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Vector mesons in meson-baryon scattering and large-N_c Quantum Chromodynamics

Vektormesonen in Meson-Baryon-Streuung und large-N_c-Quantenchromodynamik

Vom Fachbereich Physik
der Technischen Universität Darmstadt

zur Erlangung des Grades
eines Doktors der Naturwissenschaften (Dr. rer. nat.)

genehmigte Dissertation von
Hans-Friedrich Fuhrmann, M. Sc.
aus Frankfurt am Main

Referent: Apl. Prof. Dr. Matthias F. M. Lutz
Korreferent: Prof. Ph. D. Guy D. Moore

Tag der Einreichung: 25.01.2016
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Erklärung zur Dissertation

Hiermit versichere ich, die vorliegende Dissertation ohne Hilfe Dritter nur mit den angegebenen Quellen und Hilfsmitteln angefertigt zu haben. Alle Stellen, die aus Quellen entnommen wurden, sind als solche kenntlich gemacht. Diese Arbeit hat in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegen.

Darmstadt, den 21.09.2016

(Hans-Friedrich Fuhrmann)

Zusammenfassung

In dieser Doktorarbeit untersuchten wir starke Wechselwirkungen im Niederenergiebereich mit Hilfe zweier komplementärer, nichtstörungstheoretischer Ansätze: Dabei wurde das Wechselspiel zwischen der Quantenchromodynamik (QCD) im large-N_c-Bild und der chiralen Störungstheorie erforscht. Während die Entwicklung über den Parameter $1/N_c$ auf Quark- und Gluonfreiheitsgraden basiert, benutzt die chirale Störungstheorie Hadronen als effektive Freiheitsgrade. Das Hauptgewicht unserer Arbeit lag dabei auf der Erforschung der Mesonen und Baryonen, die sich aus up-, down- und strange-Quarks zusammensetzen. Wir benutzten dabei die chirale $SU(3)$-Lagrangedichte mit den ($J^P = \frac{1}{2}^+$)- und den ($J^P = \frac{3}{2}^+$)-Baryongrundzuständen als Bausteine. Im $SU(3)$-Flavourgrenzfall bilden diese Baryonzustände ein Oktett und entsprechend ein Dekuplett.

Untersuchungen in der chiralen Störungstheorie beinhalten eine Herausforderung: Die chirale Lagrangedichte umfaßt unendlich viele Beiträge. Die Behandlung der niederenergetischen Physik der Quantenchromodynamik mit Hilfe einer Störungstheorie erfordert die Anordnung dieser Beiträge nach ihrer Wichtigkeit. Wir benutzten das Wechselspiel zwischen der QCD und der chiralen Störungstheorie, um Aufschluß über die Struktur der chiralen Lagrangedichte zu geben. Im Grenzfall einer großen Farbanzahl N_c sind die Niedrigenergieparameter der chiralen Lagrangedichte miteinander verknüpft. Beispielhaft zeigt sich diese Verknüpfung darin, daß die Massen der beiden Baryonmultipletts im $SU(3)$-Flavourgrenzfall entartet sind. Diese Tatsache dient als Ausgangspunkt unserer Untersuchungen. In dieser Arbeit analysieren wir das zeitgeordnete Produkt zweier skalarer und zweier Vektorströme im Baryongrundzustand. Die Auswertung dieser Matrixelemente für $N_c \to \infty$ wurde mit entsprechenden Ergebnissen verglichen, die im Rahmen der chiralen Störungstheorie abgeleitet wurden. Daraus erhielten wir Summenregeln für einige Niederenergieparameter der chiralen Lagrangedichte. Diese Resultate für die Vektorkorrelationsfunktion wurden genutzt, um eine phänomenologische Wechselwirkung der leichten Vektormesonen mit den Baryongrundzuständen zu fixieren.

Im zweiten Teil dieser Doktorarbeit sprachen wir ein formales Problem, das bei der Zerlegung einer Partialwellenfunktion von Reaktionsamplituden für Teilchen mit nichtverschwindendem Spin auftritt, an. Dabei berücksichtigten wir im Besonderen die Vektormeson-Photoproduktion am Nukleon, wie sie gegenwärtig in [1–3] untersucht wird. Eine Zerlegung von Produktionsamplituden in kovariante Partialwellenamplituden, die sowohl frei von kinematischen Beschränkungen als auch mit der Mikrokausalitätsbedingung verträglich sind, wurde durchgeführt. Ein Mathematica-Code unter Zuhilfenahme des FeynCalc-Pakets [4] wurde geschrieben und auf einige Kontaktbeiträge und s-, u- und t-Kanalprozesse angewandt.

Contents

1 Introduction

Quantum Chromodynamics (QCD) is the accepted theory to describe the physics of strong interactions. The strong coupling constant $\alpha_S(Q)$, which depends on a momentum transfer Q, determines the strength of these interactions. The running of $\alpha_S(Q)$ can be extracted from empirical data:

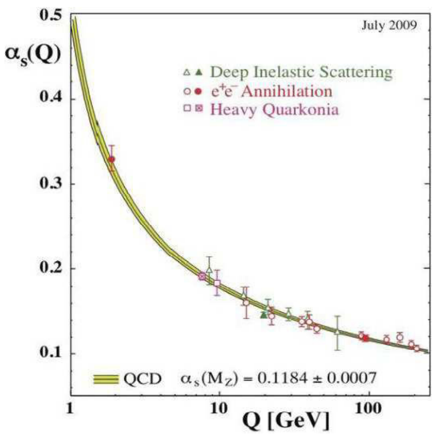


Figure 1.1.: Running of the strong coupling constant [5]

As shown in Fig. 1.1, the coupling constant is sufficiently small only for a large momentum transfer. Therefore, a strong interaction process cannot always be computed in perturbation theory. Low-energy processes like meson-baryon scattering require a non-perturbative treatment in QCD. One possible non-perturbative framework was introduced by 't Hooft [6]: he considered the number of colours N_c as a free parameter in QCD. Rather than expanding in the strong coupling constant α_S, he suggested an expansion with respect to the parameter $1/N_c$. This idea requires the replacement of the $SU(3)$ colour symmetry group by the $SU(N_c)$ group. 't Hooft found out that only planar Feynman diagrams with single gluons at the edges dominate the dynamics of QCD in this limit. There are resilient hints that the results of the $1/N_c$ expansion resembles the physical case $N_c = 3$.

A complementary approach to non-perturbative QCD at low-energies is chiral perturbation theory (χPT). Hadrons instead of quarks and gluons are identified as the relevant degrees of freedom in χPT. Effective field theories like χPT are based on the decoupling theorem [7, 8] and Wilson's renormalisation scheme for a quantum field theory [9]. In this work we consider the chiral $SU(3)$ Lagrangian constructed with the ($J^P = \frac{1}{2}^+$)- and the ($J^P = \frac{3}{2}^+$)-ground state baryons. These baryons form an octet and decuplet, respectively, in the $SU(3)$ flavour limit. The target of this Lagrangian is the low-energy physics of the up-, down- and strange quarks in QCD. The main ingredient of this effective field theory is the spontaneous breaking of the chiral symmetry of quarks. The Goldstone theorem [10, 11] predicts the formation of eight pseudoscalar bosons. The chiral Lagrangian consists of a hierarchy of interaction terms that are ordered according to their relevance. At a given order a number of low-energy parameters need to be determined either from experimental data or directly from QCD.

In this work we will correlate low-energy parameters of the chiral Lagrangian by scrutinising their expansion in $1/N_c$. A well known property of QCD at large-N_c is the degeneracy of the masses of the octet and decuplet ground state baryons in the $SU(3)$ flavour limit [6]. Further relations for the low-energy parameters are expected to hold and will be studied [12]. Parts of the chiral Lagrangian are scalar and vector external source fields which enable the computation of baryon matrix elements of these currents [13, 14]. Such matrix elements can also be studied at large-N_c QCD using the framework of [12, 15–18]. A comparison of the results provides the desired correlation of low-energy parameters. In particular we consider the time-ordered product of two scalar or two vector currents in the baryon ground states. The result from the study of the vector correlation function can also be used to correlate phenomenological interaction vertices of vector mesons with the baryon ground states. Such terms play an important role in coupled-channel dynamics of baryon resonances as for instance probed in photoproduction experiments [19].

In the second part of this thesis we address a formal problem that arises in the theoretical description of photoproduction experiments. A decomposition of reaction amplitudes into helicity-partial wave amplitudes [20] suffers from kinematical constraints that make the application of dispersion-integral representations cumbersome. Such constraints get more and more involved as the spin of the participating particles increases. In particular in the photoproduction of vector mesons this is a non-trivial issue. Several authors [21–24] developed a decomposition scheme of general on-shell scattering amplitudes into invariant amplitudes whose results are utilised to prepare covariant partial wave amplitudes compatible with coupled-channel unitarity and microcausality [25]. The on-shell scattering amplitudes are decomposed into sets of Lorentz-Dirac structures as basis vectors. The corresponding coefficients are analytic functions satisfying Mandelstam's dispersion integral representation [26]. The covariant partial wave amplitudes are constructed from those invariant amplitudes. The efficiency of the decomposition scheme for photoproduction of vector mesons is illustrated via four examples.

2 $1/N_c$ expansion

't Hooft proposed an approach to QCD [6] in which the number of colours N_c is assumed as a free parameter. The physical case is identified with $N_c = 3$. Feynman diagrams are examined in the large-N_c limit: diagrams with inner quark loops are highly suppressed in comparison to planar diagrams with single quarks at the edges of them. In this work we focus on properties of baryons in a large-N_c world. We apply a framework by Luty and March-Russell [17] supplemented by the operator reduction formalism of Dashen, Jenkins and Manohar [12, 18]. Our goal is to derive sum rules for the low-energy parameters of the chiral Lagrangian as will be introduced in chapter 3.

2.1 Introduction to the $1/N_c$ expansion

The key point is that the $1/N_c$ expansion arranges the Feynman diagrams in such a way that additional insight into the dynamics of strong interactions is obtained. Some experimental evidence supports the usefulness of the large-N_c limit: this framework predicts that internal gluon loops dominate over internal quark loops (see section 2.2). For instance, the meson-meson interaction is of the order $\mathcal{O}\left(1/N_c\right)$. As a consequence all meson states do not decay into hadrons in that limit $N_c \to \infty$. The phenomenological rule by Okubo, Zweig and Iizuka (OZI) [27–29] can be consistently explained by the $1/N_c$ expansion [30]. 't Hooft derived in [6] that planar diagrams with only quark loops at the edges of the diagrams dominate diagrams with gluonic exchanges.

2.2 The order of Feynman diagrams in terms of $1/N_c$

To explain 't Hooft's insight, we must determine a Feynman diagram's order in terms of $1/N_c$. The determination of the order is illustrated by two examples, a gluon propagator with a gluon loop,

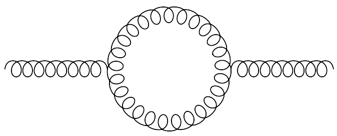


Figure 2.1.: Gluon propagator with gluon loop

and a gluon propagator with a quark loop:

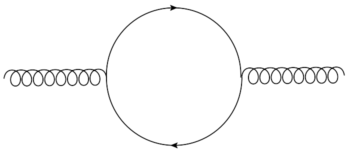


Figure 2.2.: Gluon propagator with quark loop

First, we recall the *double line notation* [30]. Each colour index of the corresponding Lagrangian leads to a solid line. A gluon is treated as a quark-antiquark $(q\bar{q})$ pair with respect to colour:

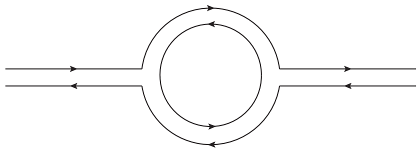


Figure 2.3.: Gluon propagator with gluon loop (double line notation)

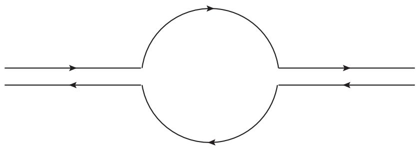


Figure 2.4.: Gluon propagator with quark loop (double line notation)

Second, we have to determine the overall N_c-factor for a Feynman diagram. Each vertex contributes by a factor of $g_s(Q)$ of the strong coupling constant. Each closed colour loop adds a factor of N_c. In the last step, we multiply as many factors $1/N_c$ to our order factor until we keep the product $g_s^2(Q)N_c$ constant. This procedure is equivalent to a rescaling of the strong coupling constant like

$$g_s(Q) \to \frac{g_s(Q)}{\sqrt{N_c}}. \tag{2.1}$$

We return to our former examples: in the case of the gluon propagator and the gluon loop, we count two vertices and one closed colour loop. This leads us to a contribution of order $1/N_c^0 = 1$:

$$g_s^2(Q)N_c = \left(g_s^2(Q)N_c\right)\frac{1}{N_c^0} \sim \mathcal{O}\left(\frac{1}{N_c^0} = 1\right). \tag{2.2}$$

In contrast to the first example, the gluon propagator with a quark loop does not contain of any closed colour loop. We conclude that it is of order $1/N_c$:

$$g_s^2(Q) = \left(g_s^2(Q)N_c\right)\frac{1}{N_c} \sim \mathcal{O}\left(\frac{1}{N_c}\right). \tag{2.3}$$

We ask for the purpose of this expansion.'t Hooft announced in his introductory article in 1974 [6] that only planar Feynman diagrams with a single quark at the edges of the diagram are relevant in first order. Planar diagrams are diagrams without any crossing propagators except at vertices. We finally conclude that Feynman diagrams with internal quark loops are suppressed in the large-N_c limit.

2.3 Inclusion of baryons

A baryon is a particle which consists of three quarks at least. If we treat the number of colours N_c as a free parameter, it carries N_c quarks. In contrast to mesons, combinatorial factors will arise if we describe a certain Feynman diagram in the $1/N_c$ expansion. Witten [30] suggested to proceed in two steps. First, he determined the interaction of quarks in the large-N_c limit. Then he combined this interaction with many-body techniques to investigate N_c-body states. The most prominent result of such an analysis is the mass degeneracy of the baryon ground states with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ quantum numbers. For instance, back at $N_c = 3$ one expects similar masses for the nucleon and the Δ resonance.

We will follow a more recent formalism by [17, 18]. A scalar mean field is assumed that will localise three bare quarks into a finite region in space. The QCD-Hamiltonian H_{QCD} is reorganised into a mean-field part H_0 and into an interaction part V:

$$H_{\text{QCD}} = H_0 + V. \tag{2.4}$$

It is then easier to study baryon matrix elements at large N_c. To do so it is necessary to construct the states of lowest energy in the scalar potential wall, that carry quantum numbers of the baryons with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, i.e. with angular momentum $L = 0$. The properties of those states will be correlated by the large-N_c analysis. It is emphasised that the mean field is needed for bookkeeping purposes only.
It is important to construct the spin-flavour structure of the baryon ground state $|\mathscr{B}_0\rangle$:

$$|\mathscr{B}_0\rangle = \mathscr{B}^{s_1 f_1 \dots s_{N_c} f_{N_c}} \epsilon^{c_1 \dots c_{N_c}} a^\dagger_{s_1 f_1 c_1} \cdots a^\dagger_{s_{N_c} f_{N_c} c_{N_c}} |0\rangle. \tag{2.5}$$

A tensor product of creation operators $a^\dagger$ with the definite spin index $s_i \in \{1, 2\}$, the flavour index $f_i \in \{1, \dots, N_F\}$ and the colour index $c_i \in \{1, \dots, N_c\}$ acts on the perturbative vacuum $|0\rangle$, the ground state of H_0. The baryon tensor $\mathscr{B}$ will provide the one-baryon state with a fixed angular momentum L and flavour quantum numbers. The creation operators $a^\dagger$ are directly related to the eigenmode expansion of the "free" field operators $\psi_I(t, \vec{x})$ in the interaction picture [18]:

$$\psi_I(t, \vec{x}) = \sum_{i=0}^{\infty} \sum_{s=1}^{2} \left(u_{i,s}(\vec{x}) e^{-iE_i t} a_{i,s} + v_{i,s}(\vec{x}) e^{iE_i t} d^\dagger_{i,s} \right), \tag{2.6}$$

where we identify $a^\dagger$ with $a^\dagger_{i=0}$.
We proceed with the time evolution of these baryon ground states as determined in the interaction picture. The unitary transformation between Heisenberg and interaction picture is given in the following way:

$$\begin{aligned} \psi(t, \vec{x}) &= U^\dagger(t) \psi_I(t, \vec{x}) U(t), \\ U(t) &= e^{iH_0 t} e^{-iH_{\text{QCD}} t} = \mathscr{T} \exp \left[-i \int_0^t \mathrm{d}t' \, \underbrace{e^{iH_0 t'} V e^{-iH_0 t'}}_{=V_I(t')} \right]. \end{aligned} \tag{2.7}$$

The baryon ground states evolve to the eigenstates of the QCD-Hamiltonian H_{QCD} with a non-zero overlap between $|\mathscr{B}_0\rangle$ and $|\mathscr{B}\rangle$ $(t \to (1 - i\epsilon)\infty)$:

$$e^{-iH_{\text{QCD}}} |\mathscr{B}_0\rangle = \langle \mathscr{B} | \mathscr{B}_0 \rangle e^{-iE_{\mathscr{B}} t} |\mathscr{B}\rangle + \dots \tag{2.8}$$

These states $|\mathscr{B}\rangle$ have the same quantum numbers like $|\mathscr{B}_0\rangle$. The states $|\mathscr{B}\rangle$ are the eigenstates of the complete Hamiltonian H_{QCD}.
The previous discussion enables us to calculate important baryon properties. We commence with the matrix elements of an arbitrary time-ordered product of space-time dependent operators $\hat{\mathscr{J}}_i(x_i)$ in the Heisenberg picture:

$$\langle \mathscr{B}'|\mathscr{T}\hat{\mathscr{J}}_1(x_1)\cdots\hat{\mathscr{J}}_m(x_m)|\mathscr{B}\rangle. \tag{2.9}$$

The corresponding matrix elements for the states $|\hat{\mathscr{B}}\rangle$ are traced back to matrix elements of the unperturbed ground states $|\hat{\mathscr{B}}_0\rangle$ in the interaction picture with the help of the expression U_I:

$$\langle \mathscr{B}'|\mathscr{T}\hat{\mathscr{J}}_1(x_1)\cdots\hat{\mathscr{J}}_m(x_m)|\mathscr{B}\rangle = \frac{\langle \mathscr{B}_0'|\mathscr{T}\hat{\mathscr{J}}_{I1}(x_1)\cdots\hat{\mathscr{J}}_{Im}(x_m)U_I|\mathscr{B}_0\rangle}{\sqrt{\langle \mathscr{B}_0'|U_I|\mathscr{B}_0'\rangle}\sqrt{\mathscr{B}_0|U_I|\mathscr{B}_0\rangle}}, \tag{2.10}$$

$$U_I = \mathscr{T}\exp\left[-i\int_{-T}^{+T}\mathrm{d}t'\; e^{iH_0t'}Ve^{-iH_0t'}\right], \tag{2.11}$$

$$U(-T)|\mathscr{B}\rangle = \frac{|\mathscr{B}_0\rangle}{\langle \mathscr{B}_0|U_I|\mathscr{B}_0\rangle^{\frac{1}{2}}} + \ldots \tag{2.12}$$

Here, the last equation refers to the unitary transformation of Eq. (2.7). The numerator of the right-hand side of the first equation consists of the sum of all possible diagrams with operators $\hat{\mathscr{J}}$ in the interaction picture. The denominator is constructed by the sum of all diagrams with combinatorial factors which are derived by the expansions of both $\sqrt{\langle \mathscr{B}_0'|U_I|\mathscr{B}_0'\rangle}$ and $\sqrt{\langle \mathscr{B}_0|U_I|\mathscr{B}_0\rangle}$. We illustrate the diagrammatic expansion by the expression $\bar{\psi}(t,\vec{x})\Gamma\psi(t,\vec{x})$ for a one-quark operator. The quantity Γ is the substitution for a general Dirac and flavour matrix. The calculation of the matrix elements of $\bar{\psi}(t,\vec{x})\Gamma\psi(t,\vec{x})$ can be illustrated by the placement of this operator on each of the N_c quark lines [16]. Dashen, Jenkins and Manohar [16] pointed out that all diagrams with an arbitrary number of planar gluons have to be taken into account at a given order $\mathscr{O}\left(1/N_c^n\right)$.

The decomposition of $\bar{\psi}(t,\vec{x})\Gamma\psi(t,\vec{x})$ into a sum of products of n-body operators $\mathscr{O}^{(n)}$, coefficients $c_k^{(n)}$ and a factor of $1/N_c^{n-1}$ reads [16]:

$$\bar{\psi}(t,\vec{x})\Gamma\psi(t,\vec{x}) = \sum_{n=0}^{N_c}\sum_k c_k^{(n)}\frac{1}{N_c^{n-1}}\hat{\mathscr{O}}_k^{(n)}. \tag{2.13}$$

The identity holds in the one-baryon states of Eq. (2.5). An operator $\hat{\mathscr{O}}_k^{(n)}$ consists of a normal-ordered product of $2n$ one-body operators $a^\dagger$ and a as introduced in Eq. (2.5). The number of $a^\dagger$ and a involved is equal. The higher eigenmode operators a_i of Eq. (2.6) with $i > 0$ can contribute to the scalar coefficients $c_k^{(n)}$ only. At finite N_c, a finite number of operators contribute only. In general for a given index n there are various operators possible. The index k runs through all possible spin and flavour combinations. The Feynman diagrams with a specific one-quark QCD operator and planar gluons illustrate the above decomposition scheme: each planar gluon adds a two-body operator with an additional factor $1/N_c$ to the decomposition.
The result (2.13) is easily generalised for the case of time-ordered products of current operators. The

combination of numerator and denominator in Eq. (2.10) leads to the following diagrammatic expansion:

$$\langle \mathscr{B}'|\mathscr{T}\hat{\mathscr{J}}_1(x_1)\cdots\hat{\mathscr{J}}_m(x_m)|\mathscr{B}\rangle = \sum_{n=1}^{N_c}\sum_k F_n^k(x_1,\ldots,x_m)\langle\mathscr{B}_0'|\hat{\mathscr{O}}_k^{(n)}|\mathscr{B}_0\rangle. \tag{2.14}$$

Here, the index N_c runs from 1 to N_c in contrast to Eq. (2.13) because we will not consider the effect of zero-body operators in our further calculations. The dependence on the spatial coordinates of the operators is explicitly stated. The coefficients F_n^k include the complete kinematical information. The second factor of this form factor is a matrix element of an n-body operator which is independent of any kinematics.

In principal the functions F_n^k can be determined from quark-gluon diagrams in QCD. Only connected diagrams contribute. Unfortunately, they are difficult to compute explicitly. Without explicit computations the large-N_c scaling of the functions F_n^k can be determined [17]. This goes in two steps: every Heisenberg operator needs at least $(n-1)$ quark-gluon exchanges, so that F_n^k is at most of order $1/N_c^{n-1}$. In addition the minimal number of quark loops, L, that are needed to form the matrix element enters. We remember 't Hooft's article [6] in which he concluded that every internal quark loop is suppressed by $1/N_c$. Thus it holds:

$$F_n^k \lesssim \frac{1}{N_c^{n-1+L}}. \tag{2.15}$$

We come back to the diagrammatic expansion in Eq. (2.14). In the following we study the general structure of the n-body operators $\hat{\mathscr{O}}_k^{(n)}$ by applying a framework developed by Dashen, Jenkins and Manohar [16]. While the large-N_c scaling of a normal-ordered n-body operators is known to be $1/N_c^n$, this is more difficult to determine for products of two normal-ordered operators. On the other hand for the evaluation of matrix elements the former are more convenient, in particular for spin and flavour properties. Therefore it is useful to work out the scaling behaviour of such products. The quintessence of [16] is that the $\hat{\mathscr{O}}_k^{(n)}$-operators can be expressed as sums over products of one-body operators. We note that such products are not normal ordered a priori. Only after a suitable linear combination normal ordering can be achieved and therewith a scaling power can be assigned to such a term. The scaling power N_c^{-n+1} holds only for a normal ordered n-body operator.

In order to facilitate such an analysis it is useful to introduce effective boson-type operators q and $q^\dagger$ that do not carry colour quantum numbers. Since the baryon state is totally antisymmetric and a colour singlet, the spin-flavour part of the wave-function must be totally symmetric. The latter will be generated by spin-flavour operators q_{sf} and $q_{sf}^\dagger$ with spin s and flavour f indices. These operators act on the vacuum $|0)$ of the spin-flavour Fock space. In contrast to the operators $\hat{\mathscr{O}}_k^{(n)}$ and the baryon state $|\mathscr{B}_0\rangle$, which are expressed in terms of the operators a and $a^\dagger$, the effective n-body operators $\mathscr{O}_k^{(n)}$ and the effective baryon state $|\mathscr{B})$ are expressed in terms of the bosonic operators q and $q^\dagger$. It holds:

$$|\mathscr{B}) = \mathscr{B}^{s_1 f_1 \ldots s_{N_c} f_{N_c}} q_{s_1 f_1}^\dagger \cdots q_{s_{N_c} f_{N_c}}^\dagger |0), \tag{2.16}$$

$$\langle\mathscr{B}_0'|\hat{\mathscr{O}}_k^{(n)}|\mathscr{B}_0\rangle = (\mathscr{B}'|\mathscr{O}_k^{(n)}|\mathscr{B}). \tag{2.17}$$

$$\left[q_{s_i f_i}, q_{s_j f_j}^\dagger\right] = \delta_{s_i s_j}\delta_{f_i f_j}, \quad \left[q_{s_i f_i}, q_{s_j f_j}\right] = \left[q_{s_i f_i}^\dagger, q_{s_j f_j}^\dagger\right] = 0. \tag{2.18}$$

We summarise the substitution rules from the fermionic to the effective bosonic quark operators:

$$
\begin{aligned}
&a_{s_i f_i c_i} \to q_{s_i f_i}, \qquad a^\dagger_{s_j f_j c_j} \to q^\dagger_{s_j f_j}, \qquad |\,\rangle \to |\,), \qquad \hat{\mathcal{O}}^{(n)} \to \mathcal{O}^{(n)}, \\
&\mathcal{O}^{(n)} = q^\dagger_{s_{j,1} f_{j,1}} \cdots q^\dagger_{s_{j,n} f_{j,n}} \mathcal{O}^{(s_{j,1} f_{j,1} \ldots s_{j,n} f_{j,n})}_{(s_{i,1} f_{i,1} \ldots s_{i,n} f_{i,n})} q_{s_{i,1} f_{i,1}} \cdots q_{s_{i,n} f_{i,n}}.
\end{aligned}
\tag{2.19}
$$

2.4 The spin-flavour structure of baryons in the $1/N_c$ expansion

The last section concludes with the diagrammatic expansion of the time-ordered product of QCD currents, properly expanded in powers of $1/N_c$. Further progress requires a systematic analysis of normal ordered n-body operators constructed in terms of effective boson-type operators q and $q^\dagger$. To investigate the spin-flavour structure of such n-body operators it is instrumental to construct irreducible representations of the contracted $SU(2N_F)$ spin-flavour symmetry group [16]. They lead to a set of operators identities, which hold in matrix elements of the baryon states. Dashen, Jenkins and Manohar [16] developed an approach which includes an arbitrary number of flavours and colours in the quark representation. It is based on the boson commutation algebra (Eq. (2.18)) with quark annihilation and creation operators. The classification of the quark operators is done with respect to the number of these single annihilation and creation operators. The following discussion focuses on the case $N_F = 3$ because the investigations of baryons in this thesis only cover particles with up-, down- and strange quarks. We make use of a basis in the decomposed representation $SU(2) \times SU(3)$ of the contracted $SU(6)$ group and introduce the unity operator $\mathbb{1}$, the spin generators J^i, the flavour generators T^a and the spin-flavour generators G^a_i [16]:

$$
\begin{aligned}
\mathbb{1} &= q^\dagger (\mathbf{1} \otimes \mathbf{1}) q, & J^i &= q^\dagger \left(\frac{\sigma^{(i)}}{2} \otimes \mathbf{1} \right) q, \\
T^a &= q^\dagger \left(\mathbf{1} \otimes \frac{\lambda^{(a)}}{2} \right) q, & G^a_i &= q^\dagger \left(\frac{\sigma^{(i)}}{2} \otimes \frac{\lambda^{(a)}}{2} \right) q.
\end{aligned}
\tag{2.20}
$$

Here, the Pauli matrices $\sigma^{(i)}$ operate in the spin space and are explicitly defined in section A.3. The Gell-Mann matrices $\lambda^{(a)}$ as generators of the $SU(3)$ flavour symmetry group are specified in Eq. (A.19). The transformation properties of $\mathbb{1}$, J^i, T^a and G^a_i under the $SU(2) \times SU(3)$ spin-flavour symmetry group are given by [16]:

$$
\begin{aligned}
\mathbb{1} &\to (0,0), & J^i &\to (1,0), \\
T^a &\to (0,\mathrm{adj}), & G^a_i &\to (1,\mathrm{adj}).
\end{aligned}
\tag{2.21}
$$

The expression "adj" refers to the adjoint representation of the $SU(3)$ group.

