 1)

b - HR reserve, i.e., difference between HR_{steady} and the HR at the start of exercise (HR_{start})

c - slope of HR curve

t - time [min]

Figure 1 illustrates the formula by means of prototypical HR responses.
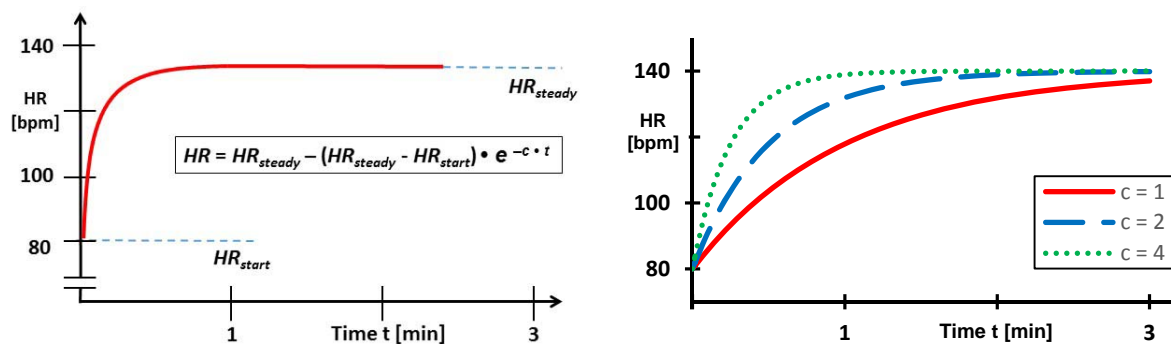

Figure. 1. Illustration of the time course of HR according to the Bunc equation (Bunc et al., 1988). Left side: prototypical HR course illustrating the Bunc equation, right side: prototypical HR courses with different c values illustrating the influence of c on the HR course

Hoffmann, Wiemeyer, and Hardy (2016) confirmed the feasibility of the Bunc formula for describing the individual HR response to the change of training load in the submaximal range, taking into account only selected time points.

In this paper, the Bunc formula (Bunc et al., 1988) was tested for the property to analyze and predict the individual responses to the change of load bouts. The aim of the study was to predict acute responses 'online', i.e., during training, as fast as possible without any prior knowledge of the change in the level of the training load or the individual HR responses to these changes. Therefore, HR responses to the change of load were recorded and analyzed over different time periods. Using this data, the slope of the course and the final HR (HR_{steady}) were incrementally calculated applying the Bunc formula. Subsequently, the difference of the calculated HRs from the measured HRs was calculated. The time point for a reliable prediction of the HR_{steady} corresponding to the load change was estimated.

Material and Methods

The study presented here was approved by the Ethics Committee of Technische Universität Darmstadt in 2016.

Participants and Apparatus

Four healthy and active adults (two males, two females) volunteered to participate in this study after having signed an informed consent. All participants declared that no contra indication for the training protocol existed (Washington et al., 1994). The participants' characteristics are described in Table 1.

All tests were performed using a cycle ergometer with a flywheel (Daum Ergometer 8008 TRS 3; Fürth, Germany). The power was controlled by the resistance at the flywheel and measured in Watts by the ergometer.

HR data was successively recorded beat by beat by a Polar V800 sport watch (Polar Electro, Kempele, Finnland). The corresponding Polar chest belt (T31) was attached to the participants prior to the training. The training protocol started after a period of 1 minute of passive sitting on the ergometer. Data recording started with the beginning of the training protocol.

Respiratory parameters were recorded during the exhaustion test and the sub test using the spiroergometry device K5 (COSMED, Rome, Italy). The mixing chamber recorded data every 10 s. First anaerobic threshold (ANT1) and second anaerobic threshold (ANT2) were automatically calculated from the respiratory parameters using the OMNIA Software (COSMED, Rome, Italy).

Table 1. Demographic and anthropometric description of the participants

	Participant 1 (P1)	Participant 2 (P2)	Participant 3 (P3)	Participant 4 (P4)	Total (n = 4)		
					Mean	SD	Range
Age [years]	31	32	32	31	31.5	0.50	1
Height [m]	168	175	182	185	177.5	6.58	17
Weight [kg]	79-80	69-70	87-90	95-97	83.1	9.35	18
Sex	Female	Female	Male	Male	---	---	---
BMI [kg/m²]	28.1	23.0	26.7	27.6	26.3	2.07	6.1
Activity time per Week [h]	3.5	2	7	3	3.875	2.17	5

Protocol

All data was obtained during a twelve-week endurance training intervention on a bike ergometer. This duration was chosen because adaptations to training can be reliably observed after this training period (Blank, 2007). Prior to and after completion of the intervention, the participants performed an all-out exhaustion test to estimate the individual maximal HR and VO_2 max.

The protocol of the exhaustion test started with a resting period with the participants sitting still on the ergometer. After 3 min the participants started pedaling for 2 min at 25 W, followed by 3 min at 50 W. After this warm-up period, the load at the ergometer was set to 100 W. The load was then successively increased by 50 W every 3 minutes until exhaustion ($load_{max}$).

Additionally, two subtests were performed in week 4 and week 8. These subtests were deployed to adapt the training intensity according to the training protocols. In the subtests, the resting and warm-up period of the exhaustion tests were repeated. Subsequently, the participants were stressed with 3 increasing load levels for three minutes each. The load was calculated to induce responses corresponding to the individual's ANT1 and ANT2 (i.e. first load ($load_{1sub}$): responses below ANT1, second load ($load_{2sub}$): responses between ANT1 and ANT2, third ($load_{3sub}$): responses above ANT2). At first, $load_{3sub}$ was calculated taking into account the load when the final exhaustion ($load_{max}$) and ANT2 ($load_{ANT2}$)was achieved. The following formula was used:

$$load_{3sub} = load_{ANT2} + \left(1 + \frac{load_{max} - load_{ANT2}}{50\ W}\right) \cdot 10\ W \qquad (2)$$

Subsequently, $load_{1sub}$ was calculated taking into account the height of the load when ANT1 was achieved ($load_{ANT1}$). If $load_{ANT1}$ was below 200 W factor x was set to 1. If $load_{ANT1}$ was above 200 W factor x was set to 2. $Load_{1sub}$ was calculated using the formula:

$$load_{1sub} = load_{ANT1} - (1 + x) \cdot 10\ W. \qquad (3)$$

To ensure a constant rise of load (e.g. increase of 40 W at each level) $load_{2sub}$ was calculated using the formula:

$$load_{2sub} = \frac{(load_{1sub} + load_{3sub})}{2} \qquad (4)$$

To give an example, the aerobic thresholds were recorded in the first exhaustion test at 250 W ($load_{ANT1}$) and at 300 W ($load_{ANT2}$). $Load_{max}$ was recorded at 350 W. Therefore, the load for the sub tests were calculated as:

- $load_{3sub} = 300\ W + (1 + \frac{350\ W - 300\ W}{50\ W}) \cdot 10\ W = 320\ W$

- $load_{1sub} = 250\ W - (1 + 2) \cdot 10\ W = 220\ W$

- $load_{2sub} = \frac{(320\ W + 220\ W)}{2} = 270\ W.$

After the 9-minute exercise period, a resting period of 5 minutes of active recovery at 25 W was applied. Subsequently, another 9-minute exercise phase and 5 min recovery phase were added, respectively.

