Chapter 8 Conclusions

With the recent advents in space based geodetic techniques, particularly in GPS, simultaneous determination of the 3D-point positions with a remarkable accuracy is now possible. In the light of these new possibilities, in the analysis of the Earth's surface deformations we are now capable to implement mathematical models that are more relevant to the 3D-nature of this phenomenon. Similar to the other measurement techniques, corresponding systematic errors in space based positioning technology are a potential source for falsifying the results of deformation analysis. Therefore, in addition to the improvement in mathematical modeling the behavior of systematic errors on the sought parameters should also be analyzed.

An attempt is made in this study to formulate the effect of different GPS systematic errors on deformation parameters. The developed deterministic models can provide us an insight to the possible deviations of strain parameters when residual or unavoidable systematic errors are still present in the GPS results.

In this work, two new approaches have been implemented and studied for analyzing the deformation of the Earth in three-dimensions. One is based on the Lagrangian representation of deformations and the other is based on its Isoparametric form. The conditioning of the problem has been analyzed analytically and numerically. This analysis has shown that the 3D-representation of the Earth's surface deformations is not continuous under infinitesimal changes of the input parameters and thereby, is an ill-posed problem. It is shown that the network configuration and the topography of the Earth's crust play a key role in this respect. Therefore, it has to be necessarily possible to analyze the sensitivity of the various parameters of the functional models in Isoparametric and Lagrangian representations of deformations (sensitivity analysis of deformation tensor) to perturbations of input parameters. Based on singular value decomposition and principal component analysis of the normal matrix a diagnosis method has been formulated for this purpose. To check the efficiency of the method simulated deformations have been used. The diagnosis method of this work can be implemented in the pre-analysis process for evaluating and selecting an optimum configuration for monitoring the deformation of an area in three-dimensions. The method of this study

for the sensitivity analysis of deformation tensor is also a suitable tool for the assessment of the efficiency of the 3D-Isoparametric approach against the Lagrangian one.

When the problem is ill-conditioned, the stability of the least-squares solution should be seriously concerned. The interrelation of the discrete Picard condition with the stability of the least-squares solution has been analyzed. If the stability of the maximum likelihood solution is not assured, regularization of the problem is inevitable.

Among different regularization techniques, truncated singular value decomposition (TSVD) is an efficient and alternative method to the standard regularization technique of Tikhonov and Philips. In addition to that, the method is easy to apply and easy to visualize. In this work, TSVD has been proposed (see Chapter 4) for regularizing the 3D-representation of deformation when the discrete Picard condition is not fulfilled in the problem. In contrary to the standard method of L-curve analysis, perturbation theory of the TSVD solution has been found as an efficient technique for finding an optimum regularization parameter. The analysis of the relative errors of different regularized solutions for the synthetic deformations of this study has shown the efficiency of the method in finding a tradeoff between the stability and the resolution in results. Moreover, the method has been successfully implemented in the 3D-Isoparametric and Lagrangian representations of real deformations in the test area of this research.

The assessment and analysis of regularization errors is a key point in any regularization process. For this purpose, external information on the sought solution is normally inevitable. This work is self-contained in this respect in the sense that regularization errors of the regularized horizontal parameters of deformation can be directly assessed by their corresponding counterparts from the two-dimensional approach. By comparing resolutions of vertical parameters of deformation to that of horizontal ones, it is possible to establish a lower and/or an upper bound limit for the regularization errors of these parameters too. Nevertheless, a realistic analysis of regularization errors is largely dependent on the redundancy of observations. When the deformation field is too inhomogeneous, the assessment of regularization errors will be problematic. This is because when the deformation monitoring network is sparse a realistic error estimate for estimated parameters of deformation would not be possible. In these cases external information on solution is inevitable.