We return at this point of the discussion to the quark operator identities which were mentioned in section 2.3. As already mentioned, Dashen, Jenkins and Manohar [16] found out that we will only have to decompose two-body operators into one- and zero-body operators to obtain the complete set of independent operator identities.

The only operator which is classified into the group of zero-body operators is the unity operator $\mathbb{1} \sim \mathcal{O}\left(1/N_c^0\right) = \mathcal{O}(1)$. Hence no operator identities are obtained in this case. The unity operator transforms as a singlet under the $SU(2) \times SU(3)$ group.

The category of one-body operators only consists of the quark number operator $q^\dagger q$. Dashen, Jenkins and Manohar derived the following operator identity [16]:

$$q^\dagger q = N_c \, \mathbb{1}. \tag{2.22}$$

The operator $q^\dagger q$ transforms as a tensor product of a quark and an antiquark representation under the $SU(6)$ group.

Every two-body operator shows a transformation pattern as a tensor product of a two-quark and a two-antiquark state under the $SU(6)$ group. Using the operator identity in Eq. (2.22), only bilinear operators $q^\dagger \Lambda^A q$ with $\Lambda^A = J^i, T^a, G_i^a$ contribute to the decomposition of two-body operators. Every product of two operators A and B can be expressed by a commutator and an anticommutator:

$$AB = \frac{1}{2}\left([A,B] + \{A,B\}\right). \tag{2.23}$$

The commutator is simplified via the $SU(6)$ commutation relations [16]:

$$\begin{aligned}
&[J_i, J_j] = i\epsilon_{ijk}J_k, \quad [T_a, T_b] = if_{abc}T_c, \quad [J_i, T_a] = 0, \\
&[J_i, G_{ja}] = i\epsilon_{ijk}G_{ka}, \qquad\qquad\qquad\quad [T_a, G_{ib}] = if_{abc}G_{ic}, \\
&[G_{ia}, G_{jb}] = \frac{i}{4}\delta_{ij}f_{abc}T_c + \frac{i}{6}\delta_{ab}\epsilon_{ijk}J_k + \frac{i}{2}\epsilon_{ijk}d_{abc}G_{kc}.
\end{aligned} \tag{2.24}$$

These relations can be concluded directly by the application of Eq. (2.18) to the operator basis in Eq. (2.20). On the other hand, one can distinguish between three different kinds of two-body operator identities for the anticommutator [16]. We state the complete set of the $SU(2) \times SU(3)$ two-body identities in the appendix C.

The study of the $1/N_c$ expansion for the baryon ground states by Dashen, Jenkins and Manohar [16] provides us with the following insights: the baryon ground states have the same mass at the order N_c and are combined to irreducible representations of the contracted $SU(6)$ group. Hence, there are $1/N_c$ corrections which are proportional to J^2 and destroy the degeneracy. Here, J refers to the one-body operator of the basis in Eq. (2.20). These results help us to describe the baryon mass spectrum and to determine the area of application of the $1/N_c$ expansion: the degenerate $SU(6)$ baryon multiplet, that is of order N_c, consists of N_c spin states. If we order these spin states from the bottom to the top with increasing spin, we obtain a mass spectrum with corrections at the top of the order $\mathcal{O}\left(1/N_c^0\right) = \mathcal{O}(1)$ and corrections at the bottom of the order $\mathcal{O}\left(1/N_c\right)$. Additional corrections from the bottom to the top of the spectrum are of the order $\mathcal{O}(N_c)$ [16]. These observations lead us to the conclusion that the $1/N_c$ expansion is only applicable to baryon ground states with a low spin value $S_p \in \left\{\frac{1}{2}, \ldots, \frac{N_c}{2}\right\}$.

We turn to the $1/N_c$ expansion of a general n-body operator as on the right-hand side of Eq. (2.14) in combination with Eq. (2.16). Our operator basis (Eq. (2.20)) enables us to decompose our n-body operators in polynomials with the components c_{lm}:

$$\mathcal{O}^{(n)} = \sum_m \sum_l c_{lm}(J^i)^m (T^a)^l (G_j^a)^{n-m-l}, \quad m, l \in \mathbb{N}. \tag{2.25}$$

The usage of some of the already discussed operator identities (see appendix C) for the three-flavour case simplifies this decomposition. The application of the relevant operator identities make it possible to formulate an operator reduction rule [16]:
"All operator products in which two flavour indices are contracted using δ_{ab}, d_{abc} or f_{abc} or two spin indices on G's are contracted using δ_{ij} or ϵ_{ijk} can be eliminated."

This operator reduction rule will be applied during the derivation of sum rules for the chiral $SU(3)$ Lagrangian in the following sections. The main idea behind these sum rules is the following: the last sections enabled us to derive a $1/N_c$ expansion of matrix elements of quark operators. On the other hand, we will introduce χPT as a complementary framework to QCD later in this work. Effective field theories such like χPT can be used to determine matrix elements of spin-flavour quark operators. Thus, it is possible to connect our results of the $1/N_c$ expansion with those of chiral perturbation theory. We will derive sum rules for one scalar current, two scalar currents and the two vector currents by a large-N_c operator analysis in combination with the chiral $SU(3)$ Lagrangian in the sections 3.3, 3.4, and 4.3, respectively. Now, the previous analysis is used to derive correlations between the low-energy parameters of the chiral and the $1/N_c$ expansion. Sum rules according to the spin-structure of the effective operators will be deduced via these correlations in the second step. Therefore, we calculate the matrix elements of our spin-flavour quark operators $\mathcal{O}^{(n)}$ both in the $1/N_c$- and in the chiral expansion. The coefficients of equivalent quark operators will be matched in a second step. To connect both kinds of expansion, we determine the action of the basis operators J, T and G from (2.20) on the baryon states $|\mathcal{B})$ in the Fock space. The spin-flavour states for $(J^P = \frac{1}{2}^+)$-baryons are denoted by

$$|p, a, \chi), \tag{2.26}$$

while the decuplet baryon states read

$$|p, klm, \chi). \tag{2.27}$$

The four-momentum of the initial baryon states is denoted by p. The three flavour indices $k, l, m \in \{1, 2, 3\}$ coincide with those in our representation of the $(J^P = \frac{3}{2}^+)$-baryons in Eq. (3.13). Due to the $SU(3)$ flavour symmetry, the flavour index a equals a natural number from 1 to 8. The projection of the baryon's spin χ on the z-direction of the spin space consists of two possible values ($\chi_{1/2} \in \{-\frac{1}{2}, \frac{1}{2}\}$) and of 4 possible values ($\chi_{3/2} \in \{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\}$) for the octet and decuplet baryons, respectively. Hence, $\chi_{1/2} = 1, 2$ and $\chi_{3/2} = 1, 2, 3, 4$. The sought actions obey the following relations [21, 31]:

$$\begin{aligned}
\mathbb{1}|p, a, \chi) &= 3|p, a, \chi), \qquad & \mathbb{1}|p, klm, \chi) &= 3|p, klm, \chi), \\
J_i|p, a, \chi) &= \frac{1}{2}\sigma^{(i)}_{\bar{\chi}\chi}|p, a, \bar{\chi}), \qquad & J_i|p, klm, \chi) &= \frac{3}{2}\left(\vec{S}\sigma_i\vec{S}^\dagger\right)_{\bar{\chi}\chi}|p, klm, \bar{\chi}), \\
T^a|p, b, \chi) &= i\, f_{bca}|p, c, \chi), \qquad & T^a|p, klm, \chi) &= \frac{3}{2}\Lambda^{a,nop}_{klm}|p, nop, \chi),
\end{aligned}$$
$$\begin{aligned}
G^a_i|p, b, \chi) &= \sigma^{(i)}_{\bar{\chi}\chi}\left(\frac{1}{2}d_{bca} + \frac{i}{3}f_{bca}\right)|p, c, \bar{\chi}) + \frac{1}{2\sqrt{2}}S^{(i)}_{\bar{\chi}\chi}\Lambda^{klm}_{ab}|p, klm, \bar{\chi}), \\
G^a_i|p, klm, \chi) &= \frac{3}{4}\left(\vec{S}\sigma_i\vec{S}^\dagger\right)_{\bar{\chi}\chi}\Lambda^{a,nop}_{klm}|p, nop, \bar{\chi}) + \frac{1}{2\sqrt{2}}\left(S^\dagger_i\right)_{\bar{\chi}\chi}\Lambda^{ab}_{klm}|p, b, \bar{\chi}).
\end{aligned} \tag{2.28}$$

The explicit representations of the Pauli matrices $\sigma^{(i)}$ of Eq. (A.3) and the "spin-$\frac{1}{2}$-to-spin-$\frac{3}{2}$"-transition matrices of Eq. (B.6) are used to extract the relevant matrix elements. We make use of the totally symmetric and antisymmetric tensors d_{abc} and f_{abc}, respectively, which are introduced in the Eqs. (A.27) and (A.26) in the appendix. Some often recurring combinations of the matrix elements of the Kronecker delta δ_{ij}, the Gell-Mann matrices $\lambda^{(a)}$ and the Levi-Civita tensor ϵ_{ijk} are given by

$$\begin{aligned} \delta^{nop}_{klm} &= \left(\delta_{kn}\delta_{lo}\delta_{mp}\right)_{\mathrm{sym}(klm)} = \left(\delta_{kn}\delta_{lo}\delta_{mp}\right)^{\mathrm{sym}(nop)}, \quad & \Lambda^{klm}_{ab} &= \left(\epsilon_{ijk}\lambda^{(a)}_{li}\lambda^{(b)}_{mj}\right)_{\mathrm{sym}(klm)}, \\ \Lambda^{a,nop}_{klm} &= \left(\lambda^{(a)}_{nk}\delta_{ol}\delta_{pm}\right)_{\mathrm{sym}(klm)}, & \Lambda^{ab}_{klm} &= \left(\epsilon_{ijk}\lambda^{(a)}_{il}\lambda^{(b)}_{jm}\right)_{\mathrm{sym}(klm)}. \end{aligned} \tag{2.29}$$

We will introduce the ($J^P = \frac{3}{2}^+$)-states of the baryon decuplet in section 3.1 of this thesis. All symmetric permutations of three particular flavour indices k, l, m refer to the same baryon state in this representation. Hence, the appropriate treatment of these baryons in a large-N_c operator analysis requires the symmetrisation of the quantities in Eq. (2.29) with respect to k, l, m. The symmetrisation of a general expression A is defined by

$$\left(A_{klm}\right)_{\mathrm{sym}(klm)} := \frac{1}{6}\left(A_{klm} + A_{lmk} + A_{mkl} + A_{lkm} + A_{mlk} + A_{kml}\right). \tag{2.30}$$

Our examination of the $1/N_c$ expansion of the baryon matrix elements (Eq. (2.16)) will require the symmetrisation with respect to both the initial and the final baryon state with the flavour indices k, l, m and n, o, p, respectively. To be able to use the transparent notation from (2.29) throughout the entire analysis we express those combinations with two sets of flavour indices by

$$\begin{aligned} \left(\delta^{nop}_{klm}\right)^{\mathrm{sym}(nop)}_{\mathrm{sym}(klm)} &= \delta^{nop}_{xyz}\delta^{xyz}_{klm} = \delta^{nop}_{klm}, \\ \left(\Lambda^{a,nop}_{klm}\right)^{\mathrm{sym}(nop)}_{\mathrm{sym}(klm)} &= \delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm} \end{aligned} \tag{2.31}$$

where we add up over all possible internal flavour indices $x, y, z \in \{1, 2, 3\}$ according to the Einstein convention. The derivation of baryon matrix elements of one scalar current, two scalar currents and two vector currents in the three sections 2.6, 2.7 and 2.8 requires the introduction of external fields. Therefore we present the approach of external fields by Gasser and Leutwyler in the next section of this chapter.

2.5 Approach of external fields

We traditionally investigate scattering processes and related amplitudes with the help of Green functions. Green functions are the vacuum expectation values of time-ordered products of operators. Lehmann, Symanzik and Zimmermann derived how scattering amplitudes are connected to Green functions according to the Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism [32]. Here, we will present the approach of external fields by Gasser und Leutwyler [13, 14]. Both authors introduce the coupling to external fields. They distinguish between a vector current $v_\mu(x)$, an axial vector current $a_\mu(x)$, a scalar density $s(x)$ and a pseudoscalar density $p(x)$. We indicate that we will denote all four kinds of external fields by the term "current" in this thesis for simplification purposes. It is possible to calculate Green functions which are related to vector, axial vector, scalar and pseudoscalar quark currents by functional differentiation of the generating functional

$$Z(v,a,s,p) = \langle 0_{\mathrm{out}}|0_{\mathrm{in}}\rangle_{v,a,s,p} = \langle 0|\mathcal{T}\exp\left(i\int \mathrm{d}^4x\ \left(\mathscr{L}_{\mathrm{QCD},0}(x)+\mathscr{L}_{\mathrm{ext}}(x)\right)\right)|0\rangle. \tag{2.32}$$

The generating functional $Z(v,a,s,p)$ depends on the previously mentioned external fields. We denote the vacuum-to-vacuum transition amplitude in the presence of external fields by $\langle 0_{\mathrm{out}}|0_{\mathrm{in}}\rangle_{v,a,s,p}$. The QCD-Lagrangian $\mathscr{L}_{\mathrm{QCD},0}(x)$ in the chiral limit is defined via the general QCD-Lagrangian

$$\begin{aligned}
\mathscr{L}_{\mathrm{QCD}}(x) &= \bar{q}(x)\left(i\gamma^{\mu}D_{\mu}(x)-M_q\right)q(x)-\frac{1}{4}G_{\mu\nu,a}(x)G^{\mu\nu,a}(x),\\
D_{\mu}(x) &= \partial_{\mu}-g_S G_{\mu}^{a}(x)\frac{\lambda_a}{2},\\
M_q &= \mathrm{diag}(m_u,m_d,m_s),\\
G_{\mu\nu,a}(x) &= \partial_{\mu}G_{\nu,a}(x)-\partial_{\nu}G_{\mu,a}(x)+g_S f_{abc}G_{\mu}^{b}(x)G_{\nu}^{c}(x).
\end{aligned} \tag{2.33}$$

We remind the reader that the investigations in this thesis only consider u-, d- and s-quarks. Therefore, $\bar{q}(x)$ and $q(x)$ consist of only three flavour components while the mass matrix M_q is a 3×3-matrix in our work. $\mathscr{L}_{\mathrm{QCD}}$ contains the quark field operator $q(x)$ and the adjoint quark field operator $\bar{q}(x)$. Both quantities are vectors with six components related to the quark flavours up (u), down (d), strange (s), charm (c), botton (b) and top (t). The introduction of the covariant derivative D_{μ} requires the strong coupling strength g_S. It is related to the strong coupling constant α_S of Fig. 1.1 via $\alpha_S(Q)=g_S^2(Q)/(4\pi)$. The fields of the gluons $G_{\mu}^{a}(x)$ depend on the colour index $a\in\{1,\ldots,8\}$. We also denote the Gell-Mann matrices λ_a of the $SU(3)$ group (Appendix A.6) with such an index a. Each matrix element m_i is identified with the current quark mass of the corresponding quark flavour i. $G_{\mu\nu,a}(x)$ are the eight gluon tensors. The totally antisymmetric structure constant of the $SU(3)$ group f_{abc} is defined in Eq. (A.26). Then, the QCD-Lagrangian $\mathscr{L}_{\mathrm{QCD},0}$ in the chiral limit is related to the former results such as follows:

$$\mathscr{L}_{\mathrm{QCD},0}(x) = \mathscr{L}_{\mathrm{QCD}}(x)\Big|_{M_q=0}. \tag{2.34}$$

The external Lagrangian $\mathscr{L}_{\mathrm{ext}}(x)$ may be expressed by [14]

$$\mathscr{L}_{\mathrm{ext}}(x) = \bar{q}(x)\gamma^{\mu}(v_{\mu}(x)+\gamma_5 a_{\mu}(x))q(x)+\bar{q}(x)(s(x)-i\gamma_5 p(x))q(x). \tag{2.35}$$

The Dirac matrices γ_5 and γ^{μ} are defined in the appendix A.5.
The chiral $SU(3)$ symmetry is explicitly broken via the quantity [33,34]

$$\chi_{\pm}(x) = \frac{1}{2}u(x)\chi_0(x)u(x)\pm\frac{1}{2}u^{\dagger}(x)\chi_0(x)u^{\dagger}(x). \tag{2.36}$$

We will discuss the non-linear parametrisation of the pseudoscalar mesons via the quantity $u(x)$ in section 3.1. We expand the objects $u(x)$ and $u^\dagger(x)$ of section 3.1 with respect to the pseudoscalar meson fields $\Phi(x)$ (Eq. (3.7)) and receive

$$\chi_+(x) = \chi_0(x) - \frac{1}{8f^2}\left\{\Phi(x), \left\{\Phi(x), \chi_0(x)\right\}\right\} + \mathcal{O}(\Phi^4(x)), \tag{2.37}$$

$$\chi_-(x) = \frac{i}{2f}\left\{\chi_0(x), \Phi(x)\right\} + \mathcal{O}(\Phi^3(x)). \tag{2.38}$$

The diagonal matrix $\chi_0(x)$ is connected with the scalar and the pseudoscalar current in the following way [31]:

$$\chi_0(x) = 2B_0\left(s(x) + ip(x)\right). \tag{2.39}$$

The order parameter of the chiral symmetry breaking in QCD determines the parameter B_0 via [31]

$$B_0 = -\frac{\langle 0|\bar{q}(x)q(x)|0\rangle}{3f}, \tag{2.40}$$

where the pion-decay constant f can be related to the pion decay $\pi^+ \to \mu^+\nu_\mu$ [35] with $f = 92.4$ MeV at leading order [34]. Later, we will identify the scalar current with the mass matrix $M_q = \mathrm{diag}(m_u, m_d, m_s)$ ($s = M_q$). Therefore, we express $\chi_0(x)$ under the assumption (2.42) by [33,34]:

$$\chi_0(x) = 2B_0\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}. \tag{2.41}$$

The expression of $\chi_0(x)$ in terms of the pion mass m_π and the kaon mass m_K at leading order underlines that we assume perfect isospin symmetry.
A simplification is done at this step in our thesis: we only consider the scalar and the vector current in our further calculations, so we assume:

$$p(x) = a_\mu(x) = 0. \tag{2.42}$$

In the case of photoproduction, the space-time dependent vector current reads [35]

$$v_\mu(x) = -eQA_\mu(x). \tag{2.43}$$

This vector current is introduced by a right- and a left-handed current [35]:

$$v_\mu(x) = \frac{1}{2}\left(r_\mu(x) + l_\mu(x)\right), \quad r_\mu(x) = l_\mu(x) = -eQA_\mu(x). \tag{2.44}$$

The current is related to the covariant electromagnetic four-potential $A_\mu(x)$. While e describes the positive elementary electric charge, the charge matrix Q contains the relevant quark charges q_u, q_d and q_s as multiples of e [35]:

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \tag{2.45}$$

We will include all relevant meson and baryon fields for the investigation of photoproduction on tree-level in chapter 3.

2.6 Baryon matrix elements of one scalar current (s) via the $1/N_c$ expansion

We examine the matrix elements of one scalar current $S^{(a)}(q)$ with a flavour index a between two $(J^P = \frac{1}{2}^+)$-baryons by a large-N_c expansion. The expansion is done at leading large-N_c order. We apply the large-N_c counting scheme of Eq. (2.15) and derive the baryon matrix elements' general order in this picture [12, 17]:

$$(\mathscr{B}'|J_i|\mathscr{B}) \sim N_c^0, \quad (\mathscr{B}'|T^a|\mathscr{B}) \sim N_c, \quad (\mathscr{B}'|G_i^a|\mathscr{B}) \sim N_c. \tag{2.46}$$

We obtain the following four contributions with the N_c-parameters $\hat{b}_1 - \hat{b}_4$:

$$(\bar{p},\dots,\bar{\chi}|S^{(a)}(q)|p,\dots,\chi) = (\bar{p},\dots,\bar{\chi}|\delta_{a0}\left(\hat{b}_1\,\mathbb{1} + \hat{b}_2\,J^2\right) + \hat{b}_3\,T^a + \hat{b}_4\,\{J^i, G_i^a\}|p,\dots,\chi). \tag{2.47}$$

The large-N_c coefficients with hats are easier distinguishable from the chiral constants in Eq. (3.28). The evaluation of all four contributions requires the matrix elements of the ten symmetric two one-body operator's combinations in appendix D. We point out that the flavour index in the decomposition scheme for s (Eq. (3.29)) runs from 0 to 8. To enable a complete matching between the chiral and the large-N_c parameters, we specify the zeroth flavour component of both T^a and G_i^a by

$$T^0 := \sqrt{\frac{1}{6}}\,\mathbb{1}, \quad G_i^0 := \sqrt{\frac{1}{6}}\,J_i \tag{2.48}$$

and receive the matrix elements of one scalar current $S^{(a)}(q)$ between two $(J^P = \frac{1}{2}^+)$- and two $(J^P = \frac{3}{2}^+)$-baryons via the large-N_c operator analysis:

$$\begin{aligned}(\bar{p},c,\bar{\chi}|S^{(a)}(q)|p,b,\chi) = \delta_{\bar{\chi}\chi}\Bigg(&\delta_{a0}\delta_{bc}\left[3\hat{b}_1 + \frac{3}{4}\hat{b}_2 + \sqrt{\frac{3}{2}}\hat{b}_3 - \frac{1}{2}\sqrt{\frac{3}{2}}\hat{b}_4\right] \\ &+ i f_{abc}\left[\hat{b}_3 + \hat{b}_4\right] + d_{abc}\left[\frac{3}{2}\hat{b}_4\right]\Bigg),\end{aligned} \tag{2.49}$$

$$(\bar{p},nop,\bar{\chi}|S^{(a)}(q)|p,klm,\chi) = \delta_{\bar{\chi}\chi}\left(\delta_{a0}\delta^{nop}_{klm}\left[3\hat{b}_1 + \frac{15}{4}\hat{b}_2\right] + \delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\left[\frac{3}{2}\hat{b}_3 + \frac{15}{4}\hat{b}_4\right]\right). \tag{2.50}$$

The object $S^{(a)}(q)$ is defined via

$$S^{(a)}(q) = i\int \mathrm{d}^4x\, e^{-iq\cdot x} S^{(a)}(x), \quad q \equiv \bar{p} - p. \tag{2.51}$$

2.7 Baryon matrix elements of two scalar currents (ss) via the $1/N_c$ expansion

The operator ansatz for the time-ordered product of two scalar currents at leading order provided by the large-N_c expansion reads

$$\begin{aligned}(\bar{p}, d, \bar{\chi}|S^{(ab)}(q)|p, c, \chi) = (\bar{p}, \dots, \bar{\chi}|\hat{c}_1\, \delta_{a0}\delta_{b0}\mathbb{1} + \hat{c}_2\, \delta_{ab}\mathbb{1} + \hat{c}_3\left(T^a\delta_{b0} + \delta_{a0}T^b\right) + \hat{c}_4\left(d_{abe}T^e + d_{ab0}T^0\right)\\ + \hat{c}_5\, \{T^a, T^b\} + \hat{c}_6\, \delta_{a0}\delta_{b0}J^2 + \hat{c}_7\left(\{J^i, G_i^a\}\delta_{b0} + \delta_{a0}\{J^i, G_i^b\}\right)|p, \dots, \chi)\end{aligned} \tag{2.52}$$

with

$$S^{(ab)}(q) = i\int \mathrm{d}^4x\, e^{-iq\cdot x}\mathscr{T}\, S^{(a)}(x)S^{(b)}(0), \quad q \equiv \bar{p} - p. \tag{2.53}$$

The flavour indices a, b, c and d run from 0 to 8. In contrast, the internal index e only runs from 1 to 8. Again, we identify the initial and final spin projection with χ and $\bar{\chi}$, respectively. To distinguish the large-N_c coefficients from the chiral constants in Eq. (3.41), we add hats to the former ones ($\hat{c}_1 - \hat{c}_7$). The symmetric two one-body operator's products of appendix D are utilised to perform the large-N_c operator analysis for the baryon octet's states:

$$\begin{aligned}(\bar{p}, d, \bar{\chi}|S^{(ab)}(q)|p, c, \chi) = \delta_{\bar{\chi}\chi}\Bigg(&\delta_{ab}\delta_{cd}\left[3\hat{c}_2 + \hat{c}_4 + \hat{c}_5\right] + \left(\delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}\right)\left[-\hat{c}_5\right]\\ &+ \delta_{a0}\delta_{b0}\delta_{cd}\left[3\hat{c}_1 + \sqrt{6}\hat{c}_3 + 6\hat{c}_5 + \frac{3}{4}\hat{c}_6 - \sqrt{\frac{3}{2}}\hat{c}_7\right]\\ &+ \left(d_{acd}\delta_{b0} + \delta_{a0}d_{bcd}\right)\left[-\sqrt{6}\hat{c}_5 + \frac{3}{2}\hat{c}_7\right]\\ &+ \left(if_{acd}\delta_{b0} + \delta_{a0}if_{bcd}\right)\left[\hat{c}_3 + \sqrt{6}\hat{c}_5 + \hat{c}_7\right]\\ &+ d_{abe}d_{ecd}\left[3\hat{c}_5\right] + d_{abe}if_{ecd}\left[\hat{c}_4\right]\Bigg).\end{aligned} \tag{2.54}$$

Corresponding matrix elements for this time-ordered current including the states of the baryon decuplet are obtained analogously:

$$\begin{aligned}(\bar{p}, nop, \bar{\chi}|S^{(ab)}(q)|p, klm, \chi) = \delta_{\bar{\chi}\chi}\delta^{nop}_{xyz}\Bigg(&\delta_{ab}\delta^{xyz}_{klm}\left[3\hat{c}_2 + \hat{c}_4\right] + \delta_{a0}\delta_{b0}\delta^{xyz}_{klm}\left[3\hat{c}_1 + \frac{15}{4}\hat{c}_6\right]\\ &+ \left(\Lambda^{a,xyz}_{klm}\delta_{b0} + \delta_{a0}\Lambda^{b,xyz}_{klm}\right)\left[\frac{3}{2}\hat{c}_3 + \frac{15}{4}\hat{c}_7\right]\\ &+ \left(\Lambda^{a,xyz}_{rst}\Lambda^{b,rst}_{klm} + \Lambda^{b,xyz}_{rst}\Lambda^{a,rst}_{klm}\right)\left[\frac{9}{4}\hat{c}_5\right] + d_{abe}\Lambda^{e,xyz}_{klm}\left[\frac{3}{2}\hat{c}_4\right]\Bigg).\end{aligned} \tag{2.55}$$

Now we turn to the $1/N_c$ expansion of the baryon matrix elements of the time-ordered product of two vector currents. The Fourier transform of such a product leads to the following ansatz as an appropriate starting point:

$$\begin{aligned}
& i \int \mathrm{d}^4 x \, e^{-iq\cdot x} \left(\bar{p}, \ldots, \bar{\chi} \left| \mathscr{T} \, V_i^{(a)}(x) V_j^{(b)}(0) \right| p, \ldots, \chi \right) \\
& = -\delta_{ij} \left(\bar{p}, \ldots, \bar{\chi} \left| \hat{g}_1 \left(\frac{1}{3} \delta_{ab} \mathbb{1} + d_{abe} T^e \right) + \frac{1}{2} \hat{g}_2 \{T^a, T^b\} \right| p, \ldots, \chi \right) \\
& + \frac{(\bar{p}+p)_i (\bar{p}+p)_j}{4M} \left(\bar{p}, \ldots, \bar{\chi} \left| \hat{g}_3 \left(\frac{1}{3} \delta_{ab} \mathbb{1} + d_{abe} T^e \right) + \frac{1}{2} \hat{g}_4 \{T^a, T^b\} \right| p, \ldots, \chi \right) \\
& + \epsilon_{ijk} f_{abe} \left(\bar{p}, \ldots, \bar{\chi} \left| \hat{g}_5 G_k^e \right| p, \ldots, \chi \right) \\
& + \left(\bar{p}, \ldots, \bar{\chi} \left| \frac{1}{2} \hat{g}_6 \{G_i^a, G_j^b\} \right| p, \ldots, \chi \right) + \left(\bar{p}, \ldots, \bar{\chi} \left| \frac{1}{2} \hat{g}_7 \{G_j^a, G_i^b\} \right| p, \ldots, \chi \right) \\
& + \mathcal{O}(1/N_c).
\end{aligned} \tag{2.56}$$

We restrict ourselves to the examination of the spatial contributions $i, j \in \{1, 2, 3\}$ instead of the general Lorentz indices μ, ν. To obtain an appropriate result to leading order N_c^0 the large-N_c parameters g_i, $i \in \{1, \ldots, 7\}$ have to belong to different large-N_c orders. We take our operator basis (Eq. (2.20)) into account and determine the large-N_c order of the one-body operator's matrix elements via Eq. (2.46). All three parameters $\hat{g}_1$, $\hat{g}_3$ and $\hat{g}_5$ which are connected to one-body operators only are of order N_c. In contrast, the parameters $\hat{g}_2$, $\hat{g}_4$, $\hat{g}_6$ and $\hat{g}_7$ are counted to the order $1/N_c$. Hence, the contributions of $\hat{g}_1$, $\hat{g}_3$ and $\hat{g}_5$ will dominate in the large-N_c limit. The specific composition of the ansatz in Eq. (2.56) arises as follows: quark-gluon diagrams where each of the flavour matrices λ_a and $lambda_b$ is connected to only a single quark line establish the one-body operator terms. This kind of diagrams is connected to the product

$$\lambda_a \lambda_b = \frac{2}{3} \delta_{ab} \mathbb{1} + \left(d_{abe} + i f_{abe} \right) \lambda_e \tag{2.57}$$

that directly follows from the commutation relationships of the SU(3) group's Lie algebra (Eq. (A.23)). Since the large-N_c expansion only consists of symmetric contributions by the one-body operators, a combination of $\delta_{ab}\mathbb{1}$ and $d_{abe}\lambda_e$ will always occur.