According to the guidelines of the WHO (2010), the training volume for the training intervention was set to 25 min of intensive training three times a week. To offer a more varying and motivating training regime three different training methods were applied: the intensive continuous method (ICM), the extensive interval method (EIM) and the intensive interval method (IIM) (Hohmann, Lames, & Letzelter, 2002). Target HRs for the training protocols were calculated using the individual HR_{max} value obtained in the first exhaustion test. Furthermore, the load in each protocol was calculated using the HR data from the first exhaustion test and the two subtests, respectively. In the first exhaustion test, the last 30 s of every load level was therefore calculated as HR corresponding to the load levels. The load evoking the target HR was linearly interpolated from the calculated data. The same procedure was used in the subtests. However, the load was linearly extrapolated in case 95% HR_{max} exceeded HR_{steady} in the third load of the subtest. Mean load of both exercise phases were calculated as load that was expected to evoke the target HR for each training protocol.

The order of the training methods was permutated twice during the intervention. The previously described subtests were conducted substituting the IIM in week 4 and 8. All protocols were automatically applied at the ergometer. The participants were advised to keep

the pedal rate (PR) constant at 80 revolutions per minutes (RPM) (Coast, Cox, & Welch, 1986).

The training protocols are displayed in Table 2 in detail. Figure 2 displays an example of the load protocol for EIM.

Data processing

The HR data for the EIM training protocol was analyzed. Compared to the other training protocols the HR of the participant is expected to stay in the submaximal range after an initial exponential increase. Therefore, we expected that the Bunc formula is valid for EIM. Additionally, the time of these load phases is expected to be sufficient for HR to reach a steady state (Kroidl, Schwarz, Lehnigk, & Fritsch, 2014).

Table 2. Protocols for the training intervention

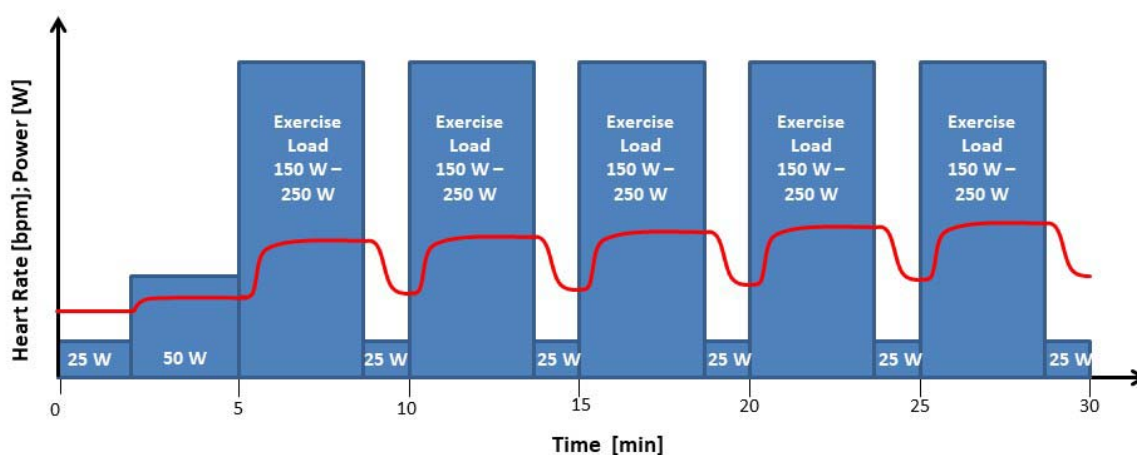|	Intensive Endurance Method (ICM)	Extensive Interval Method (EIM)	Intensive Interval Method (IIM)
Intensity	75% HR_{max}	80% HR_{max}	95% HR_{max}
Load period	25 min	3:30 min	1:00 min
Recovery Time between load intervals	0 min	1:30 min	1:30 min
Repetitions	1	5	10
Warm-up	2 min at 25 W	2 min at 25 W	2 min at 25 W
	3 min at 50 W	3 min at 50 W	3 min at 50 W
Total exercise time	30 min	30 min	30 min

Figure 2. Load protocol and prototypical HR response for EIM. Training load is varying due to the training load calculated with 80% HR_{max}.

HR data of the participants was automatically divided into exercise and recovery phases for each training protocol. The phases started with the change of the load at the ergometer. The warm-up period and the recovery phases were not part of the computations. Thus, we obtained 60 HR curves for each participant. Six curves where excluded due to measuring errors. In total, 234 data sets were processed.

The minimum value in the first 10 s of each curve was used as starting HR (HR_{start}). This was determined as the HR transition from recovering to the following exercise phase is characterized by a large variance. Therefore, miscalculation due to synchronizing errors are prevented.

Validity of the Bunc Formula

In order to gain first insights into the adaptation of the HR to the change of load bouts and to confirm the validity of the formula, the measured HR data was approximated using the previously described formula by Bunc et al. (1988.).

Expecting the HR to reach a steady state the mean HR in the last 30 s of each exercise phase was calculated as steady state HR (HR_{steady}). According to Kamath, Fallen, and McElvie (1991) a stable HR_{steady} is reached when the HR values vary less than 5 bpm for the rest of the exercise interval. Therefore, the difference of the measured HR and HR_{steady} as well as the increase of the HR values was used for the calculation of the time point when the measured HR values reached HR_{steady} (t_{steady}). In order to prevent miscalculations, the HR was preprocessed using the moving average method with a window size of 30 s. This window size was determined by testing window sizes ranging from 10 s to 60 s. The value of 30 s is considered a reasonable compromise of accuracy and noise reduction. For this calculation HR_{steady} was reached, when the difference of the averaged HR and HR_{steady} was smaller than 5 bpm and the increase of the averaged HR was smaller than 5.

The beat-to-beat HR data from the onset of exercise to t_{steady} was used for calculation of parameter c. Therefore, the data was first linearized using the formula

$$c \cdot t = -\ln \left(\frac{HR_{steady} + 1 - HR}{HR_{steady} - HR_{start}} \right) \qquad (5)$$

Parameter c was then estimated using the linear regression method.