Tensor series analysis plays an essential rule in the analysis of the Earth's surface deformations when the gradual accumulation of strain or (under certain rheological assumptions) the gradual accumulation of stress is concerned. Due to the small signal to noise ratio, the analysis of the change in deformations in a time or space series of deformation tensor requires sophisticated mathematical techniques. This work (Chapter 5), also forwards a method for analyzing the time and space variations of deformations. The devised method works in two steps: In the first step, based on the Procrustes analysis of deformations the significance of the change in deformations is statistically analyzed. In the second stage, significant changes are assigned to the normal and/or shear parameters of strain. For this purpose, Procrustean residuals are screened. The screening process is based on a modification to the BIBER robust estimator. The modified BIBER-estimator has been setup using a new loss function in which the internal reliabilities of observations take the position of the empirically determined constant threshold of BIBER-estimator for bounding the influence of larger residuals. It has been shown that for the risk level of $\alpha = 5\%$, the test power of $\beta = 10\%$ and under the conventional null hypothesis in the stochastic data screening techniques, the range of the bounding parameter in the modified BIBER-estimator coincides with the empirical range of the constant parameter in its original counterpart when the redundancy numbers are in the range of: $z_i \geq 0.6$. Moreover, this modification makes it possible to assign a certain probability to deformation parameters that are identified as the source of the change in deformations. With the new method and for the first time, not only the significance of the observed variations in deformation can be statistically analyzed but also a detailed evaluation of deformation changes will be possible. Due to the key role of Procrustes analysis in the devised method, here it has been given the name of Procrustean Statistical Inference of Deformations.

Procrustes analysis is a standard mathematical tool in the theory of statistical shape analysis. Nevertheless, the forwarded method for the statistical inference of deformation changes can also be used for the analysis of a tensor series whose tensor elements lack a geometrical implication.

Procrustean statistical inference of deformation tensors together with the forwarded approaches of this research sets out the framework of a comprehensive approach for analyzing the gradual accumulation of strain and/or stress. The individual elements of this comprehen-

sive approach and their interrelation have been discussed in the thesis. These elements and their interrelation have been summarized and illustrated in the form of a flow diagram in Appendix D for a time series of deformation tensors.

The method of this work has been implemented to analyze the kinematics of of deformation in the Kenai Peninsular as the test area. The analysis of discrete the Picard condition has proved the instability of the traditional least-squares solution for this problem. Therefore, regularized 3D-deformation tensors had to be estimated for the stations of the GPS network in this area. Sensitivity analysis of the 3D-deformation tensors showed that mainly the vertical parameters of strain are more sensitive to small perturbations of input parameters. Therefore, these parameters are expected to loose more resolution through the regularization process. Nevertheless, the conformance of the obtained pattern of vertical deformations to that of existing independent results proved that the estimated vertical parameters of deformation are not dominated by regularization errors. This also proves the efficiency of the method of this work in finding an optimum regularization parameter in the Truncated Singular Value Decomposition technique. Regularization error of the horizontal principal strains varies from 0.0050±0.2469 μ-strain on the parameter e_I in station T19D to 0.4490±0.3556 μ-strain on the parameter e_{II} in station C85G for the 3D-Isoparametric approach and from 0.0020±0.1989 μ-strain on the parameter e_I in station KIRT to 0.4470±0.1674 μ-strain on the parameter e_{II} in station DAHL for the 3D-Lagrangian approach. The significance of estimated regularization errors have been also analyzed using estimated variance-covariance matrices of 3D-deformation tensors. It is important to note that according to sensitivity analysis of deformation tensors the instability of the least-squares solution is due to discretizing the continuous deformations of the Earth by installing a specific set of survey-points on the Earth's surface.

Inhomogeneity of deformation has been treated by dividing the study area to separate partitions in each of which the deformation of the Earth can be taken to be homogeneous (Brunner, 1979; Dermains and Grafarend, 1993). Since deformation monitoring networks are normally sparse, this approach normally results in very small observational redundancies. As the result, a realistic error analysis of the estimated parameters of deformation often is impossible. Moreover, based on some physical evidences it should be necessarily possible to

justify the homogeneity of deformation in each partition too. In the approach of this work the Global Model Test (the hypothesis test of the a posteriori variance of unit weight) is used as a measure for the assessment of the adequacy of the homogeneous functional models of Chapter 3 for analyzing an inhomogeneous deformation field. The work can be further extended by implementing advanced functional models that are better adapted to the inhomogeneity of deformations. For example the domical pattern of deformation in the test area of this study can be used for enhancing the implemented functional models of this study.

The kinematic approach to the analysis of deformation also provides the required boundary conditions of the dynamic approach in which modeling of the mechanism which governs the deformation of an area is the main concern. Normally the mechanism is modeled using substances with uniform or composite properties like an elastic body or viscoelastic materials as well as some assumptions on the geometry of the model like a half-space model or a dislocation source of finite dimensions which has a definite location within the deformable medium. In the dynamics approach to the analysis of deformation, normally GPS derived velocity vectors are used as surface constraints for modeling the deformation of the Earth. The three-dimensional approaches of this study propose new surface constraints in form of 3D-deformation tensors which also allow modeling the deformation of the Earth. Estimated source parameters can be further studied to see if the new constraints can improve existing models.