Every product of two one-body operators is able to be expressed as a sum of the commutator and the anticommutator of the two operators. The $SU(6)$ Lie algebra (Eq. (2.24)) allows us to trace back every commutator of two one-body operators to a one-body operator. Lutz and Semke [18] identified in accordance with the reduction rules by Dashen, Jenkins and Manohar [12] those symmetric combinations which provide a basis for the evaluation of the matrix elements of the baryon ground state tower. All relevant symmetric combinations consist of either two T-operators, or one T- and one G-operator, or two G-operators. Hence, the terms connected to the large-N_c parameters $\hat{g}_2$, $\hat{g}_4$, $\hat{g}_6$ and $\hat{g}_7$ result from these considerations. However, combinations of one T- and one G-operator seem to be missing in our ansatz in Eq. (2.56). It is possible to prove with the help of the identity [18]

$$\{G_i^a, G_j^b\} - \{G_j^a, G_i^b\} = \epsilon_{ijk} \epsilon_{klm} \{G_l^a, G_m^b\} \tag{2.58}$$

that a linear combination of the $\hat{g}_6$- and $\hat{g}_7$-terms replaces the missing combination up to leading order. The one-body operator's actions on both the octet and decuplet baryon states (Eq. (2.28)) lead us to the baryon matrix elements of symmetric two one-body operator's products (see appendix D). Together with the identities (B.7) - (B.11) including the Pauli matrices σ_i and the "spin-$\frac{1}{2}$-to-spin-$\frac{3}{2}$"-transition matrices, these results enable us to finalise the evaluation of Eq. (2.56).

We introduce the object

$$C_{ij}^{(ab)}(q) = i \int \mathrm{d}^4x\, e^{-iq\cdot x} \mathcal{T}\, V_i^{(a)}(x) V_j^{(b)}(0), \quad q \equiv \bar{p} - p. \tag{2.59}$$

and commence with the results of the large-N_c operator analysis for the states of the baryon octet:

$$\begin{aligned}
&\left(\bar{p}, d, \bar{\chi} \left| C_{ij}^{(ab)}(q) \right| p, c, \chi\right) \\
&= \delta_{\bar{\chi}\chi}\delta_{ij}\left(\delta_{ab}\delta_{dc}\left[-\hat{g}_1 - \frac{1}{2}\hat{g}_2 + \frac{5}{24}\hat{g}_6 + \frac{5}{24}\hat{g}_7\right]\right. \\
&+\delta_{ad}\delta_{bc}\left[\frac{1}{2}\hat{g}_2 - \frac{1}{24}\hat{g}_6 - \frac{1}{24}\hat{g}_7\right] + \delta_{bd}\delta_{ca}\left[\frac{1}{2}\hat{g}_2 - \frac{1}{24}\hat{g}_6 - \frac{1}{24}\hat{g}_7\right] \\
&+d_{abe}if_{ecd}\left[-\hat{g}_1 + \frac{1}{3}\hat{g}_6 + \frac{1}{3}\hat{g}_7\right] + d_{abe}d_{ecd}\left[-\frac{3}{2}\hat{g}_2 - \frac{1}{8}\hat{g}_6 - \frac{1}{8}\hat{g}_7\right] \\
&+d_{acd}\delta_{b0}\left[\sqrt{\frac{3}{2}}\hat{g}_2 + \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_6 + \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_7\right] + d_{bcd}\delta_{a0}\left[\sqrt{\frac{3}{2}}\hat{g}_2 + \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_6 + \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_7\right] \\
&+if_{acd}\delta_{b0}\left[-\sqrt{\frac{3}{2}}\hat{g}_2 - \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_6 - \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_7\right] + if_{bcd}\delta_{a0}\left[-\sqrt{\frac{3}{2}}\hat{g}_2 - \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_6 - \frac{1}{4}\sqrt{\frac{2}{3}}\hat{g}_7\right] \\
&\left.+\delta_{a0}\delta_{b0}\delta_{dc}\left[-3\hat{g}_2 - \frac{1}{2}\hat{g}_6 - \frac{1}{2}\hat{g}_7\right]\right) \\
&+\delta_{\bar{\chi}\chi}\frac{(\bar{p}+p)_i(\bar{p}+p)_j}{4M}\left(\delta_{ab}\delta_{dc}\left[\hat{g}_3 + \frac{1}{2}\hat{g}_4\right] + \delta_{ad}\delta_{bc}\left[-\frac{1}{2}\hat{g}_4\right] + \delta_{bd}\delta_{ca}\left[-\frac{1}{2}\hat{g}_4\right]\right. \\
&+d_{abe}if_{ecd}\left[\hat{g}_3\right] + d_{abe}d_{ecd}\left[\frac{3}{2}\hat{g}_4\right] + d_{acd}\delta_{b0}\left[-\sqrt{\frac{3}{2}}\hat{g}_4\right] + d_{bcd}\delta_{a0}\left[-\sqrt{\frac{3}{2}}\hat{g}_4\right] \\
&\left.+if_{acd}\delta_{b0}\left[\sqrt{\frac{3}{2}}\hat{g}_4\right] + if_{bcd}\delta_{a0}\left[\sqrt{\frac{3}{2}}\hat{g}_4\right] + \delta_{a0}\delta_{b0}\delta_{dc}\left[3\hat{g}_4\right]\right) \\
&+(\sigma_k)_{\bar{\chi}\chi}\, i\epsilon_{ijk}\left(\delta_{ac}\delta_{bd}\left[\frac{1}{8}\hat{g}_6 - \frac{1}{8}\hat{g}_7\right] + \delta_{ad}\delta_{bc}\left[-\frac{1}{8}\hat{g}_6 + \frac{1}{8}\hat{g}_7\right]\right. \\
&\left.+if_{abe}d_{ecd}\left[-\frac{1}{2}\hat{g}_5 + \frac{1}{4}\hat{g}_6 - \frac{1}{4}\hat{g}_7\right] + f_{abe}f_{ecd}\left[\frac{1}{3}\hat{g}_5 - \frac{5}{24}\hat{g}_6 + \frac{5}{24}\hat{g}_7\right]\right).
\end{aligned} \tag{2.60}$$

The non-diagonal baryon matrix elements for two vector currents in the $1/N_c$ picture do not vanish in contrast to the two latter sections and read:

$$\begin{aligned}
&\left(\bar{p}, nop, \bar{\chi}\left|C_{ij}^{(ab)}(q)\right|p, c, \chi\right)\\
&= \left(S_i\sigma_j + S_j\sigma_i\right)_{\bar{\chi}\chi}\left(\left(d_{ace}+if_{ace}\right)\Lambda_{be}^{nop} + \left(d_{bce}+if_{bce}\right)\Lambda_{ae}^{nop}\right)\left[\frac{1}{16\sqrt{2}}\hat{g}_6 + \frac{1}{16\sqrt{2}}\hat{g}_7\right]\\
&+ \left(S_k\right)_{\bar{\chi}\chi}\epsilon_{ijk}f_{abe}\Lambda_{ce}^{nop}\left[-\frac{1}{2\sqrt{2}}\hat{g}_5 + \frac{1}{8\sqrt{2}}\hat{g}_6 - \frac{1}{8\sqrt{2}}\hat{g}_7\right]\\
&+ \left(S_k\right)_{\bar{\chi}\chi}\epsilon_{ijk}\left(\left(f_{ace}-id_{ace}\right)\Lambda_{be}^{nop} - \left(f_{bce}-id_{bce}\right)\Lambda_{ae}^{nop}\right)\left[-\frac{3}{16\sqrt{2}}\hat{g}_6 + \frac{3}{16\sqrt{2}}\hat{g}_7\right]\\
&+ \left(S_k\right)_{\bar{\chi}\chi}\epsilon_{ijk}\left(\delta_{a0}i\Lambda_{bc}^{nop} + \delta_{b0}i\Lambda_{ac}^{nop}\right)\left[\frac{1}{8\sqrt{3}}\hat{g}_6 - \frac{1}{8\sqrt{3}}\hat{g}_7\right].
\end{aligned} \tag{2.61}$$

We complete this section with the corresponding results for the baryon decuplet's states:

$$\begin{aligned}
&\left(\bar{p}, nop, \bar{\chi}\left|C_{ij}^{(ab)}(q)\right|p, klm, \chi\right)\\
&= \delta_{\bar{\chi}\chi}\delta_{ij}\left(\delta_{ab}\delta_{klm}^{nop}\left[-\hat{g}_1\right] + d_{abe}\delta_{xyz}^{nop}\Lambda_{klm}^{e,xyz}\left[-\frac{3}{2}\hat{g}_1\right]\right.\\
&\left.+\delta_{rst}^{nop}\Lambda_{xyz}^{a,rst}\Lambda_{klm}^{b,xyz}\left[-\frac{9}{8}\hat{g}_2 + \frac{9}{32}\hat{g}_6 + \frac{9}{32}\hat{g}_7\right] + \delta_{rst}^{nop}\Lambda_{xyz}^{b,rst}\Lambda_{klm}^{a,xyz}\left[-\frac{9}{8}\hat{g}_2 + \frac{9}{32}\hat{g}_6 + \frac{9}{32}\hat{g}_7\right]\right)\\
&+ \delta_{\bar{\chi}\chi}\frac{(\bar{p}+p)_i(\bar{p}+p)_j}{4M}\left(\delta_{ab}\delta_{klm}^{nop}\left[\hat{g}_3\right] + d_{abe}\delta_{xyz}^{nop}\Lambda_{klm}^{e,xyz}\left[\frac{3}{2}\hat{g}_3\right]\right.\\
&\left.+\delta_{rst}^{nop}\Lambda_{xyz}^{a,rst}\Lambda_{klm}^{b,xyz}\left[\frac{9}{8}\hat{g}_4\right] + \delta_{rst}^{nop}\Lambda_{xyz}^{b,rst}\Lambda_{klm}^{a,xyz}\left[\frac{9}{8}\hat{g}_4\right]\right)\\
&+ \epsilon_{ijk}\left(\vec{S}\sigma_k\vec{S}^\dagger\right)_{\bar{\chi}\chi}f_{abe}\delta_{xyz}^{nop}\Lambda_{klm}^{e,xyz}\left[\frac{3}{4}\hat{g}_5 - \frac{3}{16}\hat{g}_6 + \frac{3}{16}\hat{g}_7\right]\\
&+ \left(S_iS_j^\dagger + S_jS_i^\dagger\right)_{\bar{\chi}\chi}\left(\delta_{ab}\delta_{klm}^{nop}\left[\frac{1}{8}\hat{g}_6 + \frac{1}{8}\hat{g}_7\right] + d_{abe}\delta_{xyz}^{nop}\Lambda_{klm}^{e,xyz}\left[\frac{3}{16}\hat{g}_6 + \frac{3}{16}\hat{g}_7\right]\right.\\
&\left.+\delta_{rst}^{nop}\Lambda_{xyz}^{a,rst}\Lambda_{klm}^{b,xyz}\left[-\frac{9}{32}\hat{g}_6 - \frac{9}{32}\hat{g}_7\right] + \delta_{rst}^{nop}\Lambda_{xyz}^{b,rst}\Lambda_{klm}^{a,xyz}\left[-\frac{9}{32}\hat{g}_6 - \frac{9}{32}\hat{g}_7\right]\right).
\end{aligned} \tag{2.62}$$

3 Chiral perturbation theory & chiral $SU(3)$ Lagrangian

We will give a short introduction to chiral perturbation theory (χPT) in this chapter. The parts of the chiral $SU(3)$ Lagrangian which are relevant for the studies in this PhD thesis are introduced step by step. We present the building blocks of the meson ground states, the octet and the decuplet baryon fields and the photon field. The inclusion of the photon fields demands a systematic method to include external fields into the chiral Lagrangian. The approach of external fields by Gasser and Leutwyler [13,14] serves as a starting point for the derivation of the chiral Lagrangian.

Chiral perturbation theory is based on the chiral Lagrangian. The infinite number of parameters and interaction terms requests an estimate of the relevance of each term. This can be achieved by a scale separation. Hence, we need to identify a hard and a soft scale. A small quark mass or a small momentum is considered as a soft scale Q_χ. Further soft scales are the masses of the Goldstone bosons. Various authors have attempted to determine the coupling constants of the chiral $SU(3)$ Lagrangian [13, 14, 36–47]. Some of their investigations are based on quark [42–47] or hadronic models [36, 37, 40, 41] or use QCD lattice calculations [38] as a theoretical framework. Most of these examinations also use data from several experiments to fix the masses and coupling constants of χPT.

3.1 Baryon and meson fields

Our investigations require the ($J^P = \frac{1}{2}^+$)- and the ($J^P = \frac{3}{2}^+$)-baryons. The former are organised in a $SU(3)$ octet B via

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \tag{3.1}$$

This matrix representation is equivalent to the decomposition of the ($J^P = \frac{1}{2}^+$)-baryon fields into isospin multiplets:

$$\begin{aligned} B &= \frac{1}{\sqrt{2}}\left(\vec{\tau}\cdot\vec{\Sigma}(1195) + \alpha^\dagger\cdot N(939) + \Xi^T(1315)i\sigma_2\cdot\alpha + \lambda_8\Lambda(1115)\right), \\ \vec{\Sigma} &= (\Sigma_1, \Sigma_2, \Sigma_3)^T, \quad N = (p, n)^T, \quad \Xi = \left(\Xi^0, \Xi^-\right)^T, \\ \alpha^\dagger &= \frac{1}{\sqrt{2}}(\lambda_4 + i\lambda_5, \lambda_6 + i\lambda_7), \quad \vec{\tau} = (\lambda_1, \lambda_2, \lambda_3)^T. \end{aligned} \tag{3.2}$$

We introduced the three real fields of the $\vec{\Sigma}$ baryon triplet via Σ_1, Σ_2 and Σ_3. The nucleon doublet N contains the proton and the neutron field. Analogously, the Ξ baryon doublet is composed of the particle fields Ξ^0 and Ξ^-. The Lambda baryon is connected to the isospin singlet field Λ. We denote the transpose of the corresponding quantity by T. The auxiliary quantities α, $\alpha^\dagger$ and $\vec{\tau}$ are composed of some Gell-Mann matrices λ_i which are the generators of the $SU(3)$ group (Appendix A.6). Each number in parentheses specifies the mass of the particles in the corresponding multiplet in MeV.

The behaviour of the decuplet baryon fields under the chiral transformation is not fixed or unique. We choose the following transformation pattern:

$$B \to B' = RBR^{\dagger}. \tag{3.3}$$

The chiral transformation uses the elements of the chiral group $SU(3)_R \times SU(3)_L \times U(1)_V$ and is denoted by the arrow $\to$ in this section. The indices R and L refer to right- and left-handed with respect to the concept of chirality (see e.g. [35]). The matrix R is fixed by the transformation of the quantity u under the chiral transformation [31]:

$$u \to u' = h_R u R^{\dagger} = R u h_L^{\dagger}, \quad h_L \in SU(3)_L \times U(1)_V, \quad h_R \in SU(3)_R \times U(1)_V. \tag{3.4}$$

The quantity u is connected to a nonlinear parametrisation of the pseudoscalar meson octet Φ:

$$u = \exp\left(\frac{i\Phi}{2f}\right). \tag{3.5}$$

The free parameter f is identical with the pion-decay constant of Eq. (2.40). Similarly to Eq. (3.2), we introduce the fields of the pseudoscalar meson octet with the help of the following isospin decomposition:

$$\begin{aligned}
&\Phi = \vec{\tau} \cdot \vec{\pi}(140) + \alpha^{\dagger} \cdot K(494) + K^{\dagger}(494) \cdot \alpha + \lambda_8 \eta(547), \\
&\vec{\pi} = \left(\pi_1, \pi_2, \pi_3\right)^T, \quad K = \left(K^+, K^0\right)^T.
\end{aligned} \tag{3.6}$$

The pion triplet $\vec{\pi}$ is organised with the real fields π_1, π_2 and π_3 analogously to $\vec{\Sigma}$ in Eq. (3.2). The kaon isospin doublet K consists of the K^+- and the K^0 particle field. We further use the isospin singlet field η. The remaining expressions are already introduced in Eq. (3.2). The fields of the pions, kaons, anti-kaons and the η-meson are arranged in a matrix representation as the approximate Goldstone bosons:

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{3.7}$$

The invariance of the chiral Lagrangian under the $SU(3)_L \times SU(3)_R \times U(1)_V$ symmetry requires the definition of covariant derivatives. The coupling of the photon to our octet baryons in combination with the Goldstone bosons is organised by the following covariant derivative:

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B]. \tag{3.8}$$

For this purpose, we introduce the chiral connection Γ_μ in terms of the left- and right-handed current of Eq. (2.44) [31]:

$$\Gamma_\mu = \frac{1}{2}\left(u^{\dagger}(\partial_\mu - i r_\mu)u + u(\partial_\mu - i l_\mu)u^{\dagger}\right). \tag{3.9}$$

The transformed chiral connection Γ_μ is also connected to the transformation matrix R:

$$\Gamma_\mu \to \Gamma'_\mu = R\Gamma_\mu R^\dagger + R\partial_\mu R^\dagger. \tag{3.10}$$

The incoming photon may couple to either the baryons or the mesons. We ask for a representation of the pseudoscalar mesons which transforms similarly to the octet baryon fields. The vielbein U_μ with the electromagnetic four-potential A_μ satisfies this condition [34, 48]:

$$U_\mu = \frac{1}{2}u^\dagger \left(\partial_\mu \exp\left(\frac{i\Phi}{f}\right) + ieA_\mu \left[Q, \exp\left(\frac{i\Phi}{f}\right)\right]\right) u^\dagger. \tag{3.11}$$

The covariant derivative $D_\mu U_\nu$ is constructed in a similar manner to $D_\mu B$ [34, 48]:

$$D_\mu U_\nu = \partial_\mu U_\nu + [\Gamma_\mu, U_\nu]. \tag{3.12}$$

The ($J^P = \frac{3}{2}^+$)-baryons build a decuplet B^τ_{ijk} where the real particle fields are connected to it as follows:

$$\begin{aligned}
&B^\tau_{111} = \Delta^{++,\tau}, \quad B^\tau_{112} = \frac{\Delta^{+,\tau}}{\sqrt{3}}, \quad B^\tau_{122} = \frac{\Delta^{0,\tau}}{\sqrt{3}}, \quad B^\tau_{222} = \Delta^{-,\tau}, \\
&B^\tau_{113} = \frac{\Sigma^{+,\tau}}{\sqrt{3}}, \quad B^\tau_{123} = \frac{\Sigma^{0,\tau}}{\sqrt{6}}, \quad B^\tau_{223} = \frac{\Sigma^{-,\tau}}{\sqrt{3}}, \\
&B^\tau_{133} = \frac{\Xi^{0,\tau}}{\sqrt{3}}, \quad B^\tau_{233} = \frac{\Xi^{-,\tau}}{\sqrt{3}}, \\
&B^\tau_{333} = \Omega^{-,\tau}.
\end{aligned} \tag{3.13}$$

Again, we organise the coupling to the photon field and the Goldstone bosons with the help of a covariant derivative:

$$D_\mu B^\tau_{ijk} = \partial_\mu B^\tau_{ijk} + (\Gamma^B_\mu)^l_i B^\tau_{ljk} + (\Gamma^B_\mu)^l_j B^\tau_{ilk} + (\Gamma^B_\mu)^l_k B^\tau_{ijl}. \tag{3.14}$$

The description of the baryon decuplet under the chiral transformation can be stated in many ways. Similar to the baryon octet, the transformation matrix R serves for this purpose:

$$B^\tau_{ijk} \to B^{\tau'}_{ijk} = R^l_i R^m_j R^n_k B^\tau_{lmn}. \tag{3.15}$$

We do not only consider pseudoscalar but also vector mesons for the construction of an effective hadronic interaction. One important reason for this additional inclusion is our investigation of the photon's decay into a vector meson under vector dominance. To implement the symmetries which connect QCD with χPT, we will introduce the vector meson nonet in the vector representation in section 4.2.

We exclusively use the fields and mathematical structures, the building blocks, which were introduced in the former section to derive the most general chiral Lagrangian. We organise these ingredients to traces of products which are SU(3) scalars [31]. So, their invariance under the chiral transformation is secured. Additionally, the chiral $SU(3)$ Lagrangian obeys the conservation under parity transformation (P), charge conjugation (C) and time reversal (T) seperately. However, the most general chiral Lagrangian would still consist of an infinite number of contributions if these were the only restrictions. Chiral symmmetry is only approximately conserved in addition.

We introduce the parameter Q_χ that is of the size of a small hadron momentum or mass as a complementary expansion parameter [31]. The generating functionals are expanded in terms of the expansion parameter Q_χ. Both the pure meson fields Φ and the baryon fields B and B_μ do not explicitly contain any hadron momentum or mass:

$$\Phi,\ B,\ B_\tau \sim \mathcal{O}(Q_\chi^0). \tag{3.16}$$

The same applies to our quantities u and $u^\dagger$:

$$u,\ u^\dagger \sim \mathcal{O}(Q_\chi^0). \tag{3.17}$$

The derivative of each of these building blocks provides us with a small boson or fermion momentum in first approximation:

$$D_\mu\Phi,\ D_\mu u,\ D_\mu u^\dagger \sim \mathcal{O}(Q_\chi^1). \tag{3.18}$$

We point out that the masses of the ($J^P = \frac{1}{2}^+$)- and the ($J^P = \frac{3}{2}^+$)-baryons are huge in comparison to the Goldstone bosons' mass. The covariant derivative of both baryon multiplets is of the following chiral order:

$$D_\mu B,\ D_\mu B_\tau \sim \mathcal{O}(Q_\chi^0). \tag{3.19}$$

The two conclusions in the Eqs. (3.17) and (3.18) directly lead us to the order of the chiral connection Γ_μ:

$$\Gamma_\mu \sim \mathcal{O}(Q_\chi^1). \tag{3.20}$$

The vielbein U_μ contains the quantity $\partial_\mu u$ and the photon field A_μ each in one summand. It follows:

$$U_\mu \sim \mathcal{O}(Q_\chi^1). \tag{3.21}$$

We use the scalar density and the vector current for the calculation in the sections 3.3, 3.4 and 4.3. The scalar source s is linearly connected with χ_+ (Eqs. (2.37) and (2.39)). The explicit symmetry breaking term χ_+ consists of the squared pion and kaon masses (Eq. (2.39)). Since the pion and kaon mass are identified with small hadron masses, the scalar density obeys the following chiral order:

$$s \sim \mathcal{O}(Q_\chi^2). \tag{3.22}$$

The chiral connection Γ_μ contains both the left- and right-handed currents l_μ and r_μ, respectively, and the partial derivatives $\partial_\mu u$ and $\partial_\mu u^\dagger$ (Eq. (3.9)). Hence, we make the assumption that the left- and right-handed currents are counted with the same order like $D_\mu u$ and $D_\mu u^\dagger$:

$$l_\mu,\ r_\mu \sim \mathcal{O}(Q_\chi^1). \tag{3.23}$$

The vector current v_μ can be written as the linear composition in Eq. (2.44) which directly leads us to the conclusion:

$$v_\mu \sim \mathcal{O}(Q_\chi^1). \tag{3.24}$$

The external electromagnetic field does contribute both via the four-potential A_μ and via the electromagnetic strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ to the chiral Lagrangian. Due to (2.43) and (2.44), it holds:

$$A_\mu \sim \mathcal{O}(Q_\chi^1),\ F_{\mu\nu} \sim \mathcal{O}(Q_\chi^2). \tag{3.25}$$

We conclude straightforwardly for the covariant derivative $D_\mu U_\nu$:

$$D_\mu U_\nu \sim \mathcal{O}(Q_\chi^2). \tag{3.26}$$

3.3 Sum rules for one scalar current

To carry out an explicit matching between the large-N_c and the free parameters of the chiral Lagrangian, we have to perform a low-momentum expansion of its relevant terms. We remember section 2.5: the scalar current s enters the Lagrangian via the explicit symmetry breaking of the chiral symmetry. The explicit symmetry breaking enters the chiral Lagrangian via the quantities χ_+ and χ_- which are related to the non-linear representation u of Eq. (3.5) through [1]

$$\chi_\pm = \frac{1}{2} u \chi_0 u \pm \frac{1}{2} u^\dagger \chi_0 u^\dagger, \tag{3.27}$$

[1] The space dependence of all relevant quantities will be omitted for simplification.

where χ_0 is defined by Eq. (2.39). We identify five chiral symmetry breaking terms

$$\mathscr{L}_\chi^{(2)} = 2b_0\ tr(\bar{B}B)\ tr(\chi_+) + 2b_D\ tr(\bar{B}\{\chi_+, B\}) + 2b_F\ tr(\bar{B}[\chi_+, B]) \\ - 2d_0\ tr(\bar{B}_\mu \cdot B^\mu)\ tr(\chi_+) - 2d_D\ tr((\bar{B}_\mu \cdot B^\mu)\chi_+) \tag{3.28}$$

of the chiral Lagrangian at order Q_χ^2. We make use of the Gell-Mann matrices (Eq. (A.19)) as the generators of the $SU(3)$ group and establish the following decomposition scheme:

$$B = \frac{1}{\sqrt{2}}\sum_{a=1}^{8} B^a\lambda_a, \quad s = \frac{1}{2}\sum_{a=0}^{8} s^a\lambda_a. \tag{3.29}$$

The singlet case of the scalar current ($a = 0$) is organised with the additional matrix λ_0 of Eq. (A.29). It is our goal to compute the baryon matrix elements of the scalar current

$$S^{(a)}(x) \equiv \bar{q}(x)\frac{\lambda_a}{2}q(x). \tag{3.30}$$

Here, the Heisenberg quark-field operator is denoted with $q(x)$. The quantity of our interest is the Fourier transform

$$S^{(a)}(q) = i\int \mathrm{d}^4x\ e^{-iq\cdot x}\ S^{(a)}(x), \quad q \equiv \bar{p} - p. \tag{3.31}$$

To investigate the low-momentum behaviour of baryon matrix elements of $S^{(a)}(q)$, we make use of the following functional derivative of the baryon transition amplitude $\mathscr{F}$ with respect to the scalar source function $s(x)$:

$$\langle \bar{p}, \ldots | S^{(a)}(x) | p, \ldots \rangle = -i\frac{\delta}{\delta s^a(x)}\mathscr{F}(\bar{p}, p; v, a, s, p)\Bigg|_{v=0,a=0,s=M_q,p=0}. \tag{3.32}$$

Here, we identify M_q with the mass matrix $M_q = \mathrm{diag}(m_u, m_d, m_s)$ (see section 2.5). This functional derivative requires the transition amplitude between two baryon states evaluated with external fields:

$$\mathscr{F}(\bar{p}, p; v, a, s, p) \equiv \langle \vec{\bar{p}}_{\mathrm{out}} | \vec{p}_{\mathrm{in}} \rangle_{v,a,s,p}^{\mathrm{connected}}. \tag{3.33}$$