As the Bunc Formula is most suitable for the description of the HR increase from the onset of exercise to HR_{steady}, this data set was used for calculation of the coefficient of determination. Additionally, the parameters HR_{start}, HR_{steady} and c were investigated. The average value of c was calculated as baseline value for the analyzed sample.

Calculation of HR_{steady}

The second part aimed at predicting the individual HR_{steady} online while exercising. Therefore, an incremental procedure was chosen that recalculated HR_{steady} after distinct time periods.

Due to lacking knowledge about the value c in general, the average c value for all 234 HR courses was used for calculation ($c_{average}$).

Using the Bunc formula, two defined HRs (HR_1 at time point t_1 and HR_2 at time point t_2) can be described as:

$$HR_1 = HR_{steady} - (HR_{steady} - HR_{start}) \cdot e^{-c \cdot t_1} \qquad (6)$$

$$HR_2 = HR_{steady} - (HR_{steady} - HR_{start}) \cdot e^{-c \cdot t_2} \qquad (7)$$

Using the substitution method, HR_{steady} can be calculated using the formula:

$$HR_{steadyCalc} = \frac{HR_2 - HR_1 + HR_{start}\,(\,e^{-c \cdot t_1} - e^{-c \cdot t_2})}{e^{-c \cdot t_1} - e^{-c \cdot t_2}} \qquad (8)$$

According to Ricardo et al (2005), in the first 20 s after the change of load, the HR increase is linear and independent of the level of the change. To ensure a reliable inclusion of HR data that were depending on the load change HR at onset of exercise (HR_{start}) and HR at 30 s after onset of exercise (HR_{30} at t_{30}) was used as first calculation period for predicting HR_{steady}. HR_{30} was calculated as mean HR over the last 5 beats to reduce the influence of HR variability.

To test the validity of the calculated HR_{steady}, an approximated HR curve including the calculated steady state HR ($HR_{steadyCalc}$), c and HR_{start}, was calculated again. Starting with t_{50} (time point 50 s after onset of exercise) the calculated HR (HR_{calc}) was compared to the actual HR (HR_{real}; mean value over last 5 s) at the corresponding time point. If the values differed from each other of more than 2 bmp, the value c was adapted depending the tendency of the deviation and the increase of the slope of HR. In case the increase of HR was higher than 30, value c was reduced by 0.5 if the calculated HR was higher than the real data. If the calculated HR was lower than the real HR value c was increased by 0.5, respectively. In case the increase was smaller than 30, c was reduced or increased by 0.1, respectively. This method was chosen taking into account the time course of the HR.

$HR_{steadyCalc}$ was again calculated taking into account HR_{30} representing HR_1, HR_{50} representing HR_2, HR_{start} and the adapted value c. Another approximated HR curve was calculated.

This procedure was repeated every 10 s until the end of the exercise phase.

Deviation of $HR_{steadyCalc}$ and HR_{steady}

To analyze the capability of the presented algorithm for predicting HR_{steady}, the deviation of $HR_{steadyCalc}$ und HR_{steady} was calculated throughout the exercise at distinct time points. Starting with t_{30}, the deviation was calculated every 10 s to analyze the prediction performance of HR_{steady} regarding efficacy and quality of prediction.

Additionally, the time point t_{5bpm} when the difference of $HR_{steadyCalc}$ and HR_{steady} was smaller than 5 bpm respectively was calculated.

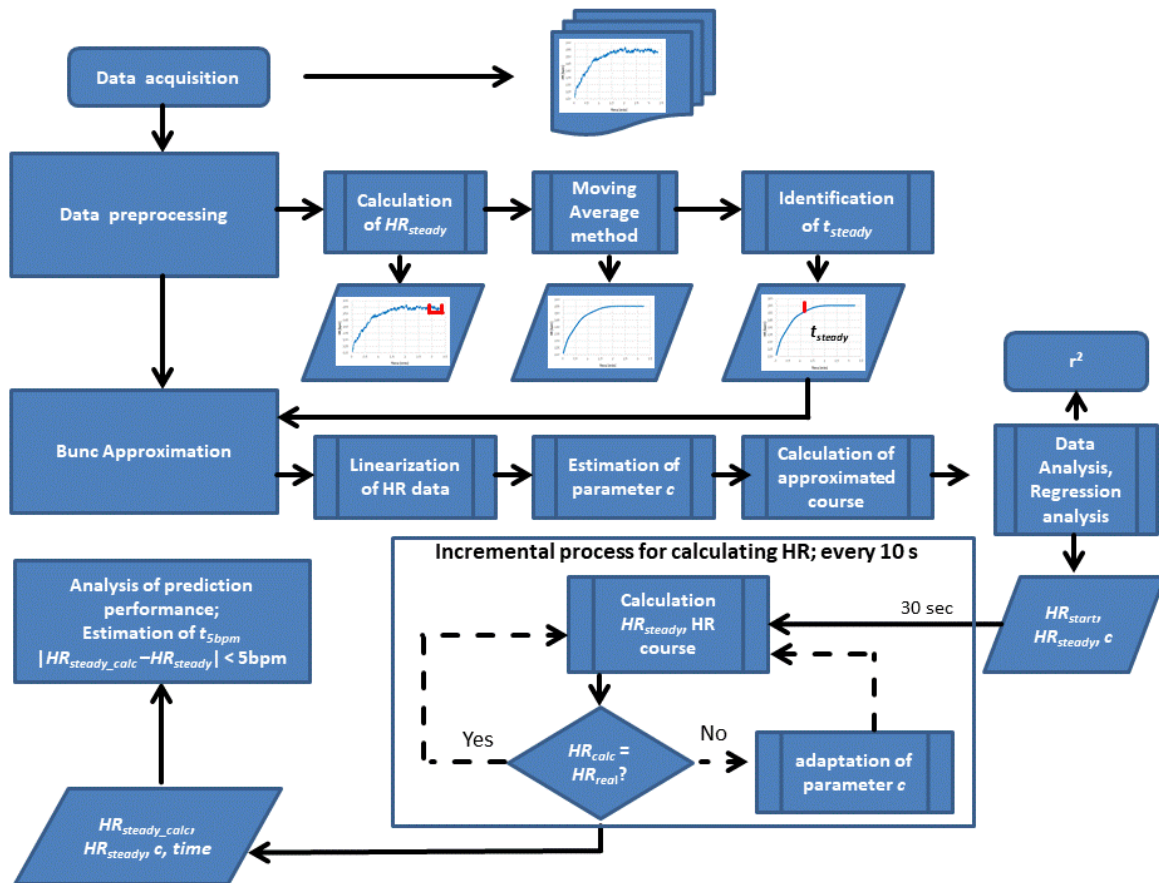The following flowchart illustrates the process (see Figure 3).