This transition amplitude is calculated via the approach with external fields from section 2.5. The three-momenta $\vec{\bar{p}}_{\mathrm{out}}$ and $\vec{p}_{\mathrm{in}}$ denote the incoming and outgoing state, respectively. Here, we utilise the generating functional $\langle \vec{\bar{p}}_{\mathrm{out}} | \vec{p}_{\mathrm{in}} \rangle_{v,a,s,p}^{\mathrm{connected}}$ which is equivalent to the vacuum-to-vacuum transition amplitude in Eq. (2.32). It enables us to calculate baryon matrix elements of certain combinations of quark field operators. We introduce the baryon octet and the baryon decuplet states

$$|p,b,\chi\rangle, \quad |p,klm,\chi\rangle \tag{3.34}$$

analogously to the corresponding states in the large-N_c picture (Eqs. (2.26) and (2.27)): While the former state is specified by the flavour index $b \in \{1,\dots,8\}$, the latter carries the flavour numbers $k,l,m \in \{1,2,3\}$. Again, the polarisation of the $(J^P = \frac{1}{2}^+)$-baryon's spin $\chi_{1/2}$ runs from 1 to 2. It holds $\chi_{3/2} \in \{1,2,3,4\}$ in the case of the decuplet state. The matrix elements of one scalar current $S^a(0)$ between $(J^P = \frac{1}{2}^+)$-baryon states are evaluated by chiral expansion in the first step:

$$\begin{aligned}\langle \bar{p},c,\bar{\chi}|S^{(a)}(q)|p,b,\chi\rangle &= \bar{u}(\bar{p},\bar{\chi})\left(\sqrt{6}b_0\ \delta_{a0}\delta_{bc} + 2b_D\ d_{abc} + 2b_F\ if_{abc}\right)u(p,\chi)\\ &= \delta_{\bar{\chi}\chi}\left(\sqrt{6}b_0\ \delta_{a0}\delta_{bc} + 2b_D\ d_{abc} + 2b_F\ if_{abc}\right).\end{aligned} \tag{3.35}$$

To obtain this result, we utilise the relationships in Eq. (A.23) which constitute the Lie algebra structure of the $SU(3)$ group. The non-relativistic expansion in the Dirac representation of the product of $\bar{u}(\bar{p},\bar{s})$ and $u(p,s)$ (Eq. (F.11)) provides us with the normalization condition $\bar{u}(p,\bar{\chi})u(p,\chi) = \delta_{\bar{\chi}\chi}$ for the baryon octet states. A similar calculation is performed for the matrix elements between two decuplet baryon states:

$$\begin{aligned}\langle \bar{p},nop,\bar{\chi}|S^{(a)}(q)|p,klm,\chi\rangle &= -\bar{u}_\alpha(\bar{p},\bar{\chi})\left(\sqrt{6}d_0\ \delta_{a0}\delta^{nop}_{klm} + d_D\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\right)u^\alpha(p,\chi)\\ &= \delta_{\bar{\chi}\chi}\left(\sqrt{6}d_0\ \delta_{a0}\delta^{nop}_{klm} + d_D\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\right).\end{aligned} \tag{3.36}$$

This calculation requires the completely symmetrised expressions δ^{nop}_{klm} and $\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}$ of Eq. (2.29) and (2.31), respectively. The normalisation condition for the spinors of the $(J^P = \frac{3}{2}^+)$-baryon states reads $\bar{u}_\alpha(p,\bar{\chi})u^\alpha(p,\chi) = -\delta_{\bar{\chi}\chi}$ and results from the non-relativistic expansion in the Eqs. (F.12) - (F.18).
The comparison of both Eq. (3.35) and Eq. (3.36) with Eq. (2.49) leads us to relationships between both kind of parameters:

$$\begin{aligned}b_0 &= \frac{1}{\sqrt{6}}\left(3\hat{b}_1 + \frac{3}{4}\hat{b}_2\right) + \frac{1}{2}\hat{b}_3 - \frac{1}{4}\hat{b}_4, \quad b_D = \frac{3}{4}\hat{b}_4, \quad b_F = \frac{1}{2}\left(\hat{b}_3 + \hat{b}_4\right),\\ d_0 &= \frac{1}{\sqrt{6}}\left(3\hat{b}_1 + \frac{15}{4}\hat{b}_2\right), \qquad d_D = \frac{3}{2}\hat{b}_3 + \frac{15}{4}\hat{b}_4.\end{aligned} \tag{3.37}$$

The correlation of the chiral parameters implies the sum rules [49]

$$b_D = 0, \quad d_0 - b_0 = -\frac{1}{3}d_D, \quad d_D = 3b_F \tag{3.38}$$

at leading order ($\hat{b}_{1,3} \neq 0, \hat{b}_{2,4} = 0$), the two relations [49]

$$\frac{1}{2}\left(b_0 - d_0\right) + b_D = \frac{1}{6}d_D, \quad d_D = 3(b_D + b_F). \tag{3.39}$$

at next-to-leading order ($\hat{b}_{1,3,4} \neq 0, \hat{b}_2 = 0$) and [49]

$$b_F + b_D = \frac{1}{3} d_D \tag{3.40}$$

at next-to-next-to-leading order ($\hat{b}_{1,2,3,4} \neq 0$).

3.4 Sum rules for two scalar currents

The following twelve symmetry-breaking terms from the chiral Lagrangian at order Q_χ^4 serve as the basis of the calculation of sum rules for two scalar currents:

$$\begin{aligned}\mathscr{L}_\chi^{(4)} = {} & c_0\; tr(\bar{B}B)\; tr(\chi_+^2) + c_1\; tr(\bar{B}\chi_+)\; tr(\chi_+ B) + c_2\; tr(\bar{B}\{\chi_+^2, B\}) \\ & + c_3\; tr(\bar{B}[\chi_+^2, B]) + c_4\; tr(\bar{B}\{\chi_+, B\})\; tr(\chi_+) \\ & + c_5\; tr(\bar{B}[\chi_+, B])\; tr(\chi_+) + c_6\; tr(\bar{B}B)(tr(\chi_+))^2 \\ & - e_0\; tr(\bar{B}_\mu \cdot B^\mu)\; tr(\chi_+^2) - e_1\; tr((\bar{B}_\mu \cdot \chi_+)(\chi_+ \cdot B^\mu)) \\ & - e_2\; tr((\bar{B}_\mu \cdot B^\mu)\chi_+^2) - e_3\; tr((\bar{B}_\mu \cdot B^\mu)\chi_+)\; tr(\chi_+) \\ & - e_4\; tr(\bar{B}_\mu \cdot B^\mu)(tr(\chi_+))^2. \end{aligned} \tag{3.41}$$

We study the baryon matrix elements of the time-ordered product of two scalar currents

$$S^{(ab)}(q) = i \int \mathrm{d}^4x\; e^{-iq\cdot x} \mathscr{T}\; S^{(a)}(x) S^{(b)}(0), \quad q \equiv \bar{p} - p. \tag{3.42}$$

To investigate the low-momentum behaviour of these baryon matrix elements, they are obtained with the help of the second functional derivative with respect to the scalar source function $s(x)$:

$$\langle \bar{p}, \ldots | \mathscr{T}\; S^{(a)}(x) S^{(b)}(y) | p, \ldots \rangle = (-i)^2 \frac{\delta}{\delta s^a(x)} \frac{\delta}{\delta s^b(y)} \mathscr{F}(\bar{p}, p; v, a, s, p) \Big|_{v=0, a=0, s=M_q, p=0}. \tag{3.43}$$

Again, we identify M_q with the mass matrix $M_q = \mathrm{diag}(m_u, m_d, m_s)$ of section 2.5. Analogously to Eq. (3.33), the second functional derivative requires the transition amplitude between two baryon states evaluated with external fields:

$$\mathscr{F}(\bar{p}, p; v, a, s, p) \equiv \langle \vec{\bar{p}}_{\text{out}} | \vec{p}_{\text{in}} \rangle^{\text{connected}}_{v,a,s,p}. \tag{3.44}$$

The chiral expansion provides us via the Lie algebra of the $SU(3)$ group (Eq. (A.23)) with the matrix elements

$$
\begin{aligned}
\langle \bar{p}, d, \bar{\chi} | S^{(ab)}(q) | p, c, \chi \rangle = \delta_{\bar{\chi}\chi} \Bigg(& \delta_{ab}\delta_{cd} \left[c_0 + \frac{2}{3} c_2 \right] + \left(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} \right) \left[\frac{1}{2} c_1 \right] + \delta_{a0}\delta_{b0}\delta_{cd} \left[3c_6 \right] \\
& + \left(d_{acd}\delta_{b0} + \delta_{a0} d_{bcd} \right) \left[\sqrt{\frac{3}{2}} c_4 \right] + \left(i f_{acd}\delta_{b0} + \delta_{a0} i f_{bcd} \right) \left[\sqrt{\frac{3}{2}} c_5 \right] \\
& + d_{abe} d_{ecd} \left[c_2 \right] + d_{abe} i f_{ecd} \left[c_3 \right] \Bigg)
\end{aligned} \tag{3.45}
$$

between two octet baryons with the initial flavour index $c = 1, \ldots, 8$ and the final flavour index d within the same range of numbers. The spin projections χ and $\bar{\chi}$ were introduced in the Eqs. (2.26) and (2.27). The examination of the current-current correlation function $S^{(ab)}(q)$ (Eq. (3.42)) between two decuplet baryon states with the initial flavour indices $k, l, m \in \{1, 2, 3\}$ and the final flavour indices $n, o, p \in \{1, 2, 3\}$ leads to

$$
\begin{aligned}
\langle \bar{p}, nop, \bar{\chi} | S^{(ab)}(q) | p, klm, \chi \rangle = \delta_{\bar{\chi}\chi} \delta^{nop}_{xyz} \Bigg(& \delta_{ab} \delta^{xyz}_{klm} \left[e_0 + \frac{1}{2} e_1 + \frac{1}{3} e_2 \right] + \delta_{a0}\delta_{b0}\delta^{xyz}_{klm} \left[3e_4 \right] \\
& + \left(\Lambda^{a,xyz}_{klm} \delta_{b0} + \delta_{a0} \Lambda^{b,xyz}_{klm} \right) \left[\frac{1}{2} \sqrt{\frac{3}{2}} e_3 \right] + d_{abe} \Lambda^{e,xyz}_{klm} \left[\frac{3}{4} e_1 + \frac{1}{2} e_2 \right] \\
& + \left(\Lambda^{a,xyz}_{rst} \Lambda^{b,rst}_{klm} + \Lambda^{b,xyz}_{rst} \Lambda^{a,rst}_{klm} \right) \left[-\frac{3}{8} e_1 \right] \Bigg) .
\end{aligned} \tag{3.46}
$$

We follow the Einstein summation convention and add up all possible expressions of a certain term if the index variables $x, y, z \in \{1, 2, 3\}$ appear twice in this term. The investigation of scalar currents and the conservation of the spin prevent the existence of non-vanishing off-diagonal transition matrix elements:

$$
\langle \bar{p}, d, \bar{\chi} | S^{(ab)}(q) | p, klm, \chi \rangle = \langle \bar{p}, nop, \bar{\chi} | S^{(ab)}(q) | p, c, \chi \rangle = 0. \tag{3.47}
$$

We match both the matrix elements from the explicit chiral symmetry breaking (left-hand side) and our large-N_c operator analysis (right-hand side):

$$
\begin{aligned}
& 3c_6 = 3\hat{c}_1 + \sqrt{6}\hat{c}_3 + 6\hat{c}_5 + \frac{3}{4}\hat{c}_6 - \sqrt{\frac{3}{2}}\hat{c}_7, \quad c_0 + \frac{2}{3} c_2 = 3\hat{c}_2 + \hat{c}_4 + \hat{c}_5, \quad c_3 = \hat{c}_4, \\
& \sqrt{\frac{3}{2}} c_5 = \hat{c}_3 + \sqrt{6}\hat{c}_5 + \hat{c}_7, \quad \frac{1}{2} c_1 = -\hat{c}_5, \quad c_2 = 3\hat{c}_5, \quad \sqrt{\frac{3}{2}} c_4 = -\sqrt{6}\hat{c}_5 + \frac{3}{2}\hat{c}_7, \\
& 3e_4 = 3\hat{c}_1 + \frac{15}{4}\hat{c}_6, \quad e_0 + \frac{1}{2} e_1 + \frac{1}{3} e_2 = 3\hat{c}_2 + \hat{c}_4, \quad \frac{1}{2}\sqrt{\frac{3}{2}} e_3 = \frac{3}{2}\hat{c}_3 + \frac{15}{4}\hat{c}_7, \\
& -\frac{3}{4} e_1 = \frac{9}{4}\hat{c}_5, \quad \frac{3}{4} e_1 + \frac{1}{2} e_2 = \frac{3}{2}\hat{c}_4.
\end{aligned} \tag{3.48}
$$

The elimination of the large-N_c constants gives rise to the seven sum rules [49]

$$
\begin{aligned}
c_1 = -\frac{2}{3}c_2 = c_4 = \frac{1}{3}e_1, \qquad c_3 = \frac{1}{2}e_1 + \frac{1}{3}e_2, \qquad c_1 + c_6 = \frac{1}{3}e_3 + e_4, \\
c_0 - \frac{1}{2}c_1 = c_3 + e_0, \qquad\qquad\qquad\qquad c_1 + c_5 = \frac{1}{3}e_3
\end{aligned}
\tag{3.49}
$$

at leading order ($\hat{c}_{1,2,3,4,5} \neq 0, \hat{c}_{6,7} = 0$) and the six relationships [49]

$$
\begin{aligned}
c_1 = -\frac{2}{3}c_2 = \frac{1}{3}e_1, \qquad c_0 + \frac{1}{3}c_2 = c_3 + e_0, \qquad \frac{1}{3}c_2 + c_4 + \frac{1}{2}c_6 = \frac{1}{6}e_3 + \frac{1}{2}e_4, \\
c_4 + c_5 = \frac{1}{3}e_3, \qquad\qquad\qquad\qquad c_3 = \frac{1}{2}e_1 + \frac{1}{3}e_2
\end{aligned}
\tag{3.50}
$$

at next-to-leading order ($\hat{c}_{1,2,3,4,5,7} \neq 0, \hat{c}_6 = 0$). At next-to-next-to-leading order ($\hat{c}_{1,2,3,4,5,6,7} \neq 0$) the sum rules [49]

$$
\begin{aligned}
c_1 = -\frac{2}{3}c_2 = \frac{1}{3}e_1, \qquad c_0 + \frac{1}{3}c_2 = c_3 + e_0, \\
c_4 + c_5 = \frac{1}{3}e_3, \qquad c_3 = \frac{1}{2}e_1 + \frac{1}{3}e_2
\end{aligned}
\tag{3.51}
$$

are obeyed.

4 Phenomenology of vector mesons

Our ultimate goal is to explore two-body scattering processes with baryons like photoproduction. According to Sakurai [50], the initial photon can also couple directly to a vector meson before interacting with the initial baryon. The conversion of a photon into a neutral vector meson is motivated by the phenomenological vector-meson dominance model (VMD). The vector mesons with neutral electromagnetic charge dominate processes where photons can couple directly to vector mesons [51]. An appropriate tool to explore systematically the validity of the VMD hypothesis and to explain why it is valid under particular circumstances would be an effective field theory formulated with vector mesons.

Ecker, Gasser, Pich and de Rafael [36] pointed out that the exchange of vector meson resonances has got a major impact on the low-energy parameters of the chiral Lagrangian. Chiral Lagrangians with vector meson fields have been constructed by several authors [15,52–55]. Such effective Lagrangians can be successfully applied to hadronic and dileptonic decays in a *resonance saturation approach* [36,37,40]. All this points to the high relevance of vector mesons as active degrees of freedom in low-energy QCD.

4.1 Hadrogenesis conjecture

Is it possible to construct a systematic chiral Lagrangian with vector mesons? The ordering of the infinite number of interaction terms requests a scale separation. Hence, we need to identify a hard and a soft scale. A small quark mass or a small momentum is considered as a soft scale Q_χ. Further soft scales are the masses of the Goldstone bosons. However, the identification of the hard scale is difficult in the case of coupled-channel dynamics. The investigation of coupled-channel dynamics requires a non-perturbative treatment of the chiral Lagrangian. As a result, dynamic scales may appear and complicate the identification of a hard scale. Here, we rely on the hadrogenesis conjecture: only the pseudoscalar meson octet as Goldstone bosons and the light vector meson nonet serve as meson basis states in the chiral Lagrangian. All other mesons are generated dynamically out of these ($J^P = 0^-$)- and ($J^P = 1^-$)-states [56]. The hadrogenesis conjecture may be justified by the appearance of a specific mass gap in the chiral limit: as the number of colours N_c in QCD increases the masses of the remaining meson states could be much larger than the masses of the light vector mesons.

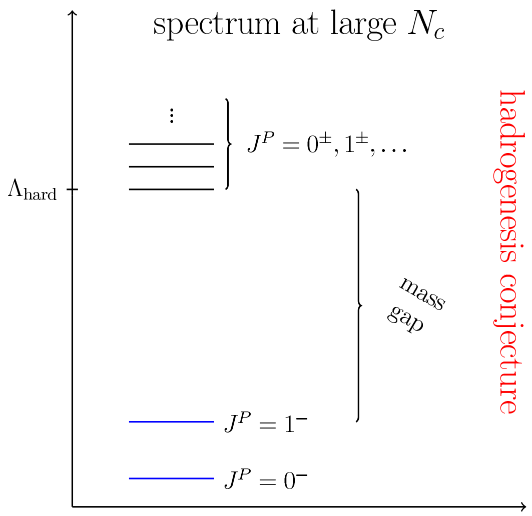


Figure 4.1.: Meson spectrum of QCD predicted by the hadrogenesis conjecture [56]

Terschluesen, Leupold and Lutz [56] estimate

$$\Lambda_{\rm hard} \geq (2-3)\mathrm{GeV} \tag{4.1}$$

if the pseudoscalar and vector meson mass m_P and m_V, respectively, is of order Q_χ^1. We adopt this estimation in this thesis and integrate the heavy meson states beyond the mass gap in the large-N_c limit out.

The hadrogenesis conjecture makes use of both the ($J^P = 0^-$)- and the ($J^P = 1^-$)-boson fields as ground states. A counting scheme for vector meson fields in the tensor representation was suggested by Terschluesen, Leupold and Lutz [56]. It is pointed out that both a consistent counting scheme for vector mesons and baryons is not developed yet and that the following discussion is of phenomenological nature.

4.2 Chiral interactions with vector mesons

In the following we will study the phenomenology of two-body interaction terms with vector mesons. The various parameters will be correlated later on by a matching to the large-N_c analysis of the two-vector current correlator of section 2.8. Such interactions are highly relevant for coupled-channel studies of baryon resonances [19]. The isospin decomposition of the vector meson nonet into the corresponding isospin multiplets is given by:

$$\begin{aligned}
V_\mu &= \vec{\tau}\cdot\vec{\rho}_\mu(770) + \alpha^\dagger \cdot K_\mu(892) + K_\mu^\dagger(892)\cdot\alpha \\
&\quad + \left(\frac{2}{3}\,\mathbb{1}_{(3\times 3)} + \frac{1}{\sqrt{3}}\lambda_8\right)\omega_\mu(782) + \left(\frac{\sqrt{2}}{3}\,\mathbb{1}_{(3\times 3)} - \sqrt{\frac{2}{3}}\lambda_8\right)\phi_\mu(1020), \\
\vec{\rho}_\mu &= (\rho_{\mu,1},\rho_{\mu,2},\rho_{\mu,3})^T, \quad K_\mu = (K_\mu^+, K_\mu^0)^T, \\
\alpha^\dagger &= \frac{1}{\sqrt{2}}\left(\lambda_4 + i\lambda_5, \lambda_6 + i\lambda_7\right), \quad \vec{\tau} = \left(\lambda_1,\lambda_2,\lambda_3\right)^T.
\end{aligned} \tag{4.2}$$

As defined in Eq. (3.2), the numbers in parentheses denote the masses of the degenerate multiplets in the unit MeV. We use the real fields $\rho_{\mu,i}$, $i = 1,2,3$ while the vector kaon doublet fields K_μ are given via particle fields. The auxiliary quantities α, $\alpha^\dagger$ and $\vec{\tau}$ are defined in Eq. (3.2). The ω_μ- and ϕ_μ fields are identified with isospin singlet fields. The matrix form of the vector meson nonet in the vector field representation is directly derived from the isospin decomposition:

$$V_\mu = \begin{pmatrix} \rho_\mu^0 + \omega_\mu & \sqrt{2}\rho_\mu^+ & \sqrt{2}K_\mu^+ \\ \sqrt{2}\rho_\mu^- & -\rho_\mu^0 + \omega_\mu & \sqrt{2}K_\mu^0 \\ \sqrt{2}K_\mu^- & \sqrt{2}\bar{K}_\mu^0 & \sqrt{2}\phi_\mu \end{pmatrix}. \tag{4.3}$$

The vector meson nonet transforms under chiral transformations like the vielbein [34,48]. The covariant derivative $D_\mu V_\alpha$, that can be explicitly written like

$$D_\mu V_\alpha = \partial_\mu V_\alpha + [\Gamma_\mu, V_\alpha] + ieA_\mu[Q, V_\alpha], \tag{4.4}$$

uses the chiral connection and preserves the validity of the chiral Ward identities in χPT.

We begin with interaction terms involving two vector fields V_μ with two $(J^P = \frac{1}{2}^+)$-baryon fields. The contributions are organised into scalar (S), tensor (T) and vector (V) parts. The scalar parts read:

$$\begin{aligned}
\mathscr{L}^{(S)}_{4,+}[V,V] = & \frac{g^{(S)}_{0,VV}}{4}\; tr(\bar{B}B)\; tr(\{V_\mu, V^\mu\}) \\
& + \frac{g^{(S)}_{F,VV}}{4}\; tr(\bar{B}[\{V_\mu, V^\mu\}, B]) + \frac{g^{(S)}_{D,VV}}{4}\; tr(\bar{B}\{\{V_\mu, V^\mu\}, B\}) \\
& + \frac{g^{(S)}_{1,VV}}{4}\; \Big(tr(\bar{B}V_\mu)\; tr(V^\mu B) + tr(\bar{B}V_\mu)\; tr(V^\mu B) \Big) \\
& + \frac{g^{(S)}_{2,VV}}{4}\; \Big(tr(\bar{B}\{V_\mu, B\})\; tr(V^\mu) + tr(\bar{B}\{V_\mu, B\})\; tr(V^\mu) \Big) \\
& + \frac{g^{(S)}_{3,VV}}{4}\; \Big(tr(\bar{B}[V_\mu, B])\; tr(V^\mu) + tr(\bar{B}[V_\mu, B])\; tr(V^\mu) \Big) \\
& + \frac{g^{(S)}_{4,VV}}{4}\; tr(\bar{B}B) \Big(tr(V_\mu)\; tr(V^\mu) + tr(V_\mu)\; tr(V^\mu) \Big) .
\end{aligned} \tag{4.5}$$

The tensor part $\mathscr{L}^{(T)}_{4,-}[V,V]$ contains of the following terms:

$$\begin{aligned}
\mathscr{L}^{(T)}_{4,-}[V,V] = & \frac{g^{(T)}_{0,VV}}{4}\; tr(\bar{B}i\sigma^{\mu\nu}B)\; tr([V_\mu, V_\nu]) \\
& + \frac{g^{(T)}_{F,VV}}{4}\; tr(\bar{B}i\sigma^{\mu\nu}[[V_\mu, V_\nu], B]) + \frac{g^{(T)}_{D,VV}}{4}\; tr(\bar{B}i\sigma^{\mu\nu}\{[V_\mu, V_\nu], B\}) \\
& + \frac{g^{(T)}_{1,VV}}{4}\; \Big(tr(\bar{B}i\sigma^{\mu\nu}V_\mu)\; tr(V_\nu B) - tr(\bar{B}i\sigma^{\mu\nu}V_\nu)\; tr(V_\mu B) \Big) \\
& + \frac{g^{(T)}_{2,VV}}{4}\; \Big(tr(\bar{B}i\sigma^{\mu\nu}\{V_\mu, B\})\; tr(V_\nu) + tr(\bar{B}i\sigma^{\mu\nu}\{V_\nu, B\})\; tr(V_\mu) \Big) \\
& + \frac{g^{(T)}_{3,VV}}{4}\; \Big(tr(\bar{B}i\sigma^{\mu\nu}[V_\mu, B])\; tr(V_\nu) + tr(\bar{B}i\sigma^{\mu\nu}[V_\nu, B])\; tr(V_\mu) \Big) .
\end{aligned} \tag{4.6}$$

Finally, we present the important vector parts of the chiral $SU(3)$ Lagrangian:

$$\begin{aligned}
\mathscr{L}^{(V)}_{4,+}[V,V] = & \frac{g^{(V)}_{0,VV}}{8}\; tr(\bar{B}i\gamma^\mu(D^\nu B))\; tr(\{V_\mu, V_\nu\}) + \text{h.c.} \\
& + \frac{g^{(V)}_{F,VV}}{8}\; tr(\bar{B}i\gamma^\mu[\{V_\mu, V_\nu\}, (D^\nu B)]) + \text{h.c.} \\
& + \frac{g^{(V)}_{D,VV}}{8}\; tr(\bar{B}i\gamma^\mu\{\{V_\mu, V_\nu\}, (D^\nu B)\}) + \text{h.c.} \\
& + \frac{g^{(V)}_{1,VV}}{8}\; \Big(tr(\bar{B}i\gamma^\mu V_\mu)\; tr(V_\nu(D^\nu B)) + tr(\bar{B}i\gamma^\mu V_\nu)\; tr(V_\mu(D^\nu B)) \Big) + \text{h.c.} \\
& + \frac{g^{(V)}_{2,VV}}{8}\; \Big(tr(\bar{B}i\gamma^\mu\{V_\mu, (D^\nu B)\})\; tr(V_\nu) + tr(\bar{B}i\gamma^\mu\{V_\nu, (D^\nu B)\})\; tr(V_\mu) \Big) + \text{h.c.}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{g^{(V)}_{3,VV}}{8}\left(tr(\bar{B}i\gamma^{\mu}[V_{\mu},(D^{\nu}B)])\;tr(V_{\nu})+tr(\bar{B}i\gamma^{\mu}[V_{\nu},(D^{\nu}B)])\;tr(V_{\mu})\right)+\text{h.c.}\\
&+\frac{g^{(V)}_{4,VV}}{8}\;tr(\bar{B}i\gamma^{\mu}(D^{\nu}B))\left(tr(V_{\mu})\;tr(V_{\nu})+tr(V_{\mu})\;tr(V_{\nu})\right)+\text{h.c.}
\end{aligned}
\tag{4.7}
$$

The signs '+' and '-' refer to the behaviour of the terms under charge conjugation. The examination of the transition baryon matrix elements require the axial vector part $\mathscr{L}^{(A)}_{4,+}[V,V]$ of the chiral $SU(3)$ Lagrangian:

$$
\begin{aligned}
\mathscr{L}^{(A)}_{4,+}[V,V]=&-\frac{h^{(A)}_{1,VV}}{4}\left(tr((\bar{B}^{\mu}\cdot\gamma_5\gamma^{\nu}B))\;tr(\{V_{\mu},V_{\nu}\})+\text{h.c.}\right)\\
&-\frac{h^{(A)}_{2,VV}}{4}\left(tr((\bar{B}^{\mu}\cdot\gamma_5\gamma^{\nu}B)\{V_{\mu},V_{\nu}\})+\text{h.c.}\right)\\
&-\frac{h^{(A)}_{3,VV}}{4}\left(tr((\bar{B}^{\mu}\cdot V_{\mu})\gamma_5\gamma^{\nu}(V_{\nu}B))+tr((\bar{B}^{\mu}\cdot V_{\nu})\gamma_5\gamma^{\nu}(V_{\mu}B))+\text{h.c.}\right)\\
&-\frac{h^{(A)}_{4,VV}}{4}\left(tr((\bar{B}^{\mu}\cdot\gamma_5\gamma^{\nu}B)V_{\mu})\;tr(V_{\nu})+tr((\bar{B}^{\mu}\cdot\gamma_5\gamma^{\nu}B)V_{\nu})\;tr(V_{\mu})+\text{h.c.}\right)\\
&-\frac{h^{(A)}_{5,VV}}{4}\;tr((\bar{B}^{\mu}\cdot\gamma_5\gamma^{\nu}B))\left(tr(V_{\mu})\;tr(V_{\nu})+tr(V_{\nu})\;tr(V_{\mu})+\text{h.c.}\right).
\end{aligned}
\tag{4.8}
$$

The relevant parts of the chiral $SU(3)$ Lagrangian which consist of two ($J^P=\frac{3}{2}^+$)-baryon states are organised in a similar way. The scalar, vector and tensor part are presented below:

$$
\begin{aligned}
\mathscr{L}^{(S)}_{4,+}[V,V]=&-\frac{h^{(S)}_{1,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B^{\mu}))\;tr(\{V_{\nu},V^{\nu}\})-\frac{h^{(S)}_{2,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B^{\mu})\{V_{\nu},V^{\nu}\})\\
&-\frac{h^{(S)}_{3,VV}}{4}\left(tr((\bar{B}_{\mu}\cdot V_{\nu})(V^{\nu}\cdot B^{\mu}))+tr((\bar{B}_{\mu}\cdot V^{\nu})(V_{\nu}\cdot B^{\mu}))\right)\\
&-\frac{h^{(S)}_{4,VV}}{4}\left(tr((\bar{B}_{\mu}\cdot B^{\mu})V_{\nu})\;tr(V^{\nu})+tr((\bar{B}_{\mu}\cdot B^{\mu})v^{\nu})\;tr(V_{\nu})\right)\\
&-\frac{h^{(S)}_{5,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B^{\mu}))\left(tr(V_{\nu})\;tr(V^{\nu})+tr(V^{\nu})\;tr(V_{\nu})\right)\\
&-\frac{h^{(S)}_{8,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B_{\nu}))\;tr(\{V^{\mu},V^{\nu}\})\\
&-\frac{h^{(S)}_{9,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B_{\nu})\{V^{\mu},V^{\nu}\})\\
&-\frac{h^{(S)}_{10,VV}}{4}\left(tr((\bar{B}_{\mu}\cdot V^{\mu})(V^{\nu}\cdot B_{\nu}))+tr((\bar{B}_{\mu}\cdot V^{\nu})(V^{\mu}\cdot B_{\nu}))\right)\\
&-\frac{h^{(S)}_{11,VV}}{4}\left(tr((\bar{B}_{\mu}\cdot B_{\nu})V^{\mu})\;tr(V^{\nu})+tr((\bar{B}_{\mu}\cdot B_{\nu})V^{\nu})\;tr(V^{\mu})\right)\\
&-\frac{h^{(S)}_{12,VV}}{4}\;tr((\bar{B}_{\mu}\cdot B_{\nu}))(tr(V^{\mu})\;tr(V^{\nu})+tr(V^{\nu})\;tr(V^{\mu})),
\end{aligned}
\tag{4.9}
$$

$$
\begin{aligned}
\mathscr{L}^{(V)}_{4,+}[V,V] = &-\frac{h^{(V)}_{1,VV}}{4}\, tr((\bar{B}_\alpha \cdot i\gamma^\mu (D^\nu B^\alpha)))\, tr(\{V_\mu, V_\nu\}) + \text{h.c.} \\
&- \frac{h^{(V)}_{2,VV}}{4}\, tr((\bar{B}_\alpha \cdot i\gamma^\mu (D^\nu B^\alpha))\{V_\mu, V_\nu\}) + \text{h.c.} \\
&- \frac{h^{(V)}_{3,VV}}{4} \Big(tr((\bar{B}_\alpha \cdot V_\mu) i\gamma^\mu (V_\nu \cdot (D^\nu B^\alpha))) + tr((\bar{B}_\alpha \cdot V_\nu) i\gamma^\mu (V_\mu \cdot (D^\nu B^\alpha)))\Big) + \text{h.c.} \\
&- \frac{h^{(V)}_{4,VV}}{4} \Big(tr((\bar{B}_\alpha \cdot i\gamma^\mu (D^\nu B^\alpha)) V_\mu)\, tr(V_\nu) + tr((\bar{B}_\alpha \cdot i\gamma^\mu (D^\nu B^\alpha)) V_\nu)\, tr(V_\mu)\Big) + \text{h.c.} \\
&- \frac{h^{(V)}_{5,VV}}{4}\, tr((\bar{B}_\alpha \cdot i\gamma^\mu (D^\nu B^\alpha))) \Big(tr(V_\mu)\, tr(V_\nu) + tr(V_\nu)\, tr(V_\mu)\Big) + \text{h.c.},
\end{aligned} \tag{4.10}
$$

$$
\begin{aligned}
\mathscr{L}^{(T)}_{4,-}[V,V] = &-\frac{h^{(T)}_{1,VV}}{4}\, tr((\bar{B}_\alpha \cdot i\sigma^{\mu\nu} B^\alpha))\, tr([V_\mu, V_\nu]) \\
&- \frac{h^{(T)}_{2,VV}}{4}\, tr((\bar{B}_\alpha \cdot i\sigma^{\mu\nu} B^\alpha)[V_\mu, V_\nu]) \\
&- \frac{h^{(T)}_{3,VV}}{4} \Big(tr((\bar{B}_\alpha \cdot V_\mu) i\sigma^{\mu\nu} (V_\nu \cdot B^\alpha)) - tr((\bar{B}_\alpha \cdot V_\nu) i\sigma^{\mu\nu} (V_\mu \cdot B^\alpha))\Big) \\
&- \frac{h^{(T)}_{4,VV}}{4} \Big(tr((\bar{B}_\alpha \cdot i\sigma^{\mu\nu} B^\alpha) V_\mu)\, tr(V_\nu) + tr((\bar{B}_\alpha \cdot i\sigma^{\mu\nu} B^\alpha) V_\nu) tr(V_\mu)\Big).
\end{aligned} \tag{4.11}
$$

4.3 Sum rules for two vector currents

We derive sum rules for the phenomenological Lagrangian of the previous section. We start with the analysis of baryon matrix elements of two vector currents. The vector quark current

$$
V^{(a)}_\mu(x) \equiv \bar{q}(x)\gamma_\mu \frac{\lambda^{(a)}}{2} q(x) \tag{4.12}
$$

is utilised to derive the baryon matrix elements of the time-ordered product of two vector currents

$$
\langle \bar{p}, \bar{\chi}, d | C^{(ab)}_{\mu\nu}(\bar{p}-p) | p, \chi, c\rangle, \quad \langle \bar{p}, \bar{\chi}, nop | C^{(ab)}_{\mu\nu}(\bar{p}-p) | p, \chi, c\rangle, \quad \langle \bar{p}, \bar{\chi}, nop | C^{(ab)}_{\mu\nu}(\bar{p}-p) | p, \chi, klm\rangle \tag{4.13}
$$

with

$$
C^{(ab)}_{\mu\nu}(q) = i \int \mathrm{d}^4 x\; e^{-iq\cdot x}\, \mathscr{T}\; V^{(a)}_\mu(x) V^{(b)}_\nu(0), \quad q \equiv \bar{p} - p. \tag{4.14}
$$

We remember that the quark spinor $q(x)$ consists of the u-, d- and s-quark fields. The Gell-Mann matrices

λ_a with the flavour index a are given in the appendix A.6. The baryon states $|p,\chi,a\rangle$ and $|p,\chi,ijk\rangle$ are already defined in (3.34). To investigate the low-momentum behaviour of the baryon matrix elements (Eq. (4.13)), these baryon matrix elements are obtained with the help of the second functional derivative with respect to the vector source function $v_\mu(x)$, analogously to the Eqs. (3.43) and (3.44):

$$\langle \bar{p},\ldots|\mathscr{T}\, V^{\mu,(a)}(x)V^{\nu,(b)}(y)|p,\ldots\rangle = (-i)^2 \frac{\delta}{\delta v_\mu^a(x)}\frac{\delta}{\delta v_\nu^b(y)}\mathscr{F}(\bar{p},p;v,a,s,p)\Bigg|_{v=0,a=0,s=M_q,p=0} . \tag{4.15}$$

The connection between the vector field V_μ and the vector current v_μ is given by [1]

$$V_\mu \ni v_\mu, \tag{4.16}$$

i. e. we simply substitute the vector meson field V_μ by the vector current v_μ in our computations.
We calculate all leading order contributions to the octet baryon matrix elements of Eq. (4.14) for two baryon octet states

$$\begin{aligned}
&\langle \bar{p},\bar{\chi},d|C_{\mu\nu}^{(ab)}(\bar{p}-p)|p,\chi,c\rangle \\
&= \bar{u}(\bar{p},\bar{\chi})g_{\mu\nu}u(p,\chi)\left(\begin{array}{c}
\delta_{ab}\delta_{dc}\left[\frac{1}{2}g_{0,VV}^{(S)}+\frac{1}{3}g_{D,VV}^{(S)}\right]+d_{abe}if_{ecd}\left[\frac{1}{2}g_{F,VV}^{(S)}\right]\\
+\,d_{abe}d_{ecd}\left[\frac{1}{2}g_{D,VV}^{(S)}\right]+\left(\delta_{da}\delta_{bc}+\delta_{db}\delta_{ac}\right)\left[\frac{1}{4}g_{1,VV}^{(S)}\right]\\
+\left(d_{acd}\delta_{b0}+d_{bcd}\delta_{a0}\right)\left[\frac{1}{2}\sqrt{\frac{3}{2}}g_{2,VV}^{(S)}\right]\\
+\left(if_{acd}\delta_{b0}+if_{bcd}\delta_{a0}\right)\left[\frac{1}{2}\sqrt{\frac{3}{2}}g_{3,VV}^{(S)}\right]\\
+\,\delta_{a0}\delta_{b0}\delta_{dc}\left[\frac{3}{2}g_{4,VV}^{(S)}\right]
\end{array}\right)\\
&+\bar{u}(\bar{p},\bar{\chi})\left(\gamma_\mu\left(p+\bar{p}\right)_\nu+\left(p+\bar{p}\right)_\mu\gamma_\nu\right)u(p,\chi)\\
&\left(\begin{array}{c}
\delta_{dc}\delta_{ab}\left[\frac{1}{8}g_{0,VV}^{(V)}+\frac{1}{12}g_{D,VV}^{(V)}\right]+d_{abe}if_{ecd}\left[\frac{1}{8}g_{F,VV}^{(V)}\right]\\
+\,d_{abe}d_{ecd}\left[\frac{1}{8}g_{D,VV}^{(V)}\right]+\left(\delta_{da}\delta_{bc}+\delta_{db}\delta_{ac}\right)\left[\frac{1}{16}g_{1,VV}^{(V)}\right]\\
+\left(d_{acd}\delta_{b0}+d_{bcd}\delta_{a0}\right)\left[\frac{1}{8}\sqrt{\frac{3}{2}}g_{2,VV}^{(V)}\right]\\
+\left(if_{acd}\delta_{b0}+if_{bcd}\delta_{a0}\right)\left[\frac{1}{8}\sqrt{\frac{3}{2}}g_{3,VV}^{(V)}\right]\\
+\,\delta_{a0}\delta_{b0}\delta_{dc}\left[\frac{3}{8}g_{4,VV}^{(V)}\right]
\end{array}\right)\\
&+\bar{u}(\bar{p},\bar{\chi})i\sigma_{\mu\nu}u(p,\chi)\left(\begin{array}{c}
f_{abe}f_{ecd}\left[-\frac{1}{2}g_{F,VV}^{(T)}\right]+if_{abe}d_{ecd}\left[\frac{1}{2}g_{D,VV}^{(T)}\right]\\
+\left(\delta_{da}\delta_{bc}-\delta_{db}\delta_{ac}\right)\left[\frac{1}{4}g_{1,VV}^{(T)}\right]
\end{array}\right),
\end{aligned} \tag{4.17}$$

for one baryon octet and one baryon decuplet state

[1] The space dependence of all relevant quantities will be omitted for simplification.

$$\begin{aligned}
&\langle\bar{p},\bar{\chi},nop|C^{(ab)}_{\mu\nu}(\bar{p}-p)|p,\chi,c\rangle \\
&=\bar{u}_\mu(\bar{p},\bar{\chi})\gamma_\nu\gamma_5 u(p,\chi)\left(\begin{array}{c}
d_{abe}\Lambda^{nop}_{ce}\left[\frac{1}{8\sqrt{2}}h^{(A)}_{2,VV}\right] \\
+\left(\left(d_{ace}+if_{ace}\right)\Lambda^{nop}_{be}+\left(d_{bce}+if_{bce}\right)\Lambda^{nop}_{ae}\right)\left[\frac{1}{16\sqrt{2}}h^{(A)}_{3,VV}\right] \\
+\left(\Lambda^{nop}_{ac}\delta_{b0}+\Lambda^{nop}_{bc}\delta_{a0}\right)\left[\frac{\sqrt{3}}{16}h^{(A)}_{4,VV}\right]
\end{array}\right) \\
&+\bar{u}_\nu(\bar{p},\bar{\chi})\gamma_\mu\gamma_5 u(p,\chi)\left(\begin{array}{c}
d_{abe}\Lambda^{nop}_{ce}\left[\frac{1}{8\sqrt{2}}h^{(A)}_{2,VV}\right] \\
+\left(\left(d_{ace}+if_{ace}\right)\Lambda^{nop}_{be}+\left(d_{bce}+if_{bce}\right)\Lambda^{nop}_{ae}\right)\left[\frac{1}{16\sqrt{2}}h^{(A)}_{3,VV}\right] \\
+\left(\Lambda^{nop}_{ac}\delta_{b0}+\Lambda^{nop}_{bc}\delta_{a0}\right)\left[-\frac{\sqrt{3}}{16}h^{(A)}_{4,VV}\right]
\end{array}\right)
\end{aligned} \tag{4.18}$$

and for two baryon decuplet states

$$\begin{aligned}
&\langle\bar{p},\bar{\chi},nop|C^{(ab)}_{\mu\nu}(\bar{p}-p)|p,\chi,klm\rangle \\
&=-\bar{u}_\alpha(\bar{p},\bar{\chi})g_{\mu\nu}u^\alpha(p,\chi)\left(\begin{array}{c}
\delta_{ab}\delta^{nop}_{klm}\left[\frac{1}{2}h^{(S)}_{1,VV}+\frac{1}{6}h^{(S)}_{2,VV}+\frac{1}{4}h^{(S)}_{3,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{4}h^{(S)}_{2,VV}+\frac{3}{8}h^{(S)}_{3,VV}\right] \\
+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{16}h^{(S)}_{3,VV}\right] \\
+\left(\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\delta_{b0}+\delta^{nop}_{xyz}\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[\frac{1}{4}\sqrt{\frac{3}{2}}h^{(S)}_{4,VV}\right] \\
+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[\frac{3}{2}h^{(S)}_{5,VV}\right]
\end{array}\right) \\
&-\left(\bar{u}_\mu(\bar{p},\bar{\chi})u_\nu(p,\chi)+\bar{u}_\nu(\bar{p},\bar{\chi})u_\mu(p,\chi)\right) \\
&\left(\begin{array}{c}
\delta_{ab}\delta^{nop}_{klm}\left[\frac{1}{4}h^{(S)}_{8,VV}+\frac{1}{12}h^{(S)}_{9,VV}+\frac{1}{8}h^{(S)}_{10,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{8}h^{(S)}_{9,VV}+\frac{3}{16}h^{(S)}_{10,VV}\right] \\
+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{32}h^{(S)}_{10,VV}\right] \\
+\delta^{nop}_{xyz}\left(\Lambda^{a,xyz}_{klm}\delta_{b0}+\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[\frac{1}{8}\sqrt{\frac{3}{2}}h^{(S)}_{11,VV}\right] \\
+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[\frac{3}{4}h^{(S)}_{12,VV}\right]
\end{array}\right) \\
&-\bar{u}_\alpha(\bar{p},\bar{\chi})\left(\gamma_\mu\left(p+\bar{p}\right)_\nu+\left(p+\bar{p}\right)_\mu\gamma_\nu\right)u^\alpha(p,\chi) \\
&\left(\begin{array}{c}
\delta_{ab}\delta^{nop}_{klm}\left[\frac{1}{4}h^{(V)}_{1,VV}+\frac{1}{12}h^{(V)}_{2,VV}+\frac{1}{8}h^{(V)}_{3,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{8}h^{(V)}_{2,VV}+\frac{3}{16}h^{(V)}_{3,VV}\right] \\
+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{32}h^{(V)}_{3,VV}\right] \\
+\left(\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\delta_{b0}+\delta^{nop}_{xyz}\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[\frac{1}{8}\sqrt{\frac{3}{2}}h^{(V)}_{4,VV}\right] \\
+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[\frac{3}{4}h^{(V)}_{5,VV}\right]
\end{array}\right) \\
&-\bar{u}_\alpha(\bar{p},\bar{\chi})i\sigma_{\mu\nu}u^\alpha(p,\chi) \\
&\left(\begin{array}{c}
if_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{4}h^{(T)}_{2,VV}\right]+\delta_{ab}\delta^{nop}_{klm}\left[\frac{1}{4}h^{(T)}_{3,VV}\right] \\
+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{3}{8}h^{(T)}_{3,VV}\right]+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{16}h^{(T)}_{3,VV}\right]
\end{array}\right).
\end{aligned} \tag{4.19}$$

The symmetrised flavour-transition tensors of Eq. (2.29) are utilised in the two latter calculations. The baryon flavour indices c and d run from 1 to 8. Although we consider both the octet and singlet contributions of two vector currents ($a,b\in\{0,\ldots,8\}$), the summation over the internal index e only covers the natural numbers from 1 to 8. Furthermore, the decuplet flavour indices obey $k,l,m,n,o,p,r,s,t\in\{1,2,3\}$. We normalise the Dirac spinors $u(p,\chi)$ and the $u_\alpha(p,\chi)$ by the conditions

$\bar{u}(p,\bar{\chi})u(p,\chi) = \delta_{\bar{\chi}\chi}$ and $\bar{u}_\alpha(p,\bar{\chi})u^\alpha(p,\chi) = -\delta_{\bar{\chi}\chi}$. Before we derive the sum rules for the vector current correlation function, the products $\bar{u}(\bar{p},\bar{\chi})\,\Gamma\,u(p,\chi)$ and $\bar{u}^\mu(\bar{p},\bar{\chi})\,\Gamma\,u^\nu(p,\chi)$ with $\Gamma = \mathbb{1},\ \gamma_\mu,\ \sigma_{\mu\nu}$ of the chiral expansion have to be expanded non-relativistically up to the order $\mathcal{O}\left(\frac{Q_\chi^2}{M^2}\right)$. The order is determined by the baryon three-momenta $\vec{\bar{p}},\vec{p}$ via $|\vec{\bar{p}}|,|\vec{p}| \sim \mathcal{O}(Q_\chi^1)$. The derivation and the final results for the relevant products are given in the appendix F. We include these results and proceed with the leading order contributions (Eqs. (4.17), (4.18), (4.19)) in the nonrelativistic case for two ($J^P = \frac{1}{2}^+$)-baryons

$$
\begin{aligned}
&\langle \bar{p},\bar{\chi},d|C_{ij}^{(ab)}(\bar{p}-p)|p,\chi,c\rangle \\
&= \delta_{\bar{\chi}\chi}\delta_{ij}\Bigg(\delta_{ab}\delta_{dc}\left[-\frac{1}{2}g_{0,VV}^{(S)}-\frac{1}{3}g_{D,VV}^{(S)}\right]+d_{abe}if_{ecd}\left[-\frac{1}{2}g_{F,VV}^{(S)}\right]+d_{abe}d_{ecd}\left[-\frac{1}{2}g_{D,VV}^{(S)}\right] \\
&+(\delta_{da}\delta_{bc}+\delta_{db}\delta_{ac})\left[-\frac{1}{4}g_{1,VV}^{(S)}\right]+(d_{acd}\delta_{b0}+d_{bcd}\delta_{a0})\left[-\frac{1}{2}\sqrt{\frac{3}{2}}g_{2,VV}^{(S)}\right] \\
&+(if_{acd}\delta_{b0}+if_{bcd}\delta_{a0})\left[-\frac{1}{2}\sqrt{\frac{3}{2}}g_{3,VV}^{(S)}\right]+\delta_{a0}\delta_{b0}\delta_{dc}\left[-\frac{3}{2}g_{4,VV}^{(S)}\right]\Bigg) \\
&+\delta_{\bar{\chi}\chi}\frac{(p+\bar{p})_i(p+\bar{p})_j}{4M}\Bigg(\delta_{ab}\delta_{dc}\left[\frac{1}{2}g_{0,VV}^{(V)}+\frac{1}{3}g_{D,VV}^{(V)}\right]+d_{abe}if_{ecd}\left[\frac{1}{2}g_{F,VV}^{(V)}\right]+d_{abe}d_{ecd}\left[\frac{1}{2}g_{D,VV}^{(V)}\right] \\
&+(\delta_{da}\delta_{bc}+\delta_{db}\delta_{ac})\left[\frac{1}{4}g_{1,VV}^{(V)}\right]+(d_{acd}\delta_{b0}+d_{bcd}\delta_{a0})\left[\frac{1}{2}\sqrt{\frac{3}{2}}g_{2,VV}^{(V)}\right] \\
&+(if_{acd}\delta_{b0}+if_{bcd}\delta_{a0})\left[\frac{1}{2}\sqrt{\frac{3}{2}}g_{3,VV}^{(V)}\right]+\delta_{a0}\delta_{b0}\delta_{dc}\left[\frac{3}{2}g_{4,VV}^{(V)}\right]\Bigg) \\
&+i\epsilon_{ijk}\,(\sigma_k)_{\bar{\chi}\chi}\Bigg(f_{abe}f_{ecd}\left[-\frac{1}{2}g_{F,VV}^{(T)}\right]+if_{abe}d_{ecd}\left[\frac{1}{2}g_{D,VV}^{(T)}\right]+(\delta_{da}\delta_{bc}-\delta_{db}\delta_{ac})\left[\frac{1}{4}g_{1,VV}^{(T)}\right]\Bigg),
\end{aligned}
\tag{4.20}
$$

for one ($J^P = \frac{1}{2}^+$)- and one ($J^P = \frac{3}{2}^+$)-baryon

$$
\begin{aligned}
&\langle \bar{p},\bar{\chi},nop|C_{ij}^{(ab)}(\bar{p}-p)|p,\chi,c\rangle \\
&= \left(S_i\sigma_j+S_j\sigma_i\right)_{\bar{\chi}\chi}d_{abe}\Lambda_{ce}^{nop}\left[\frac{1}{8\sqrt{2}}h_{2,VV}^{(A)}\right] \\
&+\left(S_i\sigma_j+S_j\sigma_i\right)_{\bar{\chi}\chi}\left((d_{ace}+if_{ace})\,\Lambda_{be}^{nop}+(d_{bce}+if_{bce})\,\Lambda_{ae}^{nop}\right)\left[\frac{1}{16\sqrt{2}}h_{3,VV}^{(A)}\right] \\
&+(S_k)_{\bar{\chi}\chi}\,i\epsilon_{ijk}\left(\Lambda_{ac}^{nop}\delta_{b0}+\Lambda_{bc}^{nop}\delta_{a0}\right)\left[-\frac{\sqrt{3}}{16}h_{4,VV}^{(A)}\right]
\end{aligned}
\tag{4.21}
$$

and for two ($J^P = \frac{3}{2}^+$)-baryons

$$
\begin{aligned}
&\langle \bar{p},\bar{\chi},nop|C_{ij}^{(ab)}(\bar{p}-p)|p,\chi,klm\rangle \\
&= \delta_{\bar{\chi}\chi}\delta_{ij}\Bigg(\delta_{ab}\delta_{klm}^{nop}\left[-\frac{1}{2}h_{1,VV}^{(S)}-\frac{1}{6}h_{2,VV}^{(S)}-\frac{1}{4}h_{3,VV}^{(S)}\right]+d_{abe}\delta_{xyz}^{nop}\Lambda_{klm}^{e,xyz}\left[-\frac{1}{4}h_{2,VV}^{(S)}-\frac{3}{8}h_{3,VV}^{(S)}\right]
\end{aligned}
$$

$$+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[\frac{3}{16}h^{(S)}_{3,VV}\right]$$
$$+\left(\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\delta_{b0}+\delta^{nop}_{xyz}\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[-\frac{1}{4}\sqrt{\frac{3}{2}}h^{(S)}_{4,VV}\right]+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[-\frac{3}{2}h^{(S)}_{5,VV}\right]\Bigg)$$
$$+\left(S_iS_j^\dagger+S_jS_i^\dagger\right)_{\bar{\chi}\chi}\Bigg(\delta_{ab}\delta^{nop}_{klm}\left[-\frac{1}{4}h^{(S)}_{8,VV}-\frac{1}{12}h^{(S)}_{9,VV}-\frac{1}{8}h^{(S)}_{10,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[-\frac{1}{8}h^{(S)}_{9,VV}-\frac{3}{16}h^{(S)}_{10,VV}\right]$$
$$+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[\frac{3}{32}h^{(S)}_{10,VV}\right]+\delta^{nop}_{xyz}\left(\Lambda^{a,xyz}_{klm}\delta_{b0}+\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[-\frac{1}{8}\sqrt{\frac{3}{2}}h^{(S)}_{11,VV}\right]$$
$$+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[-\frac{3}{4}h^{(S)}_{12,VV}\right]\Bigg)$$
$$+\delta_{\bar{\chi}\chi}\frac{(p+\bar{p})_i(p+\bar{p})_j}{4M}\Bigg(\delta_{ab}\delta^{nop}_{klm}\left[h^{(V)}_{1,VV}+\frac{1}{3}h^{(V)}_{2,VV}+\frac{1}{2}h^{(V)}_{3,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{2}h^{(V)}_{2,VV}+\frac{3}{4}h^{(V)}_{3,VV}\right]$$
$$+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{8}h^{(V)}_{3,VV}\right]$$
$$+\left(\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm}\delta_{b0}+\delta^{nop}_{xyz}\Lambda^{b,xyz}_{klm}\delta_{a0}\right)\left[\frac{1}{2}\sqrt{\frac{3}{2}}h^{(V)}_{4,VV}\right]+\delta_{a0}\delta_{b0}\delta^{nop}_{klm}\left[3h^{(V)}_{5,VV}\right]\Bigg)$$
$$+i\epsilon_{ijk}\left(\vec{S}\sigma_k\vec{S}^\dagger\right)_{\bar{\chi}\chi}\Bigg(if_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{1}{4}h^{(T)}_{2,VV}\right]+\delta_{ab}\delta^{nop}_{klm}\left[\frac{1}{4}h^{(T)}_{3,VV}\right]+d_{abe}\delta^{nop}_{xyz}\Lambda^{e,xyz}_{klm}\left[\frac{3}{8}h^{(T)}_{3,VV}\right]$$
$$+\delta^{nop}_{rst}\left(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}\right)\left[-\frac{3}{16}h^{(T)}_{3,VV}\right]\Bigg). \tag{4.22}$$

The coefficients of corresponding flavour structures from both the chiral and the large-N_c expansion are matched and provide us with the following set of equations for two octet baryon states

$$\begin{aligned}
g^{(S)}_{0,VV}&=2\hat{g}_1-\hat{g}_2-\frac{7}{6}\hat{g}_+, & g^{(S)}_{F,VV}&=2\hat{g}_1-\frac{4}{3}\hat{g}_+,\\
g^{(S)}_{D,VV}&=3\hat{g}_2+\frac{1}{2}\hat{g}_+, & g^{(S)}_{1,VV}&=-2\hat{g}_2+\frac{1}{3}\hat{g}_+,\\
g^{(S)}_{2,VV}&=-2\hat{g}_2-\frac{2}{3}\hat{g}_+, & g^{(S)}_{3,VV}&=2\hat{g}_2+\frac{2}{3}\hat{g}_+,\\
g^{(S)}_{4,VV}&=2\hat{g}_2+\frac{2}{3}\hat{g}_+, & &\\
g^{(V)}_{0,VV}&=2\hat{g}_3-\hat{g}_4, & g^{(V)}_{F,VV}&=2\hat{g}_3,\\
g^{(V)}_{D,VV}&=3\hat{g}_4, & g^{(V)}_{1,VV}&=-2\hat{g}_4,\\
g^{(V)}_{2,VV}&=-2\hat{g}_4, & g^{(V)}_{3,VV}&=2\hat{g}_4,\\
g^{(V)}_{4,VV}&=2\hat{g}_4, & &\\
g^{(T)}_{F,VV}&=-\frac{2}{3}\hat{g}_5+\frac{5}{6}\hat{g}_-, & g^{(T)}_{D,VV}&=-\hat{g}_5+\hat{g}_-,\\
g^{(T)}_{1,VV}&=-\hat{g}_-, & &
\end{aligned} \tag{4.23}$$

for one octet and one decuplet baryon state

$$
\begin{aligned}
&h^{(A)}_{2,VV} = 0, && h^{(A)}_{3,VV} = 2\hat{g}_+,\\
&h^{(A)}_{4,VV} = -\frac{4}{3}\hat{g}_-
\end{aligned}
\tag{4.24}
$$

and for two decuplet baryon states

$$
\begin{aligned}
&h^{(S)}_{1,VV} = 0, && h^{(S)}_{2,VV} = 6\hat{g}_1 + 9\hat{g}_2 - \frac{9}{2}\hat{g}_+,\\
&h^{(S)}_{3,VV} = -6\hat{g}_2 + 3\hat{g}_+, && h^{(S)}_{4,VV} = 0,\\
&h^{(S)}_{5,VV} = 0, && h^{(S)}_{8,VV} = 0,\\
&h^{(S)}_{9,VV} = 6\hat{g}_+, && h^{(S)}_{10,VV} = -6\hat{g}_+,\\
&h^{(S)}_{11,VV} = 0, && h^{(S)}_{12,VV} = 0,\\
&h^{(V)}_{1,VV} = 0, && h^{(V)}_{2,VV} = 3\hat{g}_3 + \frac{9}{2}\hat{g}_4,\\
&h^{(V)}_{3,VV} = -3\hat{g}_4, && h^{(V)}_{4,VV} = 0,\\
&h^{(V)}_{5,VV} = 0, && \\
&h^{(T)}_{2,VV} = -3\hat{g}_5 + \frac{3}{2}\hat{g}_-, && h^{(T)}_{3,VV} = 0.
\end{aligned}
\tag{4.25}
$$

Here, we make use of the quantity

$$
\hat{g}_\pm = \frac{1}{2}\left(\hat{g}_6 \pm \hat{g}_7\right)
\tag{4.26}
$$

for simplification purposes.
We eliminate the large-N_c parameters and obtain a set of sum rules which relate the chiral parameters to each other:

$$
\begin{aligned}
&g^{(S)}_{0,VV} = \frac{1}{2}g^{(S)}_{1,VV} + g^{(S)}_{F,VV}, && g^{(V)}_{0,VV} = \frac{1}{2}g^{(V)}_{1,VV} + g^{(V)}_{F,VV}, && g^{(S)}_{1,VV} = -g^{(S)}_{4,VV} - \frac{1}{6}h^{(S)}_{10,VV},\\
&g^{(T)}_{1,VV} = \frac{3}{4}h^{(A)}_{4,VV}, && g^{(V)}_{1,VV} = -\frac{2}{3}g^{(V)}_{D,VV}, && g^{(S)}_{2,VV} = -g^{(S)}_{3,VV} = -g^{(S)}_{4,VV},\\
&g^{(V)}_{2,VV} = -\frac{2}{3}g^{(V)}_{D,VV}, && g^{(V)}_{3,VV} = \frac{2}{3}g^{(V)}_{D,VV}, && g^{(S)}_{4,VV} = \frac{2}{3}g^{(S)}_{D,VV} - \frac{1}{18}h^{(S)}_{10,VV},\\
&g^{(V)}_{4,VV} = \frac{2}{3}g^{(V)}_{D,VV}, && g^{(S)}_{D,VV} = -\frac{1}{2}h^{(S)}_{3,VV} - \frac{1}{3}h^{(S)}_{10,VV}, && g^{(T)}_{D,VV} = \frac{1}{3}h^{(T)}_{2,VV} - \frac{3}{8}h^{(A)}_{4,VV},\\
&g^{(V)}_{D,VV} = -h^{(V)}_{3,VV}, && g^{(S)}_{F,VV} = \frac{1}{3}h^{(S)}_{2,VV} + \frac{1}{2}h^{(S)}_{3,VV} + \frac{2}{9}h^{(S)}_{10,VV}, && g^{(T)}_{F,VV} = \frac{2}{9}h^{(T)}_{2,VV} - \frac{3}{8}h^{(A)}_{4,VV},\\
&g^{(V)}_{F,VV} = \frac{2}{3}h^{(V)}_{2,VV} + h^{(V)}_{3,VV}, && && h^{(S)}_{9,VV} = -h^{(S)}_{10,VV} = 3h^{(A)}_{3,VV},
\end{aligned}
$$
$$
h^{(S)}_{1,VV} = h^{(S)}_{4,VV} = h^{(S)}_{5,VV} = h^{(S)}_{8,VV} = h^{(S)}_{11,VV} = h^{(S)}_{12,VV} = h^{(V)}_{1,VV} = h^{(V)}_{4,VV} = h^{(V)}_{5,VV} = h^{(T)}_{3,VV} = h^{(A)}_{2,VV} = 0.
\tag{4.27}
$$

5 On-shell scattering amplitudes & decomposition

A short introduction into the basics of scattering theory will be given at the beginning of this chapter. Two-body scattering reactions will be decomposed leading to Mandelstam's dispersion integral representation. We also confront this representation with complementary decomposition schemes. In accordance to Mandelstam's approach, we will present the resulting invariant functions $G_n(s,t)$ for different combinations of spin. The motivation for and the introduction into a modified decomposition scheme for the on-shell scattering amplitudes of photoproduction processes is presented in the following section. A stepwise derivation of the appropriate projection algebra for the decomposition of our photoproduction scattering amplitudes is presented. Four examples for the application of this projection algebra conclude this chapter.