Figure 3. Flow Chart of the calculations). After the acquisition of HR data, all HR courses were preprocessed in three steps. HR courses were afterwards approximated using the Bunc-Formula and distinct parameters essential for calculation determined. HR_{steady} representing the steady state HR evoked by the load was predicted and the corresponding HR course modeled. After distinct time periods, the calculated HR (HR_{calc}) was compared to the measured HR (HR_{real}). In case of deviation the parameters were adapted and recalculated to fit the measured data. $HR_{steadyCalc}$ was compared to HR_{steady} throughout the modeling process. The prediction performance of the algorithm was analyzed using the obtained data. The period of calculation (t_{5bpm}) when HR_{steady} was adequately predicted was determined.

Results

As expected, a strong increase and decrease of HR depending on the load protocol can be observed in all participants. The difference from HR_{start} to HR_{steady} was higher than 20 bpm in all participants except for 2 increases of 19 bpm in week 6 in participant 2 (Overall: *Mean* = 38.8 bpm; *SD* = 8.54; *Max* = 60 bpm; *Min* = 19 bpm). An example of the HR adaptation corresponding to the induced load is illustrated in Figure 4.
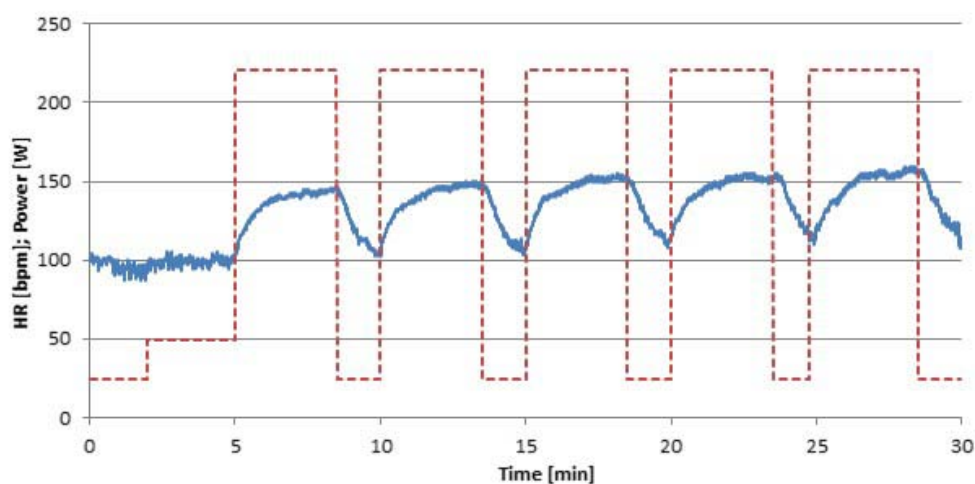
Figure 4. Prototypical HR course of participant 3 in week 3; the solid blue line represents the HR values calculated from the RR-intervals; the dotted red line represents the load protocol.

Statistical analysis of HR_{start} and HR_{steady} for all exercise phases is displayed in Table 3.

In all participants, the HR_{start} increased during the training session. The highest increase was found in participant 4 (see Table 3). No difference of HR_{start} was observed depending on the week of the training. The differences of HR_{start} were highly significant in all participants except for participant 2 and 4 ($t = 0.925$; $p = 0.357$; see Figure 5).

In contrast, the differences of HR_{steady} were less pronounced in all participants. Whereas an increase of HR_{steady} during the training intervention could also be found in all participants (see Table 3), the difference between the participants was significant only between P1 and P3 and between P2 and P3 (see Figure 6). Again, no overall trend of HR_{steady} depending on the week of training was observed.

Table 3. HR_{start} and HR_{steady} for all participants, HR_{max}: maximal heart rate achieved in an all-out exhaustion test.

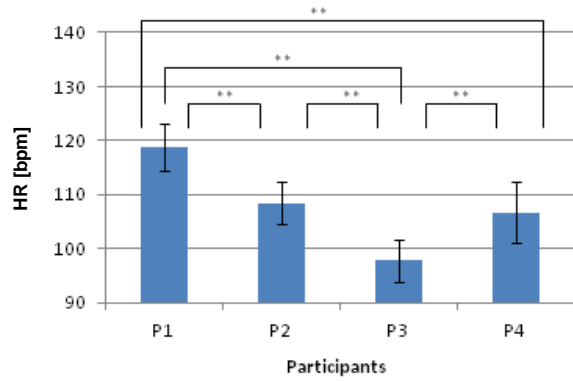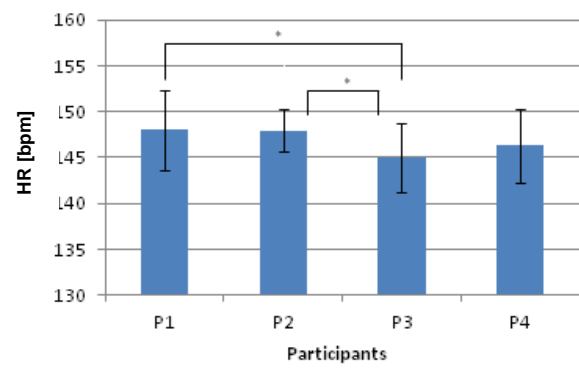| | Participant 1 (P1) | | Participant 2 (P2) | | Participant 3 (P3) | | Participant 4 (P4) | |
	HR_{start} [bpm]	HR_{steady} [bpm]	HR_{start} [bpm]	HR_{steady} [bpm]	HR_{start} [bpm]	HR_{steady} [bpm]	HR_{start} [bpm]	HR_{steady} [bpm]
HR_{max}	189		185		188		193	
Mean	118.7	148.0	108.4	147.9	97.8	144.9	106.7	146.3
SD	8.9	8.8	7.8	4.6	7.9	7.5	11.2	8.0
Max	140	168	123	156	119	165	135	168
Min	97	132	82	131	81	128	86	127
Interval 1 (I₁)	116	143	103	143	91	134	92	138
Interval 5 (I₅)	122	151	111	151	103	150	116	153
Difference I₁-I₅	7	8	7	8	13	11	24	12

Figure 5. Mean HR_{start} in for all participants. ** p<0.01

Figure 6. Mean HR_{steady} for all participants. * p<0.05

Validity of the Bunc formula

The mean coefficient of determination between the calculated Bunc formula and the measured HR data for all 234 HR data sets was $r^2 = 0.962$ ($SD = 0.025$; $Max = 0.991$, $Min = 0.702$.