5.1 Fundamentals of scattering theory

We commence this section with the introduction of a general scattering matrix S which satisfies the unitarity condition:

$$SS^\dagger = S^\dagger S = \mathbb{1}. \tag{5.1}$$

Unitarity ensures that the probability density of the scattered state is an invariant under scattering [57]:

$$\langle f|f\rangle = \langle i|S^\dagger S|i\rangle = \langle i|i\rangle. \tag{5.2}$$

The initial state $|i\rangle$ is observed in the limit $t \to -\infty$. It is transformed after the scattering into the final state $|f\rangle = S|i\rangle$ for $t \to +\infty$. We relate S with the transition amplitude T with the help of

$$S = \mathbb{1} + iT. \tag{5.3}$$

The scattering matrix S is calculated via the equation of motion in the interaction picture [58]:

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \mathscr{H}_I(t)|\Psi(t)\rangle. \tag{5.4}$$

The scattered state $|\Psi(t)\rangle$ is observed at a discrete time. The Hamiltonian in the interaction picture $\mathscr{H}_I(t)$ is related to the Hamiltonian in the Schroedinger picture $\mathscr{H}_I^S$ through [58]

$$\mathscr{H}_I(t) = \exp\left(iH_0^S(t-t_0)\right)\mathscr{H}_I^S \exp\left(-iH_0^S(t-t_0)\right). \tag{5.5}$$

The equation of motion is transformed in combination with the initial condition

$$|\Psi(t=-\infty)\rangle \equiv |i\rangle \tag{5.6}$$

to the integral equation [58]

$$|\Psi(t)\rangle = |i\rangle + (-i)\int_{-\infty}^{t} \mathrm{d}t_1\; \mathscr{H}_I(t_1)|\Psi(t_1)\rangle. \tag{5.7}$$

We iteratively solve this equation with respect to the time t_i and arrive at the Dyson expansion as solution for the scattering matrix S [58]:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \ldots \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 \ldots \mathrm{d}^4x_n \mathscr{T}\{\mathscr{H}_I(x_1)\mathscr{H}_I(x_2)\ldots\mathscr{H}_I(x_n)\}. \tag{5.8}$$

The integrals cover the complete space-time while $\mathscr{T}$ describes the time-ordering operator. It acts on a product of Hamiltonian densities $\mathscr{H}_I(x_i)$ in the interaction picture at the spatial coordinate x_i.

5.2 Decomposition into invariant amplitudes

The scattering matrix S describes the transition between the incoming two-body state $|\alpha\rangle$ and the outgoing state $|\beta\rangle$:

$$S_{\alpha\beta} = \langle\beta|S|\alpha\rangle. \tag{5.9}$$

The Dyson expansion of the scattering matrix in Eq. (5.8) is used to calculate matrix elements of the transition amplitude T which we introduced in (5.9) for e.g. photoproduction:

$$\langle\bar{q}\bar{p}|T|qp\rangle = (2\pi)^4\delta^4(\bar{q}+\bar{p}-q-p)T_{S_qS_p\to S_{\bar{q}}S_{\bar{p}}}. \tag{5.10}$$

While q and p represent the 4-momentum of the initial boson and fermion, respectively, we name the 4-momenta of the final boson and fermion by $\bar{q}$ and $\bar{p}$, respectively. Both energy and 3-momentum conservation are assured by the δ^4-function. We distinguish the invariant amplitudes $T_{S_qS_p\to S_{\bar{q}}S_{\bar{p}}}$ by the Spin S_i of the corresponding particle with the momentum i. The investigations in this thesis make it necessary to develop a decomposition scheme for these invariant amplitudes. We would like to point out that the choices we make regarding the decomposition scheme are not unique. Chew, et al. [59] constructed a decomposition scheme for on-shell pion photoproduction based on the CGLN amplitudes $A(s,t)$, $B(s,t)$, $C(s,t)$ and $D(s,t)$. Six amplitudes $A^{BT}_{1-6}(s,t)$ were derived by Bardeen and Tung [60] for the decomposition of Compton scattering amplitudes. Other schemes for particles with arbitrary spin were also derived (see e.g. [61]). All of these cited sets of amplitudes are free from kinematical constraints. Nevertheless, we avoid these approaches because they are unsystematically derived each for a specific reaction process. Additionally, the results on this field are mostly restricted to particles with lower spin like [62] and cannot be extended systematically to the particle multiplets we use in this thesis.

We will rather derive systematically spin-dependent decomposed expressions for our on-shell scattering amplitudes step by step. It is pointed out that the further discussion is restricted to two-body processes with both an incoming and an outgoing pair of one boson and one fermion. This decomposition leads into a set of analytic functions. Additionally, the freedom from kinematical constraints is demanded from our decomposition scheme. The analytic functions $G_n(s,t)$ are expressed in terms of the two Mandelstam variables s and t.
First, we recall the Mandelstam variables s, u and t and their relationship to the initial and final boson masses m and $\bar{m}$, and the initial and final fermion masses M and $\bar{M}$:

$$\begin{aligned} s &= (p+q)^2 = (\bar{p}+\bar{q})^2, \\ u &= (p-\bar{q})^2 = (\bar{p}-q)^2, \\ t &= (p-\bar{p})^2 = (\bar{q}-q)^2, \\ s+u+t &= m^2 + M^2 + \bar{m}^2 + \bar{M}^2. \end{aligned} \tag{5.11}$$

At the beginning, we treat elastic scattering of a ($J^P = 0^-$)-boson off a ($J^P = \frac{1}{2}^+$)-fermion with the same kind of particles in the outgoing channel. The dimension of every set of invariant functions $G_n(s,t)$ equals the total number of independent amplitudes in the helicity basis which reads

$$\frac{1}{2}\,(2S_q+1)\,(2S_p+1)\,(2S_{\bar{q}}+1)\,(2S_{\bar{p}}+1). \tag{5.12}$$

The factor $\frac{1}{2}$ arises from the assumption of parity conservation. Hence, the corresponding helicity space for the decomposition is two-dimensional according to Eq. (5.12). We choose two Lorentz-Dirac structures with a minimal number of momenta to be free from kinematical constraints. The unity operator and the quantity $\not{w} \equiv \gamma_\mu w^\mu = (\not{p}+\not{q}) = (\bar{\not{p}}+\bar{\not{q}})$ serve for our purposes [63]:

$$T_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\bar{\lambda})(G_1(s,t)\,\mathbb{1} + G_2(s,t)\,\not{w})u(p,\lambda). \tag{5.13}$$

The spinors $u(p,\lambda)$ and $\bar{u}(\bar{p},\bar{\lambda})$ of the initial and final fermion are expressed with the help of the helicities λ and $\bar{\lambda}$. Instead of $\not{w}$, any other combination of the fermion and boson momenta $\not{p}$, $\bar{\not{p}}$ and $\not{q}$, $\bar{\not{q}}$, respectively, could have been used for the decomposition. The decomposition procedure in this PhD thesis is based on the Mandelstam's dispersion integral representation [26]. Once we fix the spins of both incoming and outgoing particles, it provides us with a set of invariant functions $G_n(s,t)$ with transparent analytic properties [63]. Each of these functions $G_n(s,t)$ are only non-analytic at a finite number of dynamical singularities. These singularities are directly connected to the corresponding s-, u- and t-channel processes:

$$\begin{aligned} G_n(s,t) = {} & \frac{1}{\pi}\int \mathrm{d}s' \frac{\rho_s^n(s')}{s'-s} + \frac{1}{\pi}\int \mathrm{d}t' \frac{\rho_t^n(t')}{t'-t} + \frac{1}{\pi}\int \mathrm{d}u' \frac{\rho_u^n(u')}{u'-u} \\ & + \frac{1}{\pi^2}\int \mathrm{d}u' \int \mathrm{d}s' \frac{\rho_{us}^n(u',s')}{(u'-u)(s'-s)} \\ & + \frac{1}{\pi^2}\int \mathrm{d}u' \int \mathrm{d}t' \frac{\rho_{ut}^n(u',t')}{(u'-u)(t'-t)} \\ & + \frac{1}{\pi^2}\int \mathrm{d}s' \int \mathrm{d}t' \frac{\rho_{st}^n(s',t')}{(s'-s)(t'-t)}. \end{aligned} \tag{5.14}$$

We point out at this step that we only gain from this representation in combination with the following decomposition in Lorentz-Dirac structures. For instance, a perturbative treatment of the spectral functions $\rho_i^n(i')$ and $\rho_{ij}^n(i',j')$ would destroy the unitarity condition for the scattering matrix S.
We continue with the examination of the elastic scattering of ($J^P = 1^-$)-bosons off ($J^P = \frac{1}{2}^+$)-fermions with ($J^P = 0^-$)-bosons and ($J^P = \frac{1}{2}^+$)-fermions as outgoing particles. Although our decomposition should be applied to photoproduction, the following derivation considers massive ($J^P = 1^-$)-bosons. Since a photon may convert into a $\rho_{\mu\nu}$-, a $\omega_{\mu\nu}$- or a $\phi_{\mu\nu}$-meson of the vector meson nonet under vector meson dominance, our results can be applied straightforwardly to photoproduction. The determination of the six invariant functions $G_n(s,t)$ becomes difficult. An appropriate set of $G_n(s,t)$ is identified after Heo [63]: possible combinations of γ^μ, w^μ, $\not{w}$ and the final boson momentum $\bar{q}$ are constructed. We do not include the boson momentum q. Its on-shell condition requires the validity of the Ward identity:

$$q^\mu \epsilon_\mu(q,\alpha) = 0. \tag{5.15}$$

Here, the polarisation vector ϵ_μ is a function of the polarisation α. The Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$ and the expression $i\gamma_5$ are added to guarantee the same parity for each Lorentz-Dirac structure. We conclude with an eight-dimensional, i. e. an over-complete, collection of functions $G_n(s,t)$ and Lorentz-Dirac structures [63]:

$$\begin{aligned} T_{1\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\bar{\lambda})(&G_1(s,t)\,\gamma^\mu\, i\,\gamma_5 + G_2(s,t)\, w^\mu\, i\,\gamma_5 + G_3(s,t)\,\bar{q}^\mu\, i\,\gamma_5 \\ &+ G_4(s,t)\epsilon^{\mu\nu\rho\sigma}\bar{q}_\nu w_\rho q_\sigma + G_5(s,t)\,\gamma^\mu\not{w}\, i\,\gamma_5 + G_6(s,t)\, w^\mu\not{w}\, i\,\gamma_5 \\ &+ G_7(s,t)\,\bar{q}^\mu\not{w}\, i\,\gamma_5 + G_8(s,t)\,\epsilon^{\mu\nu\rho\sigma}\bar{q}_\nu w_\rho q_\sigma\not{w})\epsilon_\mu(q,\alpha)\, u(p,\lambda). \end{aligned} \tag{5.16}$$

It was proven that two of the eight functions $G_n(s,t)$ are linear combinations of the remaining six [63]. We finally arrive at [63]:

$$\begin{aligned} T_{1\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\bar{\lambda})\,(&G_1(s,t)\,\gamma^\mu\, i\,\gamma_5 + G_2(s,t)\, w^\mu\, i\,\gamma_5 + G_3(s,t)\,\bar{q}^\mu\, i\,\gamma_5 \\ &+ G_5(s,t)\,\gamma^\mu\not{w}\, i\,\gamma_5 + G_6(s,t)\, w^\mu\not{w}\, i\,\gamma_5 + G_7(s,t)\,\bar{q}^\mu\not{w}\, i\,\gamma_5)\,\epsilon_\mu(q,\alpha)\, u(p,\lambda). \end{aligned} \tag{5.17}$$

The principle of a minimal number of momenta manifests itself through our choice again: only Lorentz-Dirac structures with zero, one or two momenta appear while those with three or four momenta are linearly decomposed in terms of the former. The investigation of reactions such as photoproduction with vector mesons ($\gamma B \to VB$) forces us to discuss the elastic scattering with ($J^P = 1^-$)-bosons and ($J^P = \frac{1}{2}^+$)-fermions both in the initial and final state. We derive the corresponding decomposition scheme similarly to the former. Due to the two polarisation vectors $\epsilon^*_{\bar{\mu}}(\bar{q},\bar{\alpha})$ and $\epsilon_\mu(q,\alpha)$, we combine the objects $g^{\bar{\mu}\mu}$, γ^μ, w^μ, $\bar{q}^\mu$, q^μ and $\not{w}$. Again, the Levi-Civita tensor and the γ_5-matrix ensure an equal parity for all terms. We find out that we have to neglect all contributions including the Levi-Civita tensor [63]. However, we fail to construct a decomposition scheme which satisfies both analyticity and freedom from kinematical constraints. A kinematical singularity at $s = 0$ remains in our final solution [63]. This singularity indicates that our set of analytic amplitudes $G_n(s,t)$ are somehow connected to each other at the singularity point [63]. Hence, we will introduce a modified decomposition scheme which consists of analytic functions $F_n^\pm(\sqrt{s},t)$ instead of $G_n(s,t)$. A specific symmetry will enable us to relate the former to the latter.

5.3 Decomposition scheme for photoproduction

The modified decomposition scheme is derived similarly to the previous chapter: we start with a decomposition for $T_{0\frac{1}{2}\to 0\frac{1}{2}}$, extend our result to the on-shell scattering amplitude $T_{1\frac{1}{2}\to 0\frac{1}{2}}$ and finally decompose $T_{1\frac{1}{2}\to 1\frac{1}{2}}$. First, we introduce some quantities frequently used in the decomposition algebra [24]:

$$
\begin{aligned}
k^\mu &= \frac{1}{2}(q^\mu - p^\mu), & \bar{k}^\mu &= \frac{1}{2}(\bar{q}^\mu - \bar{p}^\mu), \\
r^\mu &= k^\mu - \frac{1}{2}\frac{q^2 - p^2}{s} w^\mu, & \bar{r}^\mu &= \bar{k}^\mu - \frac{1}{2}\frac{\bar{q}^2 - \bar{p}^2}{s} w^\mu,
\end{aligned} \tag{5.18}
$$

in which it holds $w^\mu = q^\mu + p^\mu = \bar{q}^\mu + \bar{p}^\mu$ as introduced in Eq. (5.13). The following products will serve us to present the later derived projection algebra in an efficient manner [24]:

$$
\begin{aligned}
r \cdot r &= -p_{\mathrm{cm}}^2, & w \cdot r &= 0, \quad w \cdot w = s, \\
\bar{r} \cdot r &= -\bar{p}_{\mathrm{cm}} p_{\mathrm{cm}} \cos\theta, & \bar{r} \cdot w &= 0, \quad \bar{r} \cdot \bar{r} = -\bar{p}_{\mathrm{cm}}^2.
\end{aligned} \tag{5.19}
$$

p_{cm} and $\bar{p}_{\mathrm{cm}}$ are the initial and final fermion three-momenta in the center-of-momentum system. The related four-momenta p and $\bar{p}$ are determined by the scattering angle θ [24]. Again, we identify the dimension of the corresponding helicity space with

$$
\frac{1}{2}\,(2S_q + 1)\,(2S_p + 1)\,(2S_{\bar{q}} + 1)\,(2S_{\bar{p}} + 1). \tag{5.20}
$$

To derive the appropriate projection algebra for photoproduction, we use the elastic scattering of a $(J^P = 0^-)$-boson off a $(J^P = \frac{1}{2}^+)$-fermion with the same kind of particles in the final state as a starting point. After the exclusion of the adjoint spinor $\bar{u}(\bar{p}, \bar{\lambda})$ for the final fermion and the spinor $u(p, \lambda)$ for the initial fermion [24],

$$
T_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar{q}, q, w) = \bar{u}(\bar{p}, \bar{\lambda})\; \bar{T}_{0\frac{1}{2}\to 0\frac{1}{2}}\; u(p, \lambda), \tag{5.21}
$$

the remaining on-shell scattering amplitude is decomposed into the invariant functions $F_1^{\pm}(\sqrt{s}, t)$ and the projection matrices $P_\pm$ [24]:

$$
\bar{T}_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar{q}, q, w) = F_1^{+}(\sqrt{s}, t)\; P_+ + F_1^{-}(\sqrt{s}, t)\; P_- . \tag{5.22}
$$

We have to identify two Dirac structures for the construction of $P_\pm$. The number of momenta within these Dirac structures should be minimised, too. So, we opt for the unity operator $\mathbb{1}$ and w^μ again. The projection matrices can be fixed in the following way [24]:

$$
P_\pm = \frac{1}{2}\left(\mathbb{1} \pm \frac{\not{w}}{\sqrt{s}}\right), \quad P_\pm P_\pm = P_\pm, \quad P_\pm P_\mp = 0. \tag{5.23}
$$

The MacDowell symmetry [64] elucidates our choice: the invariant functions $F_1^{\pm}(\sqrt{s},t)$ can be expressed in dependence of the already introduced functions $G_1(s,t)$ and $G_2(s,t)$ of Eq. (5.13) [24]:

$$F_1^{\pm}(\sqrt{s},t) = G_1(s,t) \pm \sqrt{s}\, G_2(s,t). \tag{5.24}$$

Obviously, this decomposition satisfies the MacDowell symmetry relationship [64]:

$$F_1^{+}(-\sqrt{s},t) = F_1^{-}(\sqrt{s},t). \tag{5.25}$$

The projection algebra for elastic scattering of a $(J^P = 0^-)$-boson off a $(J^P = \frac{1}{2}^+)$-fermion with the same kind of particles in the final state is fixed by [24]

$$\frac{1}{2}\, tr(P_{\pm}\Lambda Q_{\pm}\bar{\Lambda}) = 1. \tag{5.26}$$

The two-dimensional basis of the helicity space [24]

$$Q_{\pm} = \frac{s}{v^2}\left((\bar{r}\cdot r)P_{\mp} - \bar{E}_{\mp}E_{\mp}P_{\pm}\right) \tag{5.27}$$

consists of the useful auxiliary quantities [24]

$$E_{\pm} = \frac{\sqrt{s}}{2}\left(1 - \frac{m^2 - M^2}{s}\right) \pm M, \quad \bar{E}_{\pm} = \frac{\sqrt{s}}{2}\left(1 - \frac{\bar{m}^2 - \bar{M}^2}{s}\right) \pm \bar{M},$$
$$v^{\mu} = \epsilon^{\mu\alpha\tau\beta}\bar{k}_{\alpha}w_{\tau}k_{\beta} \tag{5.28}$$

the already introduced projection matrices $P_{\pm}$ and the scalar product $(\bar{r}\cdot r)$. The quantities Λ and $\bar{\Lambda}$ with the definitions

$$\Lambda = \not{p} + M, \quad \bar{\Lambda} = \not{\bar{p}} + \bar{M}, \tag{5.29}$$

ensure that the on-shell conditions

$$p^2 = M^2, \quad \bar{p}^2 = \bar{M}^2 \tag{5.30}$$

are valid for the initial and final fermion, respectively. We use the elastic scattering of a $(J^P = 1^-)$-boson off a $(J^P = \frac{1}{2}^+)$-fermion for our investigations of photoproduction. It is pointed out that there is an important difference between the treatment of a massive $(J^P = 1^-)$-boson and a massless photon in

Quantum Field Theory: The Proca equations of the associated four-vector field of the former demonstrate that only three components of this four-vector field can be chosen freely (see e.g. [65], p. 135). In contrast, the field of the latter carries two degrees of freedom which are related to the two physical transverse polarisation states (see e.g. [65], p. 132). Nevertheless, we make an attempt to apply the following decomposition scheme to our on-shell scattering amplitudes due to a lack of alternatives. First, we focus on photoproduction processes with pseudoscalar mesons ($\gamma B \to PB$). The appropriate decomposition scheme and algebra is derived on the basis of the former decomposition. However, we extend the notation of Eq. (5.10): the initial massive ($J^P = 1^-$)-boson is substituted by a massless photon γ. Hence, we denote the initial photon by γ instead of its spin 1.

In contrast to the former two-dimensional decomposition, the total number of helicity amplitudes is now six (Eq. (5.12)). We exclude the final and initial spinor $\bar{u}(\bar{p},\bar{\lambda})$ and $u(p,\lambda)$, respectively, and the polarisation vector $\epsilon_\mu(q,\alpha)$ of the initial photon:

$$T_{\gamma\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\bar{\lambda})\; \bar{T}^{\mu}_{\gamma\frac{1}{2}\to 0\frac{1}{2}}\; \epsilon_\mu(q,\alpha)\; u(p,\lambda). \tag{5.31}$$

Similar to the previous considerations, we specify the decomposition by

$$\bar{T}^{\mu}_{\gamma\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \sum_{\pm}\sum_{n=1}^{3}\left(F_n^{\pm}(\sqrt{s},t)\; T^{\mu}_{\pm,n}\right). \tag{5.32}$$

The six Lorentz-Dirac basis vectors $T^{\mu}_{\pm,i}$ are composed of our former projection matrices $P_\pm$ and three additional expressions w^μ, $\bar{r}^\mu$ and $\hat{\gamma}^\mu$,

$$\begin{aligned} T^{\mu}_{\pm,1} &= \gamma_5\; i\; P_\pm \hat{\gamma}^\mu, \\ T^{\mu}_{\pm,2} &= \gamma_5\; i\; P_\pm w^\mu, \\ T^{\mu}_{\pm,3} &= \gamma_5\; i\; P_\pm \bar{r}^\mu, \end{aligned} \tag{5.33}$$

where the latter denotes

$$\hat{\gamma}^\mu = \gamma^\mu - \frac{1}{s}\not{w} w^\mu. \tag{5.34}$$

We defined the quantities w^μ and r^μ at the beginning of this section so that $\bar{r}^\mu$ is orthogonal to w^μ:

$$\bar{r}^\mu w_\mu = \bar{r}\cdot w = 0. \tag{5.35}$$

The Lorentz-Dirac structure $\hat{\gamma}^\mu$ is defined so that it is also orthogonal to w^μ:

$$\hat{\gamma}^\mu w_\mu = \hat{\gamma}\cdot w = 0. \tag{5.36}$$

These orthogonality relations simplify the corresponding projection algebra [24]

$$\frac{1}{2}\, tr(T^{\mu}_{a,n}\Lambda Q^{b,k}_{\mu}\bar{\Lambda}) = \delta_{nk}\delta_{ab}, \quad q^{\mu}Q^{\pm,k}_{\mu} = 0 \tag{5.37}$$

with the projection quantities

$$\begin{aligned}
Q^{\pm,1}_{\mu} &= \mp\frac{\sqrt{s}}{v^2}P_{\pm}v_{\mu},\\
Q^{\pm,2}_{\mu} &= -\bar{R}_{\pm}\; i\,\gamma_5\bar{w}_{\rfloor,\mu} + \frac{\delta+1}{2}\frac{\sqrt{s}}{v^2}\bar{E}_{\pm}Q_{\pm}v_{\mu},\\
Q^{\pm,3}_{\mu} &= -\bar{R}_{\pm}\; i\,\gamma_5\bar{r}_{\rfloor\lfloor,\mu} + \frac{\sqrt{s}}{v^2}E_{\mp}Q_{\mp}v_{\mu},
\end{aligned} \tag{5.38}$$

$$\bar{R}_{\pm} = \frac{s}{v^2}\left(E_{\mp}\bar{E}_{\pm}P_{\pm} - (\bar{r}\cdot r)P_{\mp}\right), \quad \delta = \frac{m^2-M^2}{s}. \tag{5.39}$$

The Ward identity $q^{\mu}Q^{\pm,k}_{\mu} = 0$ ensures that only on-shell scattering amplitudes contributes to the decomposition. To obtain the clearly arranged expressions in Eq. (5.38), we substitute certain linear combinations of r, $\bar{r}$ and w by the four-vectors $r_{\rfloor\lfloor,\mu}$, $w_{\lfloor,\mu}$ and $w_{\rfloor,\mu}$. The latter are determined by the following scalar products [23]:

$$\begin{aligned}
r_{\rfloor\lfloor}\cdot r = 1, &\quad r_{\rfloor\lfloor}\cdot\bar{r} = r_{\rfloor\lfloor}\cdot w = 0,\\
w_{\rfloor}\cdot w = 1, &\quad w_{\rfloor}\cdot r = w_{\rfloor}\cdot\bar{q} = 0,\\
w_{\lfloor}\cdot w = 1, &\quad w_{\lfloor}\cdot r = w_{\lfloor}\cdot\bar{p} = 0.
\end{aligned} \tag{5.40}$$

As the two last relations indicate, a vector with the index $\rfloor$ is orthogonal to the boson four-momentum while a vector with the index $\lfloor$ is orthogonal to the fermion four-momentum. Lutz and Vidaña [23] introduce the explicit form of the quantities $r_{\rfloor\lfloor,\mu}$, $w_{\lfloor,\mu}$ and $w_{\rfloor,\mu}$ with the help of

$$r_{\rfloor\lfloor,\mu} \equiv r_{\bar{r}\;w,\mu}, \quad w_{\lfloor,\mu} \equiv w_{r\;\bar{p},\mu}, \quad w_{\rfloor,\mu} \equiv w_{r\;\bar{q},\mu} \tag{5.41}$$

where the auxiliary quantity $a_{b\;c,\mu}$ is constructed by the vectors a_{μ}, b_{μ} and c_{μ} in the following way [23]:

$$\begin{aligned}
\frac{a_{b\;c,\mu}}{a_{b\;c}\cdot a_{b\;c}} &= a_{\mu} - \frac{a\cdot c}{c\cdot c}c_{\mu} - \frac{a\cdot\left(b-\frac{c\cdot b}{c\cdot c}c\right)}{\left(b-\frac{c\cdot b}{c\cdot c}\right)^2}\left(b_{\mu} - \frac{c\cdot b}{c\cdot c}c_{\mu}\right),\\
a_{b\;c,\mu}a^{\mu} \equiv a_{b\;c}\cdot a = 1, &\quad a_{b\;c,\mu}b^{\mu} \equiv a_{b\;c}\cdot b = 0, \quad a_{b\;c,\mu}c^{\mu} \equiv a_{b\;c}\cdot c = 0.
\end{aligned} \tag{5.42}$$

The quantities $\bar{r}_{\rfloor\lfloor,\mu}$, $\bar{w}_{\rfloor,\mu}$ and $\bar{w}_{\lfloor,\mu}$ are defined analogously to Eq. (5.40):

$$\bar{r}_{\rfloor\lfloor,\mu} \equiv \bar{r}_{r\ \ w,\mu}, \quad \bar{w}_{\rfloor,\mu} \equiv w_{\bar{r}\ \ q,\mu}, \quad \bar{w}_{\lfloor,\mu} \equiv w_{\bar{r}\ \ p,\mu}. \tag{5.43}$$

Heo connects the analytic functions $G_n(s,t)$ of Eq. (5.17) with $F_n^{\pm}(\sqrt{s},t)$ in the following way [63]: he constructs a decomposition scheme for the reaction $1\frac{1}{2} \to 0\frac{1}{2}$ and makes use of the MacDowell symmetry. The new scheme consists of the invariant amplitudes $G_n^{\pm}(\sqrt{s},t)$ which are related to the analytic functions $G_n(s,t)$ of Eq. (5.17) via [63]

$$G_n^{\pm}(\sqrt{s},t) = G_n^{\mp}(-\sqrt{s},t) = G_n(s,t) \pm \sqrt{s}\ G_{n+4}(s,t), \quad n \in \{1,2,3\}. \tag{5.44}$$

Finally, he can derive correlations between $G_n^{\pm}(\sqrt{s},t)$ and the invariant functions $F_n^{\pm}(\sqrt{s},t)$ [63]:

$$F_1^{\pm} = G_1^{\pm}, \quad F_2^{\pm} = G_2^{\pm} \pm \frac{G_1^{\pm}}{\sqrt{s}} + \frac{(w\cdot q)G_3^{\pm}}{s}, \quad F_3^{\pm} = G_3^{\pm}. \tag{5.45}$$

Again, the MacDowell symmetry relations connect corresponding invariant functions $F_i^{+}(\sqrt{s},t)$ and $F_i^{-}(\sqrt{s},t)$ together:

$$F_i^{+}(-\sqrt{s},t) = F_i^{-}(\sqrt{s},t). \tag{5.46}$$

Our investigation does not only cover the processes of the kind of $\gamma B \to PB$ but also $\gamma B \to VB$. The total number of helicity amplitudes is now 18 according to Eq. (5.12). We turn to the elastic scattering of a photon off a ($J^P = \frac{1}{2}^+$)-fermion with a ($J^P = 1^-$)-boson and a ($J^P = \frac{1}{2}^+$)-fermion in the final state. First, we exclude both the final polarisation vector $\epsilon^*_{\bar{\mu}}(\bar{q},\bar{\alpha})$ of the final vector meson and the initial polarisation vector $\epsilon_\mu(q,\alpha)$, and the final spinor $\bar{u}(\bar{p},\bar{\lambda})$ and the initial spinor $u(p,\lambda)$ for the incoming and outgoing octet baryon, respectively:

$$T_{\gamma\frac{1}{2}\to 1\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\bar{\lambda})\ \epsilon^*_{\bar{\mu}}(\bar{q},\bar{\alpha})\ \bar{T}^{\bar{\mu}\mu}_{\gamma\frac{1}{2}\to 1\frac{1}{2}}\ \epsilon_\mu(q,\alpha)\ u(p,\lambda). \tag{5.47}$$

$\bar{T}^{\bar{\mu}\mu}_{\gamma\frac{1}{2}\to 1\frac{1}{2}}$ is decomposed into nine invariant functions $F_i^{+}(\sqrt{s},t)$ and their nine counterparts $F_i^{-}(\sqrt{s},t)$:

$$\bar{T}^{\bar{\mu}\mu}_{\gamma\frac{1}{2}\to 1\frac{1}{2}}(\bar{q},q,w) = \sum_{\pm}\sum_{n=1}^{9}\left(F_n^{\pm}(\sqrt{s},t)\ T^{\bar{\mu}\mu}_{\pm,n}\right). \tag{5.48}$$

Again, the Lorentz-Dirac structures from our former considerations serve to construct the 18-dimensional basis

$$\begin{aligned}
T^{\bar{\mu}\mu}_{\pm,1} &= \hat{g}^{\bar{\mu}\mu}P_{\pm}, & T^{\bar{\mu}\mu}_{\pm,2} &= \hat{\gamma}^{\bar{\mu}}P_{\pm}\hat{\gamma}^{\mu}, \\
T^{\bar{\mu}\mu}_{\pm,3} &= \hat{\gamma}^{\bar{\mu}}P_{\pm}w^{\mu}, & T^{\bar{\mu}\mu}_{\pm,4} &= w^{\bar{\mu}}P_{\pm}\hat{\gamma}^{\mu}, \\
T^{\bar{\mu}\mu}_{\pm,5} &= \hat{\gamma}^{\bar{\mu}}P_{\pm}\bar{r}^{\mu}, & T^{\bar{\mu}\mu}_{\pm,6} &= r^{\bar{\mu}}P_{\pm}\hat{\gamma}^{\mu}, \\
T^{\bar{\mu}\mu}_{\pm,7} &= w^{\bar{\mu}}P_{\pm}\bar{r}^{\mu}, & T^{\bar{\mu}\mu}_{\pm,8} &= r^{\bar{\mu}}P_{\pm}w^{\mu}, \\
T^{\bar{\mu}\mu}_{\pm,9} &= w^{\bar{\mu}}P_{\pm}w^{\mu},
\end{aligned} \tag{5.49}$$

in which we defined the Lorentz-Dirac structure $\hat{g}^{\bar{\mu}\mu}$ perpendicular to w_μ:

$$\hat{g}^{\bar{\mu}\mu} = g^{\bar{\mu}\mu} - \frac{w^{\bar{\mu}} w^{\mu}}{s}, \quad \hat{g}^{\bar{\mu}\mu} w_\mu = 0. \tag{5.50}$$

In addition to the following trace equation [24],

$$\frac{1}{2} \, tr(T^{\bar{\mu}\mu}_{a,n} \Lambda Q^{b,k}_{\bar{\mu}\mu} \bar{\Lambda}) = \delta_{ab} \delta_{nk}, \tag{5.51}$$

two Ward identities complete our projection algebra for this case:

$$\bar{q}^{\bar{\mu}} Q^{\pm,k}_{\bar{\mu}\mu} = 0, \quad q^{\mu} Q^{\pm,k}_{\bar{\mu}\mu} = 0. \tag{5.52}$$

We present two of the 18 projections $Q^{\pm,k}_{\bar{\mu}\mu}$ on the basis vectors as examples [24]:

$$\begin{aligned} Q^{\pm,2}_{\bar{\mu}\mu} &= r_{\rfloor\lfloor,\bar{\mu}} (P_\pm - 2(\bar{r} \cdot r) Q_\mp) \bar{r}_{\rfloor\lfloor,\mu} - (\bar{r}_{\rfloor\lfloor} \cdot r_{\rfloor\lfloor}) \frac{1}{v^2} v_{\bar{\mu}} (P_\pm - 2(\bar{r} \cdot r) Q_\mp) v_\mu \\ &+ \bar{E}_\pm \, i \, \gamma_5 \frac{\sqrt{s}}{v^2} v_{\bar{\mu}} (P_\mp + 2(\bar{r} \cdot r) R_\pm) \bar{r}_{\rfloor\lfloor,\mu} - E_\pm \, i \, \gamma_5 \frac{\sqrt{s}}{v^2} r_{\rfloor\lfloor,\bar{\mu}} (P_\pm + 2(\bar{r} \cdot r) R_\mp) v_\mu. \end{aligned} \tag{5.53}$$

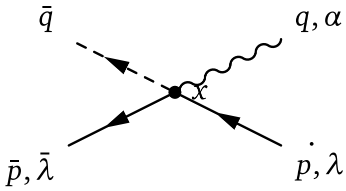
These basis vectors are linear compositions of the components v^μ (Eq. (5.28)), $r^{\bar{\mu}}_{\rfloor\lfloor}$ (Eq. (5.40)), $(\bar{r} \cdot r)$ (Eq. (5.19)), $P_\pm$ (Eq. (5.23)), $Q_\pm$ (Eq. (5.27)), $E_\pm$ (Eq. (5.28)), $\bar{E}_\pm$ (Eq. (5.28)) and $R_\pm$ (Eq. (5.39)). The objects $\bar{r}_{\rfloor\lfloor,\mu}$, $\bar{w}_{\rfloor,\mu}$ and $\bar{w}_{\lfloor,\mu}$ are introduced in Eq. (5.43).

We present the complete set of the 18 projection objects $Q^{\pm,k}_{\bar{\mu}\mu}$ in the appendix E.

5.4 Examples of invariant functions for photoproduction

We decompose the kinematical part of on-shell scattering amplitudes on tree-level for the photoproduction processes $\gamma B \to PB$ and $\gamma B \to VB$. While a photon γ and the states B of the $(J^P = \frac{1}{2}^+)$-baryon octet appear in the initial channel, the final meson-baryon state may consist of a baryon octet state and of either a state of the pseudoscalar (P) or the vector mesons (V). We present examples of invariant functions for the contact term processes and the s-, u- and t-channel processes with the corresponding counterterms of the chiral $SU(3)$ Lagrangian. It is pointed out that our calculations are done under the assumption of massless Goldstone bosons.

The contact term process which is related to the scattering reaction $\gamma B \to PB$ is illustrated by the following Feynman diagram:



(5.54)

Figure 5.1.: Contact interaction of the reaction $\gamma B \rightarrow PB$

The initial (final) boson and fermion momentum is denoted by q and p ($\bar{q}$ and $\bar{p}$), respectively. The polarization α of the initial photon is determined as well as the helicities λ and $\bar{\lambda}$ of the initial and the final baryon. We turn to the basis vector (Eq. (5.33))

$$T^{\mu}_{+,2} = \gamma_5 \; i \; P_+ w^{\mu} \tag{5.55}$$

with the corresponding projection onto this basis vector (Eq. (E.3))

$$Q^{+,2}_{\mu} = -\bar{R}_+ \; i \; \gamma_5 \bar{w}_{\rfloor,\mu} + \frac{\delta+1}{2}\frac{\sqrt{s}}{v^2}\bar{E}_+ Q_+ v_{\mu} \tag{5.56}$$

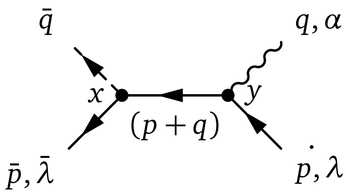
and present the corresponding invariant function $F^+_2(\sqrt{s}, t)$ which was calculated via a Mathematica Code:

$$F^+_2(\sqrt{s}, t) = -\frac{i}{\sqrt{s}}. \tag{5.57}$$

The complete set of invariant functions $F^{\pm}_n(\sqrt{s}, t)$ is given in the appendix G. We derive the corresponding on-shell scattering amplitude via the following parts of the chiral $SU(3)$ Lagrangian (for details, see appendix G):

$$\left(-\frac{ieF_A}{2f}\right) \; tr\left(\bar{B}\gamma^{\mu}\gamma_5\left[\left[QA_{\mu}, \Phi\right], B\right]\right), \quad \left(-\frac{ieD_A}{2f}\right) \; tr\left(\bar{B}\gamma^{\mu}\gamma_5\left\{\left[QA_{\mu}, \Phi\right], B\right\}\right). \tag{5.58}$$

The s-channel process with a ($J^P = \frac{1}{2}^+$)-baryon as the intermediate particle that is related to the scattering reaction $\gamma B \rightarrow PB$ is illustrated by the following Feynman diagram:



(5.59)

Figure 5.2.: S-channel process of the reaction $\gamma B \rightarrow PB$

The four-momentum $(p+q)$ of the intermediate baryon is enforced by the conservation of the four-momentum. The application of our decomposition scheme of section 5.3 to the basis vector of Eq. (5.33)

$$T^{\mu}_{+,1} = \gamma_5 \; i \; P_+ \hat{\gamma}^{\mu} \tag{5.60}$$

with the corresponding projection on this basis vector (Eq. (E.3))

$$Q^{+,1}_{\mu} = -\frac{\sqrt{s}}{v^2} P_+ v_{\mu} \tag{5.61}$$

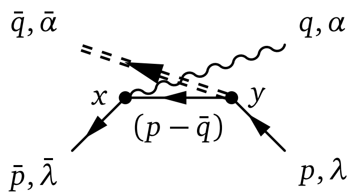
leads to the invariant function $F^+_1(\sqrt{s}, t)$:

$$F^+_1(\sqrt{s}, t) = \frac{\sqrt{s} m_B + s - 2w \cdot q}{s - m_B^2}. \tag{5.62}$$

Again, the complete set of invariant functions $F^{\pm}_n(\sqrt{s}, t)$ is given in the appendix G. The $(J^P = \frac{1}{2}^+)$-baryon mass is denoted by m_B. Here, the utilised Lagrangians read (for details, see appendix G):

$$\begin{aligned} & i \; e \; tr\left(\bar{B}\gamma^{\mu}\left[QA_{\mu}, B\right]\right), \\ & \left(-\frac{F_A}{2f}\right) \; tr\left(\bar{B}\gamma_{\mu}\gamma_5\left[\partial^{\mu}\Phi, B\right]\right), \quad \left(-\frac{D_A}{2f}\right) \; tr\left(\bar{B}\gamma_{\mu}\gamma_5\left\{\partial^{\mu}\Phi, B\right\}\right). \end{aligned} \tag{5.63}$$

The reaction $\gamma B \to VB$ with the focus on the u-channel process with a $(J^P = \frac{1}{2}^+)$-baryon as the intermediate particle



(5.64)

Figure 5.3.: U-channel process of the reaction $\gamma B \to VB$

together with the basis vector (Eq. (5.49))

$$T^{\bar{\mu}\mu}_{+,4} = w^{\bar{\mu}} P_+ \hat{\gamma}^{\mu} \tag{5.65}$$

and the corresponding projection (Eq. (E.3))

$$Q^{+,4}_{\bar{\mu}\mu} = \frac{\sqrt{s}}{v^2} w^{\bar{\mu}}_{\rfloor} P_+ \; i \; \gamma_5 v^{\mu} + \frac{1}{2}(\bar{\delta} + 1)\frac{s}{v^2}((\bar{r} \cdot r)E_+ Q^{\bar{\mu}\mu}_{-,2} - (r \cdot r)\bar{E}_- Q^{\bar{\mu}\mu}_{+,2}) \tag{5.66}$$

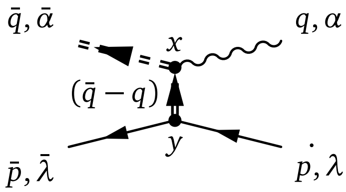
is related to the exemplified invariant function $F^+_4(\sqrt{s}, t)$:

$$F_4^{+}(\sqrt{s},t) = \frac{(2m_B - \sqrt{s})\left(m_B^2 - s + 2w\cdot\bar{q}\right)}{\left(m_B^2 + 2\bar{q}\cdot q - s\right)\sqrt{s}}. \tag{5.67}$$

All relevant invariant functions $F_n^{\pm}(\sqrt{s},t)$ are given in the appendix G. The desired reaction amplitudes are computed via the following Lagrangian contributions with vector meson fields in the tensor representation (for details, see appendix G):

$$\begin{aligned}
& i\, e\, tr\left(\bar{B}\gamma^{\mu}\left[QA_{\mu},B\right]\right),\\
& \left(-\frac{F_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^{\alpha}\left[\partial^{\beta}V_{\alpha\beta},B\right]\right),\quad \left(-\frac{D_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^{\alpha}\left\{\partial^{\beta}V_{\alpha\beta},B\right\}\right),\\
& \left(-\frac{G_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^{\alpha}B\right)\, tr\left(\partial^{\beta}V_{\alpha\beta}\right),\\
& \left(-\frac{F_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}\left[V_{\alpha\beta},B\right]\right),\quad \left(-\frac{D_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}\left\{V_{\alpha\beta},B\right\}\right),\\
& \left(-\frac{G_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}B\right)\, tr\left(V_{\alpha\beta}\right).
\end{aligned} \tag{5.68}$$

The t-channel process of $\gamma B \to VB$ with a $(J^P = 1^-)$-meson as the intermediate particle is illustrated by



(5.69)

Figure 5.4.: T-channel process of the reaction $\gamma B \to VB$

The helicity of the outgoing vector meson is denoted by $\bar{\lambda}$. The intermediate particle carries the four-momentum $(\bar{q}-q)$. We choose the basis vector (Eq. (5.49))

$$T^{\bar{\mu}\mu}_{-,7} = w^{\bar{\mu}} P_{\_}\bar{r}^{\mu} \tag{5.70}$$

with the corresponding projection on the basis vector (Eq. (E.3))

$$\begin{aligned}
Q^{-,7}_{\bar{\mu}\mu} = {} & Q_{_}(w^{\bar{\mu}}_{\rfloor}\bar{r}^{\mu}_{\rfloor\lfloor} - (w_{\rfloor}\cdot\bar{r}_{\rfloor\lfloor})\frac{1}{v^2}v^{\bar{\mu}}v^{\mu}) - (r\cdot r)\bar{E}_{+}\frac{s}{v^2}(Q^{\bar{\mu}\mu}_{+,4} - \frac{1}{2}(\bar{\delta}+1)Q^{\bar{\mu}\mu}_{-,5})\\
& + (\bar{r}\cdot r)E_{+}\frac{s}{v^2}(Q^{\bar{\mu}\mu}_{-,4} + \frac{1}{2}(\bar{\delta}+1)Q^{\bar{\mu}\mu}_{+,5}).
\end{aligned} \tag{5.71}$$

The application of our Mathematica Code leads to the invariant amplitude $F_7^{-}(\sqrt{s},t)$

$$F_7^{-}(\sqrt{s},t) = -\frac{2i}{\sqrt{s}\left(2 - \frac{m_{1^-}^2}{m_B^2 - \bar{q}\cdot q - s + w\cdot(\bar{q}+q)}\right)}. \tag{5.72}$$

Like for the other examples, the complete set of the corresponding analytic functions is presented in the appendix G. We denote the vector meson mass by m_{1^-} to distinguish it from the chiral parameter m_V. Here, we obtained our result via the following parts of the chiral $SU(3)$ Lagrangian (for details, see appendix G):

$$
\begin{aligned}
&\left(-\frac{ie}{4}\right)\, tr\left(\partial^\rho V_{\rho\sigma}\left[QA_\mu, V^{\mu\sigma}\right]\right), \quad \left(-\frac{ie}{4}\right)\, tr\left(\left[QA_\mu, V^{\mu\sigma}\right]\partial^\rho V_{\rho\sigma}\right),\\
&\left(-\frac{ie_M}{4}\right)\, tr\left(\left[V_\rho^\nu, V^{\rho\mu}\right] QF_{\nu\mu}\right),\\
&\left(-\frac{F_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^\alpha\left[\partial^\beta V_{\alpha\beta}, B\right]\right), \quad \left(-\frac{D_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^\alpha\left\{\partial^\beta V_{\alpha\beta}, B\right\}\right),\\
&\left(-\frac{G_V}{2m_V}\right)\, tr\left(\bar{B}\gamma^\alpha B\right)\, tr\left(\partial^\beta V_{\alpha\beta}\right),\\
&\left(-\frac{F_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}\left[V_{\alpha\beta}, B\right]\right), \quad \left(-\frac{D_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}\left\{V_{\alpha\beta}, B\right\}\right),\\
&\left(-\frac{G_T m_V}{8f}\right)\, tr\left(\bar{B}\sigma^{\alpha\beta}B\right)\, tr\left(V_{\alpha\beta}\right).
\end{aligned}
\tag{5.73}
$$

6 Conclusions

We examined strong interactions in the low-energy regime in terms of two complementary non-perturbative approaches: the interplay of large-N_c QCD and chiral perturbation theory (χPT) was studied. While the expansion in the parameter $1/N_c$ is based on quark and gluon degrees of freedom, χPT uses hadrons as effective degrees of freedom. The focus of our work was the investigation of mesons and baryons composed from up-, down- and strange quarks. We used the chiral $SU(3)$ Lagrangian with ($J^P = \frac{1}{2}^+$)- and ($J^P = \frac{3}{2}^+$)-baryon ground states as building blocks. In the $SU(3)$-flavour limit the latter form an octet and a decuplet, respectively.

Studies in χPT hold a challenge: the chiral Lagrangian consists of an infinite number of terms. The treatment of low-energy QCD physics via a perturbation theory requires the ordering of these terms according to their relevance. We used the interplay between large-N_c QCD and χPT to shed light on the structure of the chiral Lagrangian. In the limit of large-N_c the low-energy parameters of the chiral Lagrangian are correlated. For instance the masses of the two baryon multiplets turn degenerate in the $SU(3)$-flavour limit. This serves as the starting point of our investigations. In this work we analysed the time-ordered product of two scalar and two vector currents in the baryon ground state. The examination of these matrix elements at large-N_c was compared to corresponding results derived in χPT. From this we obtained sum rules for some low-energy parameters of the chiral Lagrangian. The results for the vector correlation function were used to constrain a phenomenological interaction of light vector mesons with the baryon ground states.

In the second part of this thesis we addressed a formal problem which arises in a partial wave decomposition of reaction amplitudes for particles with non-vanishing spin. In particular we considered the vector meson photoproduction off the nucleon as it is currently studied in [1–3]. A decomposition of on-shell production amplitudes into covariant partial wave amplitudes which are both free from kinematical constraints and compatible with the microcausality condition was achieved. A Mathematica code using the FeynCalc package [4] was written and applied to some tree-level contact terms and s-, u- and t-channel processes.

A Conventions

A.1 Natural units

The examinations throughout this thesis are done in natural units, i. e.

$$\hbar = c = k_B = 1. \tag{A.1}$$

The definition contains the reduced Planck constant $\hbar = \frac{h}{2\pi}$, the speed of light in vacuum c and the Boltzmann constant k_B.

A.2 Notation

The commutator and the anticommutator of two arbitrary operators A and B are expressed throughout this thesis as follows:

$$[A,B] \equiv [A,B]_- \equiv AB - BA, \tag{A.2}$$
$$\{A,B\} \equiv [A,B]_+ \equiv AB + BA. \tag{A.3}$$

A.3 Pauli matrices

The linear vector space of all complex 2×2-matrices $\mathbb{C}_{(2\times 2)}$ is spanned by the three Pauli matrices σ_1, σ_2, σ_3 in combination with the two-dimensional unit matrix σ_0. The Hermitian Pauli matrices are commonly expressed through

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.4}$$

They are traceless and satisfy the properties [31]

$$\begin{gathered} \det \sigma_i = -1, \quad \sigma_i^t = \sigma_i^* = -\sigma_2 \sigma_i \sigma_2, \\ [\sigma_i, \sigma_j] = 2i\, \epsilon_{ijk} \sigma_k, \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij}. \end{gathered} \tag{A.5}$$

A.4 Metric & derivatives

The metric of our relativistic framework is specified with the help of the metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.6}$$

It can be used to define products of two contravariant or covariant quantities:

$$g_{\mu\nu}V^\mu W^\nu = V_\nu W^\nu = V^\nu W_\nu = g^{\mu\nu}V_\mu W_\nu = V\cdot W = V^0W^0 - \vec{V}\cdot\vec{W} = V^0W^0 - V^iW^i. \qquad \text{(A.7)}$$

We introduced the Einstein convention in the last step: the summation over both correspondent Lorentz (Greek) and spacial (Latin) indices is done without explicit notation. To derive scattering amplitudes, we need derivatives of both contravariant and covariant spacial coordinates x^μ and x_μ, respectively:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \qquad \partial^\mu \equiv \frac{\partial}{\partial x_\mu}. \qquad \text{(A.8)}$$

The explicit expressions for both derivatives read:

$$\partial_\mu = \begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}, \qquad \partial^\mu = \begin{pmatrix} \frac{\partial}{\partial t} \\ -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} \\ -\frac{\partial}{\partial z} \end{pmatrix}. \qquad \text{(A.9)}$$

The investigation of scattering amplitudes also requires the specification of the totally antisymmetric Levi-Civita tensor:

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{if } \{\mu,\nu,\rho,\sigma\} \text{ is an even permutation of } \{0,1,2,3\} \\ -1 & \text{if it is an odd permutation} \\ 0 & \text{otherwise} \end{cases}. \qquad \text{(A.10)}$$

A.5 Dirac matrices

In this thesis, we make use of the four-component γ matrices which satisfy the following anticommutation relations [65]:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \qquad \text{(A.11)}$$

The Hermitian component γ^0 and the anti-Hermitian components γ^i of γ^μ are used to define γ_5 through [65]

$$\begin{aligned} \gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 &= -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \\ &= -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^3\gamma^2\gamma^1\gamma^0 = \gamma_5^\dagger, && \text{(A.12)} \\ \gamma_5^2 &= \mathbb{1}, && \text{(A.13)} \\ \{\gamma_5, \gamma^\mu\} &= 0. && \text{(A.14)} \end{aligned}$$

We introduce the antisymmetric quantity $\sigma^{\mu\nu}$ as the commutator of the γ matrices [65]:

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu], \tag{A.15}$$
$$\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}, \tag{A.16}$$
$$[\gamma_5, \sigma^{\mu\nu}] = 0, \tag{A.17}$$
$$\gamma_5\sigma^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}. \tag{A.18}$$

A.6 SU(3) group

The $SU(3)$ group is described by the eight Hermitian Gell-Mann matrices λ_i, $i = 1, \ldots 8$:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{A.19}$$

These matrices obey the relationships

$$\lambda_a = \lambda_a^\dagger, \tag{A.20}$$
$$tr(\lambda_a\lambda_b) = 2\delta_{ab}, \tag{A.21}$$
$$tr(\lambda_a) = 0. \tag{A.22}$$

They are traceless and fulfil the commutation relationships of a Lie algebra [35]:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i\, f_{abc}\frac{\lambda_c}{2}, \tag{A.23}$$
$$\{\lambda_a, \lambda_b\} = \frac{4}{3}\delta_{ab} + 2d_{abc}\lambda_c. \tag{A.24}$$

f_{abc} is the totally antisymmetric tensor and takes the value 0 except for the cases

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}, \quad f_{156} = f_{367} = -\frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{A.25}$$

It is connected to the Gell-Mann matrices of the $SU(3)$ group via

$$f_{abc} = \frac{1}{4i}\, tr([\lambda_a, \lambda_b]\lambda_c). \tag{A.26}$$

The totally symmetric tensor d_{abc} is obtained by

$$d_{abc} = \frac{1}{4}\, tr(\{\lambda_a, \lambda_b\}\lambda_c). \tag{A.27}$$

The non-vanishing d_{abc} symbols of $SU(3)$ are given in the following:

$$\begin{gathered} d_{118} = d_{228} = d_{338} = \frac{1}{\sqrt{3}}, \quad d_{146} = d_{157} = d_{256} = d_{344} = d_{355} = \frac{1}{2}, \quad d_{247} = d_{366} = d_{377} = -\frac{1}{2}, \\ d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}, \quad d_{888} = -\frac{1}{\sqrt{3}}. \end{gathered} \tag{A.28}$$

We introduced the vector meson nonet in this thesis which requires the definition of an additional matrix λ_0 [35]:

$$\lambda_0 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A.29}$$

A possible basis for the linear vector space of all 3×3-matrices is given by λ_i, $i = 0, \ldots, 8$ [35].