P1: $r^2 = 0.946$ $SD = 0.028$; $Max = 0.986$, $Min = 0.881$;

P2: $r^2 = 0.967$ $SD = 0.022$; $Max = 0.991$, $Min = 0.898$;

P3: $r^2 = 0.975$ $SD = 0.013$; $Max = 0.990$, $Min = 0.915$;

P4: $r^2 = 0.959$ $SD = 0.039$; $Max = 0.985$, $Min = 0.702$).

The value c representing the slope of the adaptation course was varying between all participants (see Table 4).

Table 4. Value c for all participants

	Participant 1	**Participant 2**	**Participant 3**	**Participant 4**
Mean	1.42	1.74	1.55	1.08
SD	0.506	0.452	0.224	0.186
Max	3.04	2.86	2.29	1.57
Min	0.67	0.83	1.22	0.72

To illustrate the influence of *c* on the HR course two exemplary courses with a high and low *c* value are displayed in Figure 7.
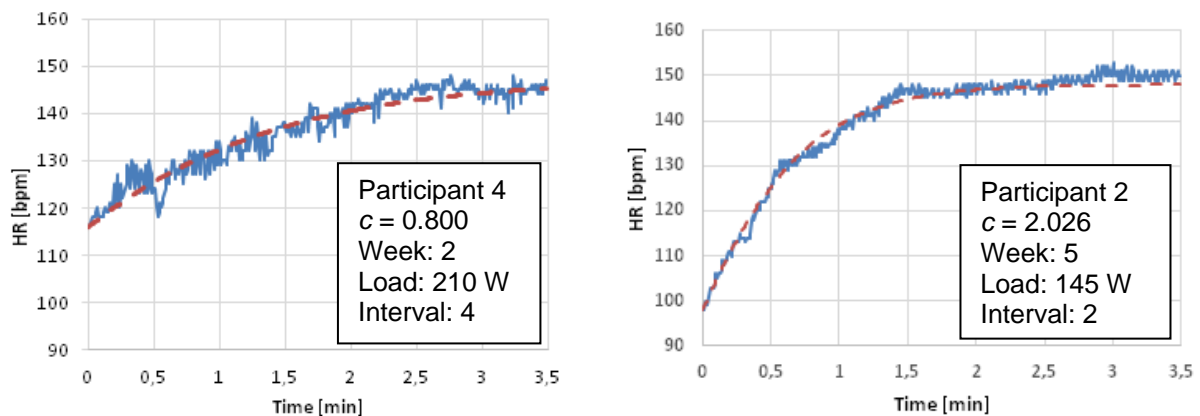

Figure 7. Left side: prototypical HR course of participant 4 with a low *c* value (*c* = 0.800); Right side; prototypical HR course of participant 2 with a high *c* value (*c* = 2.026). Solid blue line represents the HR values calculated using the RR-intervals; intermittent red line represents the approximated course using the Bunc formula.

Significant differences of the mean *c* value were found in all participants except for P1 and P3 (see Figure 8).
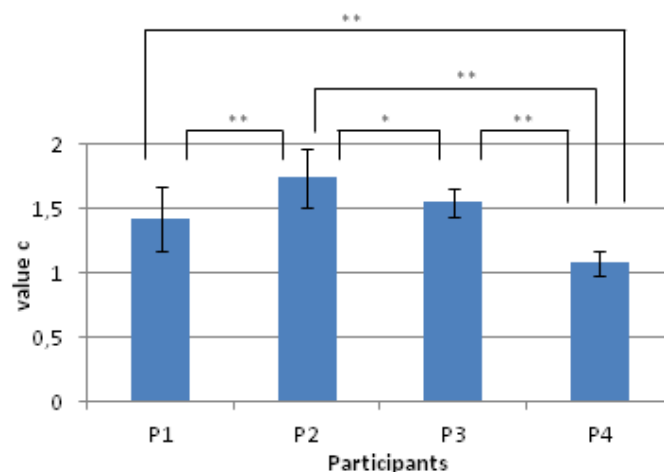

Figure 8. Calculated c (representing the slope of the HR course) for all participants. * p< 0.05, ** p < 0.01.

Additionally, *c* is varying irrespective of the week of training intervention. No overall trend was observed within twelve weeks. Value *c* was also varying irrespective of the load interval within each training session.

The difference of HR_{steady} to HR_{start} is not correlated to value *c* (r^2 = 0.011; *N* = 234; *p* = 0.12) (see Figure 9).

All calculated data is displayed in Table 5 and Table 6.

Table 5. Value *c* depending on the week of the training intervention (week 1 to week 12) averaged for all participants.

Week	*Mean*	*SD*	*Max*	*Min*
1	1.300	0.201	1.544	1.045
2	1.325	0.234	1.687	1.073
3	1.306	0.253	1.644	1.042
4	1.585	0.425	2.193	1.129
5	1.422	0.381	1.952	1.054
6	1.565	0.278	1.900	1.221
7	1.611	0.419	2.263	1.270
8	1.573	0.299	1.935	1.208
9	1.337	0.168	1.529	1.147
10	1.312	0.268	1.653	0.986
11	1.487	0.222	1.729	1.235
12	1.499	0.353	2.043	1.181

Table 6. Value *c* depending on the particular training interval during the training session, averaged for all participants

Training interval	*Mean*	*SD*	*Max*	*Min*
1	1.503	0.366	2.216	0.986
2	1.329	0.295	1.894	0.911
3	1.472	0.333	2.046	0.999
4	1.445	0.399	2.272	0.990
5	1.493	0.271	2.007	1.078

Deviation of $HR_{steadyCalc}$ to HR_{steady}

Prototypical calculation processes including the adaptation of $HR_{steadyCalc}$ throughout incremental procedure are displayed in Figure 10.

The prediction quality of the algorithm was increasing throughout the calculation process. In total, all HR_{steady} values were correctly predicted after 150 s when the difference of measured to calculated HR was less than 5 bpm. Already after 30 s, HR_{steady} was correctly predicted in 95 out of the 234 courses. The amount of correct recognition increased to 161 (out of 234 courses; 69%) after 60 s and to 186 (out of 234 courses; 80%) after 90 s, respectively.
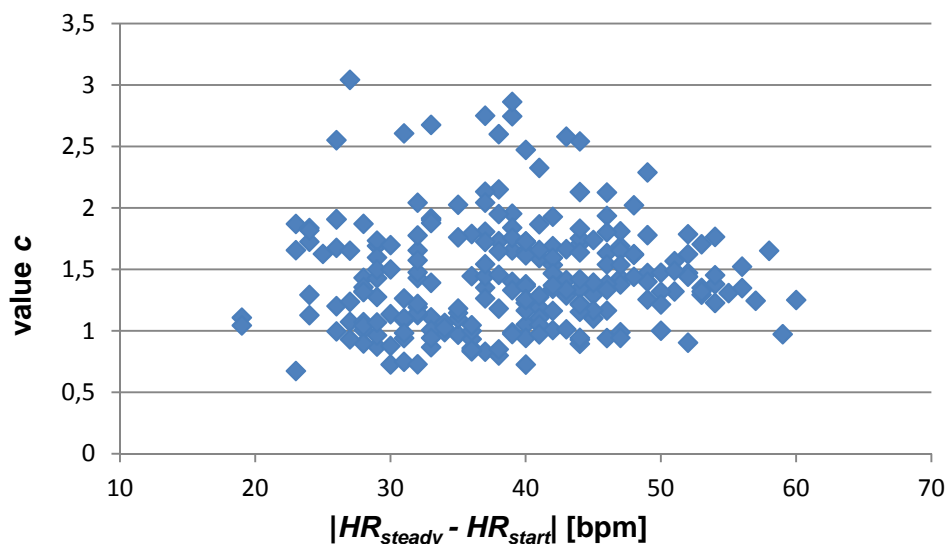
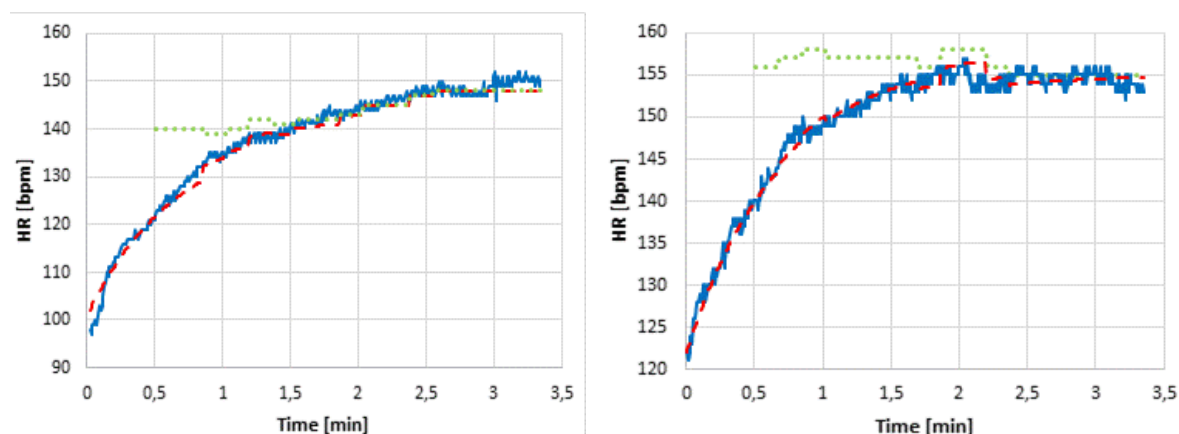Figure 9. Correlation of the difference of $|HR_{steady} - HR_{start}|$ and value c.

Figure 10. Prototypical courses of calculation of $HR_{steadyCalc}$. Left side: Poor estimation of HF_{steady_calc} within 149 s (1.5 min) after onset of exercise. A strong underestimation can be observed (HR_{steady} = 148 bpm). Right side: good estimation of $HR_{steadyCalc}$ within 30 s (0.5 min) after onset of exercise (HR_{steady} = 154 bpm). Although $HR_{steadyCalc}$ is varying during the calculation process, the deviation of HR_{steady} and $HR_{steadyCalc}$ stays within the range of 5 bpm. Blue Solid line represents the HR values calculated using the RR-intervals. Red intermittent line represents the approximated course using the Bunc formula. Green dotted line represents $HR_{steadyCalc}$.

However, the correct recognition was varying between the participants. Already after 60 sec, the correct HR_{steady} was predicted in 48 out of 60 courses (80%) in participant 1, in 47 out of 60 courses (78%) in participant 2 and in 47 out of 55 courses (85%) in participant 3. After 90 s, the deviation of HR_{steady} and $HR_{steadyCalc}$ was smaller than 5 bpm in 53 out of 60 courses (88%) in participant 1, in 54 out of 60 courses (91%) in participant 2 and in 54 out of 55 courses (98%) in participant 3. In contrast, the amount of correct recognition was only 32 % (19 out of 59 courses) and 42 % (25 out of 59 courses) in participant 4 after 60 s and 90 s, respectively.

All data of correct prediction of HR_{steady} when the difference of measured to calculated HR was less than 5 bpm after distinct time periods and cumulative recognition of $HR_{steadyCalc}$ for all participants are illustrated in figure 11.
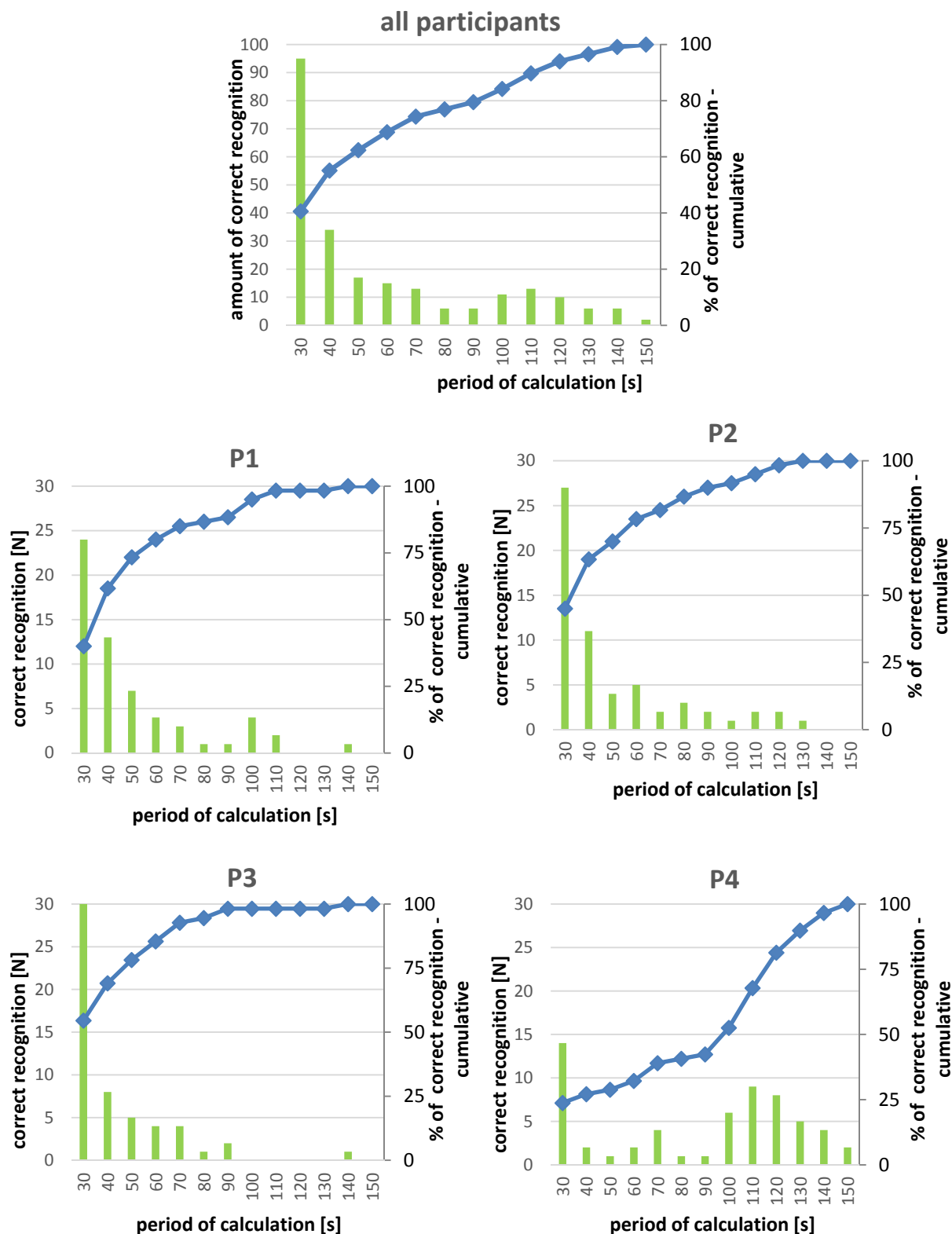
Figure 11. Amount of correct calculations of HR_{steady} after distinct periods of calculation illustrated for all participants and each participant individually; dark blue: amount of cumulative correct calculations in %; light green: amount of correct calculation itemized for each time point. For example, P1 showed 24 correct estimations of HR_{steady} when HR data from t0 to t30 were included (green bars); additional 13 correct calculations were achieved when HR data from t0 to t40 were included. Cumulative detection rate increased from 40 to 61.6 per cent (blue line).

Considering the mean deviation of $HR_{steadyCalc}$ and HR_{steady} after the given calculation periods for all participants a decrease in absolute deviation and corresponding standard deviation can be observed. All data is displayed in Table 7.

Table 7. Mean deviation of HR_{steady} and $HR_{steadyCalc}$ for all participants

Period of calculation [s]	30	40	50	60	70	80	90	100	110	120	130	140	150
Mean [bpm]	6.14	6.11	5.29	4.80	4.65	4.22	4.19	3.67	3.27	2.78	2.42	2.25	1.94
SD	5.17	5.15	4.60	4.08	3.99	3.59	3.60	3.03	2.88	2.46	2.15	1.91	1.69

In general, the final HR was predicted after 57.8 s (*SD* = 34.77, *Max* = 150 s, *Min* = 30 s). The poorest calculation of $HR_{steadyCalc}$ was found in participant 4. Whereas in the other participants $HR_{steadyCalc}$ was correctly calculated in average after 48.1 s, the calculation for participant 4 took 86.8 s (P1: *Mean* = 49.5 s, *SD* = 26.13; P2: *Mean* = 50.0 s, *SD* = 27.62; P3: *Mean* = 44.8 s; *SD* = 24.39; P4: *Mean* = 86.8 s; *SD* = 41.19).

Discussion

In this study, the monoexponential formula developed by Bunc et al. (1988) was tested for the validity to analyze and predict the individual responses to the change of load bouts. The HR data clearly shows that even though a high variation in HR_{start} can be observed, the algorithm performed fast in most HR courses and provided sufficient results for HR_{steady} without knowing the level of the change in load or the individual HR responses to these changes.

Considering the amount of correct recognitions and the mean deviation of $HR_{steadyCalc}$ and HR_{steady} for all participants, the algorithm performs reasonable in most participants after a calculation time of 60 s. However, the prediction quality after a calculation period of 90 s is with almost 80% of correct recognition and a mean deviation of 4.19 bpm (*SD* = 3.60) increased.

Analysis revealed that the c value is independent of the week of the training intervention, the load intervals in each training session, or the difference of HR_{start} and HR_{steady}. Rather, *c* seems to be very individual due to significant differences among the participants. Further research is required to answer the question, if an influence of long-term training can be verified or if further influences on c can be detected.

Additionally, the signal processing of the HR data calculated from the RR intervals is challenging. As the heart rate variability is highly modulated by internal influencing factors (i.e. venous return flow, breathing of the participant), it is very challenging to take into account all possible influencing factors on the HR. Therefore, the preprocessing of the signal needs to be improved. One possibility might be the averaging of the HR data. However, determining the optimal window size is still challenging to balance accuracy and noise reduction (i.e. HR_{start}, HR_{max}). Additionally, the specific analysis of distinct parameters might improve the prediction. For example, one of these parameters is the time point when the HR course changes from linear to exponential increase. As proposed by Engelen, Porszas, Riley, Wasserman, Maehara, and Barstow, 2013) a small plateau of HR can be observed. Plateaus were found throughout the HR course due to variances (81 out of 234 courses – 34%). In these cases, the Bunc algorithm might lead to a misinterpretation of a transiently steady HR as HR_{steady}. Additionally, the incremental calculation might incorrectly adapt the value *c*.

As the baseline value for *c* was calculated over all participants, the algorithm performed best when the value *c* for the analyzed data set matched $c_{average} = 1.5$ calculated prior to the analyses. Additionally, a fast recognition was observed with *c* values higher than $c_{average}$. ($1.5 < c$). The poorest results were achieved when the *c* value calculated for a particular phase was less than 0.8.

This was especially apparent in participant 4. Due to the delayed correct calculation, HR_{steady} was often underestimated. As the algorithm was developed to fit the $HR_{steadyCalc}$ to the data especially when a strong increase can be observed, the small *c* might lead to only small adaptations of the value *c*. This might provoke a delay as $HR_{steadyCalc}$ is recalculated only every 10 s. Therefore, the parameters for the adaptation of the value *c* during the calculation process require further refinement. Especially, deflection points when the increase of the HR flattens should be integrated in the calculation. Including more than the currently used two time points might lead to a faster and more reliable recognition. Additionally, the baseline for value *c* requires refinement und needs to be estimated in a wider group.

Further analyzing the HR data of participant 4 revealed that the HR might not reach a stable steady state but keeps increasing beyond the submaximal range. Therefore, future research should address the question if the formula is feasible for predicting HR that exceeds the submaximal range. Additionally, future investigations should also examine other training protocols (i.e. intensive or extensive training methods) or the HR course during regeneration phases.

Furthermore, the validity of predicting $HR_{steadyCalc}$ should be investigated using and comparing further formulas or procedures (i.e. Le et al. 2008; Cheng et al., 2008) that were described earlier. This might give insight if more parameters are essential for a more reliable and faster prediction.

Additionally, further research needs to address the question how the load has to be adapted in case the load is predicted to be insufficient or overstraining for optimal training adaptation.

Conclusion

The monoexponential formula from Bunc has the potential to be used as a method for predicting individual strain without knowledge of the change in level of load. However, the prediction algorithm requires further refinement to improve the quality and the speed of the prediction. Especially, HR courses with a slow increase were not predicted sufficiently. More parameters of the HR reaction should be included in the calculation, i.e. distinct deflection points or plateaus of the HR course. Furthermore, the signal processing of the HR needs to be improved to prevent miscalculations due to variations of the HRs.

Acknowledgements

References

Åstrand, P.O., & Rodahl, K. (1970). *Textbook of Work Physiology*. New York: McGraw-Hill.
Blank, M. (2007). *Dimensionen und Determinanten der Trainierbarkeit konditioneller Fähigkeiten*. [Dimensions and determinants of training conitioning abilities] Hamburg: Czwalina.

Baig, D. E. Z. (2014). *Physiological control of Human Heart Rate and Oxygen Consumption during Rhythmic Exercises.* Research Paper, The University of New South Wales, Sydney, Australia.

Bunc, V. P., Heller, J., Leso, J. (1988). Kinetics of heart rate response to exercise. *Journal of Sports Science, 6* (1), 39-48.

Cheng, T. M., Savkin, A. V., Celler, B. G., Su, S. W., Wang, L., & others. (2008). Nonlinear modeling and control of human heart rate response during exercise with various work load intensities. *IEEE Transactions on biomedical engineering*, *55*(11), 2499–2508.

Coast, J.R., Cox, R. H., & Welch, H. (1986). Optimal pedaling rate in prolonged bouts of cycle ergometry. *Medicine and Science in Sport & Exercise, 18* (2), 225-230.

Engelen, M., Porszasz, J., Riley, M., Wasserman, K., Maehara, K., & Barstow, T. J. (1996). Effects of hypoxic hypoxia on O 2 uptake and heart rate kinetics during heavy exercise. *Journal of applied physiology*, *81*(6), 2500-2508.

Hoffmann, K., Wiemeyer, J., & Hardy, S. (2014). Comparison of two procedures to predict the individual Hear Rate Reaction. In A. Baca, & M. Stöckl (Eds.) *Proceedings of the 10th Symposium on Computer Science in Sport of the German Society of Sport Science (dvs), Vienna, Austria, September 12-14.* (pp. 105 - 110). Hamburg: Czwalina.

Hoffmann, K., Wiemeyer, J., & Hardy, S. (2016). Prediction and control of the individual Heart Rate response in Exergames. In P. Chung, A. Soltoggio, C. W Dawson, Q. Meng, & M. Pain (Eds.) *Proceedings of the 10th International Symposium on Computer Science in Sports. ISCSS. Loughborough, United Kingdom, September 9-11* (pp. 171-178). Springer International Publishing.

Hohmann, A., Lames, M., & Letzelter, M. (2002). *Einführung in die Trainingswissenschaft.* Wiebelsheim: Limpert Verlag.

Kamath, M. V., Fallen, E. L., & Mckelvie, R. (1991). Effects of steady state exercise on the power spectrum of heart rate variability. *Medicine and science in sports and exercise*, *23*(4), 428-434.

Kroidl, R., Schwarz, S., Lehnigk, B., & Fritsch, J. (Eds.). (2014). *Kursbuch Spiroergometrie: Technik und Befundung verständlich gemacht*. Georg Thieme Verlag.

Le, A., Jaitner, T., Tobias, F., & Litz, L. (2008). A dynamic heart rate prediction model for training optimization in cycling (p83). In M. Estivalet, & P. Brisson (Eds.), *The Engineering of Sport 7* (pp. 425–433). Springer.

Ludwig, M., Sundaram, A. M., Füller, M., Asteroth, A. & Prassler, E. (2015). On modeling the cardiovascular system and predicting the human heart rate under strain. In A. Holzinger, C. Röcker, M. Helfert, A. Fred, J. O'Donoghue, & M. Ziefle (Eds.), *Proceedings of the 1st International Conference on Information and Communication Technologies for Ageing Well and e-Health. Lisbon, Portugal May 20-22, 2015*, Scitepress.

Ricardo, D.R., de Almeida, M.B., Franklin, B.A. & Araujo, C.G.S. (2005). Initial and Final Exercise Heart Rate – Influence of Gender, Aerobic Fitness, and Clinical Status. *Chest, 127* (1), 318 – 327.

Saykrs, B. McA. (1973). Analysis of Heart Rate Variability, *Ergonomics*, *16*(1), 17-32.

Stirling, J. R., Zakynthinaki, M., Refoyo, I., & Sampedro, J. (2008). A model of heart rate kinetics in response to exercise. *Journal of Nonlinear Mathematical Physics*, *15*(sup3), 426–436.

Sumida, M., Mizumoto, T., & Yasumoto, K. (2013). Estimating heart rate variation during walking with smartphone. In F. Mattern, S. Santini, Canny, J. F., Langheinrich, M., & J. Rekimoto (Eds.), *Proceedings of the 2013 ACM international joint conference on Pervasive and ubiquitous computing. Zurich, Switzerland September 08-12, 2013*, ACM.

Washington, R. L., Bricker, J. T., Alpert, B. S., Daniels, S. R., Deckelbaum, R. J., Fisher, E. A., Gidding, S. S., Isabel-Jones, J., Kavey, R.-E. W., Marx, G. R., Strong, W. B., Teske DW, Wilmore JH, Winston M (1994). Guidelines for exercise testing in the pediatric age group. From the Committee on Atherosclerosis and Hypertension in Children, Council on Cardiovascular Disease in the Young, the American Heart Association. *Circulation 90* (2), 2166-2179.

World Health Organization WHO. (2010). *Global Recommendations on Physical Activity for Health.* Geneva: World Health Organization.

Xiaro, F., Chen, Y., Yuchi, M., Ding, M., & Jo, J. (2010). Heart rate prediction model based on physical activities using evolutionary neural network. In J. S. Pan, X. Li, T. S. Pan, W. M. Zheng & X. Wang (Eds.), *Fourth International Conference on Genetic and Evolutionary Computing (ICGEC), 2010* (pp. 198–201). IEEE.