B S-matrices

The construction of our coupled-channel notation in isospin-strangeness basis requires the definition of the "spin-$\frac{1}{2}$-to-spin-$\frac{3}{2}$"-transition matrices S_1, S_2 and S_3. We need these transition matrices to derive the spin-$\frac{3}{2}$ spinors $u^\mu(p,\lambda)$ directly from the spin-$\frac{1}{2}$ spinors $u(p,s)$. The spin-$\frac{1}{2}$ spinors read [31]

$$u(p,s) = \sqrt{\frac{E_p + M}{2M}} \begin{pmatrix} \mathbb{1}_{(2\times 2)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} \end{pmatrix} \chi_s^{(1/2)}, \quad \chi_1^{(1/2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_2^{(1/2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{B.1}$$

This result together with the coupling of spin-1 polarisation and the spin-$\frac{1}{2}$ spinors leads to the spin-$\frac{3}{2}$ spinors [31]

$$u^\mu(p,s) = \sum_{\lambda',s} C\left(1\lambda\ \frac{1}{2}s' | \frac{3}{2}s\right) \epsilon^\mu(p,\lambda) u(p,s'). \tag{B.2}$$

The quantities λ' and s' represent the eigenvalues of the spin projection operator in z-direction in the spin space, so we conclude

$$\lambda' \in \{-1,0,1\}, \quad s' \in \{-\tfrac{1}{2}, \tfrac{1}{2}\}. \tag{B.3}$$

We can identify an equation for the spin-$\frac{3}{2}$ spinors similar to (B.1),

$$\begin{gathered} u^\mu(p,s) = \sqrt{\frac{E_p + M}{2M}} \begin{pmatrix} S^{\mu,\dagger}(p) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} S^{\mu,\dagger}(p) \end{pmatrix} \chi_s^{(3/2)}, \\ \chi_1^{(3/2)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_2^{(3/2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_3^{(3/2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi_4^{(3/2)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \end{gathered} \tag{B.4}$$

in which the components of the contravariant expression $S^{\mu,\dagger}(p)$ are connected to the S-matrices S_1, S_2 and S_3 [31]:

$$\begin{gathered} S^{0,\dagger}(p) = \frac{|\vec{p}|}{M} S_3^T, \quad S^{1,\dagger}(p) = S_1^T, \\ S^{2,\dagger}(p) = S_2^T, \quad S^{3,\dagger}(p) = \frac{E_p}{M} S_3^T \end{gathered} \tag{B.5}$$

with

$$S_1 = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad S_2 = \begin{pmatrix} \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{6}} \\ \frac{i}{\sqrt{6}} & 0 \\ 0 & \frac{i}{\sqrt{2}} \end{pmatrix}, \quad S_3 = \begin{pmatrix} 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 \\ 0 & \sqrt{\frac{2}{3}} \\ 0 & 0 \end{pmatrix}, \qquad \vec{S} = (S_1, S_2, S_3)^T. \tag{B.6}$$

They obey certain properties like [31]

$$\vec{S}^\dagger \cdot \vec{S} = 2\, \mathbb{1}_{(2\times2)}, \quad \vec{S} \cdot \vec{S}^\dagger = \mathbb{1}_{(4\times4)}, \quad \vec{\sigma} \cdot \vec{S}^\dagger = 0, \quad \vec{S} \cdot \vec{\sigma} = 0, \tag{B.7}$$

$$S_i^\dagger S_j = \delta_{ij} \cdot \mathbb{1}_{(2\times2)} - \frac{1}{3}\sigma_i\sigma_j = \delta_{ij} \cdot \mathbb{1}_{(2\times2)} - \frac{1}{3}(\delta_{ij} + i\epsilon_{ijk}\sigma_k) = \frac{2}{3}(\delta_{ij} \cdot \mathbb{1}_{(2\times2)} - i\epsilon_{ijk}\,\frac{1}{2}\sigma_k), \tag{B.8}$$

$$S_i\sigma_j - S_j\sigma_i = -i\epsilon_{ijk}T_k, \quad S_i\sigma_k\sigma_i = 2S_k, \quad i\epsilon_{ijk}S_i\sigma_j = \sigma_k, \tag{B.9}$$

$$\epsilon_{ijk}S_iS_j^\dagger = i\vec{S}\sigma_k\vec{S}^\dagger, \quad \epsilon_{ijk}(\vec{S}\sigma_i\vec{S}^\dagger)(\vec{S}\sigma_j\vec{S}^\dagger) = \frac{2}{3}\epsilon_{ijk}S_iS_j^\dagger = \frac{2}{3}\, i\vec{S}\sigma_k\vec{S}^\dagger, \tag{B.10}$$

$$(\vec{S}\sigma_k^*\vec{S}^\dagger) = (\vec{S}\sigma_k\vec{S}^\dagger)^t, \quad \sum_i (\vec{S}\sigma^{(i)}\vec{S}^\dagger)(\vec{S}\sigma^{(i)}\vec{S}^\dagger) = \frac{5}{3}\mathbb{1}_{(4\times4)}. \tag{B.11}$$

All of these properties were proven explicitly with the help of Mathematica 10.

C $SU(2) \times SU(3)$ operator identities

We present the complete set of operator identities for the contracted $SU(6)$ group ($N_F = 3$). The entries in the column on the right determine the transformation behaviour of each identity under the $SU(2) \times SU(3)$ group [16]:

$2\ \{J^i, J^i\} + 3\ \{T^a, T^a\} + 12\ \{G^a_i, G^a_i\} = 5\ N_c\ (N_c + 6)$	$(0,0)$
$d_{abc}\ \{G^a_i, G^b_i\} + \frac{2}{3}\ \{J^i, G^c_i\} + \frac{1}{4}\ d_{abc}\ \{T^a, T^b\} = \frac{2}{3}\ (N_c + 3)\ T^c$	$(0,8)$
$\{T^a, G^a_i\} = \frac{2}{3}\ (N_c + 3)\ J^i$	$(1,0)$
$\frac{1}{3}\ \{J^k, T^c\} + d_{abc}\ \{T^a, G^b_k\} - \epsilon_{ijk} f_{abc}\ \{G^a_i, G^b_j\} = \frac{4}{3}\ (N_c + 3)\ G^c_k$	$(1,8)$
$-12\ \{G^a_i, G^a_i\} + 27\ \{T^a, T^a\} - 32\ \{J^i, J^i\} = 0$	$(0,0)$
$d_{abc}\ \{G^a_i, G^b_i\} + \frac{9}{4} d_{abc}\ \{T^a, T^b\} - \frac{10}{3}\ \{J^i, G^c_i\} = 0$	$(0,8)$
$4\ \{G^a_i, G^b_i\} = \{T^a, T^b\} \qquad (27)$	$(0,27)$
$\epsilon_{ijk}\ \{J^i, G^c_j\} = f_{abc}\ \{T^a, G^b_k\}$	$(1,8)$
$3 d_{abc}\ \{T^a, G^b_k\} = \{J^k, T^c\} - \epsilon_{ijk} f_{abc}\ \{G^a_i, G^b_j\}$	$(1,8)$
$\epsilon_{ijk}\ \{G^a_i, G^b_j\} = f_{acg} d_{bch}\ \{T^g, G^h_k\} \qquad (10 + \overline{10})$	$(1, 10 + \overline{10})$
$3\ \{G^a_i, G^a_j\} = \{J^i, J^j\} \qquad (J = 2)$	$(2,0)$
$3 d_{abc}\ \{G^a_i, G^b_j\} = \{J^i, G^c_j\} \qquad (J = 2)$	$(2,8)$

Table C.1.: $SU(2) \times SU(N_F)$ operator identities for $(N_F = 3)$ with corresponding transformation pattern [16]

D Baryon matrix elements of symmetric products of two one-body operators

The baryon matrix elements of the symmetric products of two one-body operators from [18] were carefully cross-checked. We present the results with two slight corrections:

$$(d,\bar{\chi}|\{J_i,J_j\}|c,\chi) = \frac{1}{2}\delta_{ij}\delta_{\bar{\chi}\chi}\delta_{cd}, \tag{D.1}$$

$$(d,\bar{\chi}|\{J_i,T^a\}|c,\chi) = \sigma^{(i)}_{\bar{\chi}\chi} i f_{cda}, \tag{D.2}$$

$$(d,\bar{\chi}|\{J_i,G^a_j\}|c,\chi) = \delta_{ij}\delta_{\bar{\chi}\chi}\left(\frac{1}{2}d_{cda}+\frac{i}{3}f_{cda}\right), \tag{D.3}$$

$$(d,\bar{\chi}|\{T^a,T^b\}|c,\chi) = \delta_{\bar{\chi}\chi}\left(\delta_{ab}\delta_{dc}-(\delta_{ad}\delta_{bc}+\delta_{bd}\delta_{ca})+3d_{abe}d_{ecd}\right), \tag{D.4}$$

$$\begin{aligned}(d,\bar{\chi}|\{T^a,G^b_i\}|c,\chi) = \sigma^{(i)}_{\bar{\chi}\chi}\Bigg(&\frac{1}{3}\delta_{ab}\delta_{dc}-\frac{1}{3}(\delta_{ad}\delta_{bc}+\delta_{bd}\delta_{ca})+d_{abe}\left(d_{ecd}+\frac{i}{2}f_{ecd}\right)\\ &-\frac{i}{2}(d_{abe}f_{bce}+f_{ade}d_{bce})\Bigg),\end{aligned} \tag{D.5}$$

$$\begin{aligned}(d,\bar{\chi}|\{G^a_i,G^b_j\}|c,\chi) = {}&\delta_{ij}\delta_{\bar{\chi}\chi}\left(\frac{5}{12}\delta_{ab}\delta_{dc}-\frac{1}{12}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})-\left(\frac{1}{4}d_{cde}-\frac{2}{3}if_{cde}\right)d_{abe}\right)\\ &+i\epsilon_{ijk}\sigma^{(k)}_{\bar{\chi}\chi}\left(\frac{1}{4}(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})+\left(\frac{1}{2}d_{dce}+\frac{5}{12}if_{cde}\right)if_{abe}\right),\end{aligned} \tag{D.6}$$

$$(nop,\bar{\chi}|\{J_i,G^a_j\}|c,\chi) = \frac{1}{4\sqrt{2}}\left(3i\epsilon_{ijk}S_k+S_i\sigma_j+S_j\sigma_i\right)_{\bar{\chi}\chi}\Lambda^{nop}_{ac}, \tag{D.7}$$

$$(nop,\bar{\chi}|\{T^a,G^b_i\}|c,\chi) = \frac{i}{2\sqrt{2}}S^{(i)}_{\bar{\chi}\chi}\left(f_{abd}\Lambda^{nop}_{dc}+2f_{acd}\Lambda^{nop}_{bd}\right), \tag{D.8}$$

$$\begin{aligned}(nop,\bar{\chi}|\{G^a_i,G^b_j\}|c,\chi) = {}&i\epsilon_{ijk}S^{(k)}_{\bar{\chi}\chi}\frac{1}{8\sqrt{2}}\Bigg(-\left(d_{cdb}+\frac{2}{3}if_{cdb}\right)\Lambda^{nop}_{ad}+\frac{5}{3}\left(if_{abd}\Lambda^{nop}_{dc}+if_{acd}\Lambda^{nop}_{bd}\right)\\ &-(a\longleftrightarrow b)\Bigg)\\ &+\left(S_i\sigma_j+S_j\sigma_i\right)_{\bar{\chi}\chi}\frac{1}{8\sqrt{2}}\left(\Lambda^{nop}_{ad}(d_{bcd}+if_{bcd})+(a\longleftrightarrow b)\right),\end{aligned} \tag{D.9}$$

$$(nop,\bar{\chi}|\{J_i,J_j\}|klm,\chi) = \left(\frac{9}{2}\delta_{ij}\delta_{\bar{\chi}\chi}-3\left(S_iS^\dagger_j+S_jS^\dagger_i\right)_{\bar{\chi}\chi}\right)\delta^{nop}_{klm}, \tag{D.10}$$

$$(nop,\bar{\chi}|\{J_i,T^a\}|klm,\chi) = \frac{9}{2}\left(\vec{S}\sigma_i\vec{S}^\dagger\right)_{\bar{\chi}\chi}\Lambda^{a,nop}_{klm}, \tag{D.11}$$

$$(nop,\bar{\chi}|\{J_i,G^a_j\}|klm,\chi) = \left(\frac{9}{4}\delta_{ij}\delta_{\bar{\chi}\chi}-\frac{3}{2}\left(S_iS^\dagger_j+S_jS^\dagger_i\right)_{\bar{\chi}\chi}\right)\Lambda^{a,nop}_{klm}, \tag{D.12}$$

$$(nop,\bar{\chi}|\{T^a,T^b\}|klm,\chi) = \frac{9}{4}\delta_{\bar{\chi}\chi}\left(\Lambda^{a,nop}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,nop}_{xyz}\Lambda^{a,xyz}_{klm}\right), \tag{D.13}$$

$$(nop,\bar{\chi}|\{T^a,G^b_i\}|klm,\chi) = \frac{9}{8}\left(\vec{S}\sigma_i\vec{S}^\dagger\right)_{\bar{\chi}\chi}\left(\Lambda^{a,nop}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,nop}_{xyz}\Lambda^{a,xyz}_{klm}\right), \tag{D.14}$$

$$(nop,\bar{\chi}|\{G^a_i,G^b_j\}|klm,\chi) = \delta_{ij}\delta_{\bar{\chi}\chi}\left(\frac{9}{16}\Lambda^{a,nop}_{xyz}\Lambda^{b,xyz}_{klm}+(a\longleftrightarrow b)\right)$$

$$
\begin{aligned}
&+ i\epsilon_{ijk}\left(\vec{S}\sigma_k\vec{S}^\dagger\right)_{\bar{\chi}\chi}\left(\frac{1}{16}\Lambda^{nop}_{ac}\Lambda^{bc}_{klm} + \frac{3}{16}\Lambda^{a,nop}_{xyz}\Lambda^{b,xyz}_{klm} - (a \longleftrightarrow b)\right)\\
&+ \left(S_iS_j^\dagger + S_jS_i^\dagger\right)_{\bar{\chi}\chi}\left(\frac{1}{16}\Lambda^{nop}_{ac}\Lambda^{bc}_{klm} - \frac{3}{8}\Lambda^{a,nop}_{xyz}\Lambda^{b,xyz}_{klm} + (a \longleftrightarrow b)\right).
\end{aligned}
\tag{D.15}
$$

E Projection algebra for $\gamma + \frac{1}{2}^+ \to 1^- + \frac{1}{2}^+$

We introduce the decomposition for the scattering of photons off ($J^P = \frac{1}{2}^+$)-fermions via the projection algebra

$$\frac{1}{2}\, tr(T^{\bar\mu\mu}_{a,n} \Lambda Q^{b,k}_{\bar\mu\mu} \bar\Lambda) = \delta_{ab}\delta_{nk}, \tag{E.1}$$

which is completed by the following Ward identities:

$$\bar q^{\bar\mu} Q^{\pm,k}_{\bar\mu\mu} = 0, \quad q^{\mu} Q^{\pm,k}_{\bar\mu\mu} = 0. \tag{E.2}$$

While $T^{\bar\mu\mu}_{a,n}$ denote the basis vectors, the projections of the on-shell scattering amplitude on this basis, $Q^{\pm,k}_{\bar\mu\mu}$, are given as follows [24]:

$$\begin{aligned}
Q^{\pm,1}_{\bar\mu\mu} &= \frac{1}{v^2} v_{\bar\mu} Q_\pm v_\mu - Q^{\mp,2}_{\bar\mu\mu},\\
Q^{\pm,2}_{\bar\mu\mu} &= r_{\rfloor\lfloor,\bar\mu}\left(P_\pm - 2(\bar r\cdot r)Q_\mp\right)\bar r_{\rfloor\lfloor,\mu} - \left(\bar r_{\rfloor\lfloor}\cdot r_{\rfloor\lfloor}\right)\frac{1}{v^2} v_{\bar\mu}\left(P_\pm - 2(\bar r\cdot r)Q_\mp\right) v_\mu\\
&\quad + \bar E_\pm\, i\,\gamma_5 \frac{\sqrt{s}}{v^2} v_{\bar\mu}\left(P_\mp + 2(\bar r\cdot r)R_\pm\right)\bar r_{\rfloor\lfloor,\mu} - E_\pm\, i\,\gamma_5 \frac{\sqrt{s}}{v^2} r_{\rfloor\lfloor,\bar\mu}\left(P_\pm + 2(\bar r\cdot r)R_\mp\right) v_\mu,\\
Q^{\pm,3}_{\bar\mu\mu} &= \mp\frac{\sqrt{s}}{v^2} v_{\bar\mu}\, i\,\gamma_5 P_\pm \bar w_{\rfloor,\mu} \pm \frac{1}{2}(\delta+1)\frac{s}{v^2}\left((\bar r\cdot r)\bar E_\pm Q^{\mp,2}_{\bar\mu\mu} - (\bar r\cdot\bar r)E_\mp Q^{\pm,2}_{\bar\mu\mu}\right),\\
Q^{\pm,4}_{\bar\mu\mu} &= \pm\frac{\sqrt{s}}{v^2} w_{\rfloor,\bar\mu} P_\pm\, i\,\gamma_5 v_\mu \pm \frac{1}{2}\left(\bar\delta+1\right)\frac{s}{v^2}\left((\bar r\cdot r)E_\pm Q^{\mp,2}_{\bar\mu\mu} - (r\cdot r)\bar E_\mp Q^{\pm,2}_{\bar\mu\mu}\right),\\
Q^{\pm,5}_{\bar\mu\mu} &= \mp\frac{\sqrt{s}}{v^2} v_{\bar\mu}\, i\,\gamma_5 P_\pm \bar r_{\rfloor\lfloor,\mu} \pm \frac{s}{v^2}\left((r\cdot r)\bar E_\pm Q^{\mp,2}_{\bar\mu\mu} - (\bar r\cdot r)E_\mp Q^{\pm,2}_{\bar\mu\mu}\right),\\
Q^{\pm,6}_{\bar\mu\mu} &= \pm\frac{\sqrt{s}}{v^2} r_{\rfloor\lfloor,\bar\mu} P_\pm\, i\,\gamma_5 v_\mu \pm \frac{s}{v^2}\left((\bar r\cdot\bar r)E_\pm Q^{\mp,2}_{\bar\mu\mu} - (\bar r\cdot r)\bar E_\mp Q^{\pm,2}_{\bar\mu\mu}\right),\\
Q^{\pm,7}_{\bar\mu\mu} &= Q_\pm\left(w_{\rfloor,\bar\mu}\bar r_{\rfloor\lfloor,\mu} - \left(w_\rfloor\cdot\bar r_{\rfloor\lfloor}\right)\frac{1}{v^2} v_{\bar\mu} v_\mu\right) \pm (r\cdot r)\bar E_\mp \frac{s}{v^2}\left(Q^{\mp,4}_{\bar\mu\mu} - \frac{1}{2}\left(\bar\delta+1\right)Q^{\pm,5}_{\bar\mu\mu}\right)\\
&\quad \mp (\bar r\cdot r)E_\mp \frac{s}{v^2}\left(Q^{\pm,4}_{\bar\mu\mu} \mp \frac{1}{2}\left(\bar\delta+1\right)Q^{\mp,5}_{\bar\mu\mu}\right),\\
Q^{\pm,8}_{\bar\mu\mu} &= Q_\pm\left(r_{\rfloor\lfloor,\bar\mu}\bar w_{\rfloor,\mu} - \left(r_{\rfloor\lfloor}\cdot\bar w_\rfloor\right)\frac{1}{v^2} v_{\bar\mu} v_\mu\right) \pm (\bar r\cdot\bar r)E_\mp \frac{s}{v^2}\left(Q^{\mp,3}_{\bar\mu\mu} - \frac{1}{2}(\delta+1)Q^{\pm,6}_{\bar\mu\mu}\right)\\
&\quad \mp (\bar r\cdot r)\bar E_\mp \frac{s}{v^2}\left(Q^{\pm,3}_{\bar\mu\mu} - \frac{1}{2}(\delta+1)Q^{\mp,6}_{\bar\mu\mu}\right),\\
Q^{\pm,9}_{\bar\mu\mu} &= Q_\pm\left(w_{\rfloor,\bar\mu}\bar w_{\rfloor,\mu} - \left(w_\rfloor\cdot\bar w_\rfloor - \frac{1}{s}\right)\frac{1}{v^2} v_{\bar\mu} v_\mu\right) - \frac{1}{4}\left(\bar\delta+1\right)(\delta+1)\frac{s}{v^2}\left((\bar r\cdot r)Q^{\mp,2}_{\bar\mu\mu} - \bar E_\mp E_\mp Q^{\pm,2}_{\bar\mu\mu}\right)\\
&\quad \pm\frac{1}{2}\left(\bar\delta+1\right)\frac{s}{v^2}\left((\bar r\cdot r)E_\mp Q^{\mp,3}_{\bar\mu\mu} - (r\cdot r)\bar E_\mp Q^{\pm,3}_{\bar\mu\mu}\right) \pm \frac{1}{2}(\delta+1)\frac{s}{v^2}\left((\bar r\cdot r)\bar E_\mp Q^{\mp,4}_{\bar\mu\mu} - (\bar r\cdot\bar r)E_\mp Q^{\pm,4}_{\bar\mu\mu}\right).
\end{aligned} \tag{E.3}$$

F Spinors for $(J^P = \frac{1}{2}^+)$- and $(J^P = \frac{3}{2}^+)$-baryons

The momentum and spin dependence of the spin-1/2 spinors $u(p,s)$ are given by

$$u(p,s) = \sqrt{\frac{E_p + M}{2M}} \begin{pmatrix} \mathbb{1}_{(2\times 2)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} \end{pmatrix} \chi_s^{(1/2)}, \quad \chi_1^{(1/2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_2^{(1/2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{F.1}$$

where the energy E_p, the three-momentum $\vec{p}$ and the four-momentum p are connected through

$$E_p^2 = \vec{p}^2 + M^2, \quad p^2 = M^2. \tag{F.2}$$

To investigate $(J^P = \frac{3}{2}^+)$-baryons, we introduce similarly the spin-3/2 spinor

$$u^\mu(p,s) = \sqrt{\frac{E_p + M}{2M}} \tilde{S}^{\mu\dagger}(p)\chi_s^{(3/2)}, \quad \chi_1^{(3/2)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_2^{(3/2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_3^{(3/2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi_4^{(3/2)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{F.3}$$

The spin-transition matrices $S^{i\dagger}(p)$, $i = 0, \dots, 3$, are expressed in the (4×4)-matrix space:

$$\begin{aligned} S^{0\dagger} &= \frac{|\vec{p}|}{M} \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}, \qquad S^{1\dagger}(p) = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \\ S^{2\dagger}(p) &= \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{6}} & 0 \\ 0 & -\frac{i}{\sqrt{6}} & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}, \quad S^{3\dagger}(p) = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}. \end{aligned} \tag{F.4}$$

We connect the quantity $\tilde{S}^{\mu\dagger}(p)$ of Eq. (F.3) with these spin-transition matrices via

$$S^{\mu\dagger}(p) = \begin{pmatrix} S^{\mu\dagger}(p) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} \end{pmatrix}. \tag{F.5}$$

The spinor sum over all possible spin projections s is identified with the projection operator $\Lambda_+^{\mu\nu}(p)$ for the positive energy states. It shows the subsequent kinematical dependency:

$$\Lambda_+^{\mu\nu}(p) \equiv \sum_s u^\mu(p,s)\bar{u}^\nu(p,s) = -\frac{\not{p} + M}{2M} \left(g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p^\mu p^\nu}{M^2} + \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3M} \right). \tag{F.6}$$

The insertion of $\tilde{S}^{\mu\dagger}(p)$ into Eq. (F.6) provides us with

$$\tilde{S}^{\mu\dagger}(p)\tilde{S}^{\nu}(p) = -\frac{\not{p}+M}{E_p+M}\left(g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p^\mu p^\nu}{M^2} + \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3M}\right),$$
$$\tilde{S}^{0\dagger}(p)S^{0}(p) = \frac{2}{3}\frac{\vec{p}^2}{m^2}\mathbb{1}_{(2\times 2)}. \tag{F.7}$$

To derive the normalisation condition both for the spin-1/2 and the spin-3/2 spinors, we utilise Eqs. (F.1) and (F.3), respectively, and start with the following equations:

$$\bar{u}(p',s')\,\Gamma\,u(p,s) = \chi_{s'}^{\dagger(1/2)}\,\tilde{\Gamma}\,\chi_s^{(1/2)},$$
$$\bar{u}^\mu(p',s')\,\Gamma\,u^\nu(p,s) = \chi_{s'}^{\dagger(3/2)}\left(S^\mu(p')\,\tilde{\Gamma}\,S^{\nu\dagger}(p)\right)\chi_s^{(3/2)}. \tag{F.8}$$

It is the purpose of these equations to perform a non-relativistic expansion with $\tilde{\Gamma}$ up to the order $\mathcal{O}(Q^2/M^2)$ with $|\vec{p}|, |\vec{p}'| \sim \mathcal{O}(Q)$ and $M' = M$. In the case of the normalisation conditions, we obtain for

$$\Gamma = \mathbb{1}, \quad \tilde{\Gamma} = N_{p'}N_p\left(\mathbb{1} - \frac{\vec{\sigma}\cdot\vec{p}'}{E_{p'}+M}\frac{\vec{\sigma}\cdot\vec{p}}{E_p+M}\right) \tag{F.9}$$

the expansion's result

$$\tilde{\Gamma}_{\text{n.r.}} = \mathbb{1}. \tag{F.10}$$

This directly leads to the normalisation conditions:

$$\bar{u}(p,s')u(p,s) = \delta_{s's}, \quad \bar{u}_\mu(p,s')u^\mu(p,s) = -\delta_{s's}. \tag{F.11}$$

The non-relativistic expansion for several other quantities of $\tilde{\Gamma}$, that are relevant for our calculations in section 4.3, are presented in the following:

$$\Gamma = \gamma_0, \qquad \tilde{\Gamma} = N_{\bar{p}}N_p\left(\mathbb{1} + \frac{\vec{\sigma}\cdot\vec{\bar{p}}}{E_{\bar{p}}+M}\frac{\vec{\sigma}\cdot\vec{p}}{E_p+M}\right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \mathbb{1}, \tag{F.12}$$

$$\Gamma = \gamma_i, \qquad \tilde{\Gamma} = N_{\bar{p}}N_p\left(\sigma_i\frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} + \frac{\vec{\sigma}\cdot\vec{\bar{p}}}{E_{\bar{p}}+M}\sigma_i\right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \frac{1}{2M}\left((p+\bar{p})_i + i\epsilon_{jki}(p-\bar{p})_j\sigma_k\right), \tag{F.13}$$

$$\Gamma = \gamma_5, \qquad \tilde{\Gamma} = N_{\bar{p}}N_p\left(\frac{\vec{\sigma}\cdot\vec{p}}{E_p+M} - \frac{\vec{\sigma}\cdot\vec{\bar{p}}}{E_{\bar{p}}+M}\right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \frac{1}{2M}\vec{\sigma}\cdot(\vec{p}-\vec{\bar{p}}), \tag{F.14}$$

$$\Gamma = \gamma_0\gamma_5, \quad \tilde{\Gamma} = N_{\bar{p}}N_p\left(\frac{\vec{\sigma}\cdot\vec{\bar{p}}}{E_{\bar{p}}+M} + \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M}\right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \frac{1}{2M}\vec{\sigma}\cdot(\vec{p}+\vec{\bar{p}}), \tag{F.15}$$

$$\Gamma = \gamma_i \gamma_5, \quad \tilde{\Gamma} = N_{\bar{p}} N_p \left(\sigma_i + \frac{\vec{\sigma} \cdot \vec{\bar{p}}}{E_{\bar{p}} + M} \sigma_i \frac{\vec{\sigma} \cdot \vec{p}}{E_p + M} \right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \sigma_i, \tag{F.16}$$

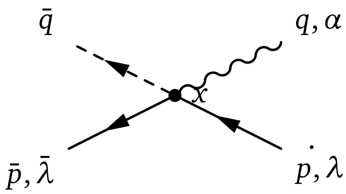
$$\Gamma = \sigma_{0j}, \quad \tilde{\Gamma} = N_{\bar{p}} N_p i \left(\sigma_j \frac{\vec{\sigma} \cdot \vec{p}}{E_p + M} - \frac{\vec{\sigma} \cdot \vec{\bar{p}}}{E_{\bar{p}} + M} \sigma_j \right), \qquad \tilde{\Gamma}_{\text{n.r.}} = \frac{i}{2M} \left((p - \bar{p})_j + i \epsilon_{kij} (p + \bar{p})_k \sigma_i \right), \tag{F.17}$$

$$\Gamma = \sigma_{ij}, \quad \tilde{\Gamma} = N_{\bar{p}} N_p \epsilon_{ijk} \left(\sigma_k - \frac{\vec{\sigma} \cdot \vec{\bar{p}}}{E_{\bar{p}} + M} \sigma_k \frac{\vec{\sigma} \cdot \vec{p}}{E_p + M} \right), \quad \tilde{\Gamma}_{\text{n.r.}} = \epsilon_{ijk} \sigma_k. \tag{F.18}$$

Here, we identify the Latin indices i, j with the spatial indices $1, 2$ and 3.

G Complete sets of the invariant functions $F_n^{\pm}(\sqrt{s},t)$ for the examples in section 5.4

Contact term process which is related to the scattering reaction $\gamma B \to PB$:



(G.1)

Figure G.1.: Contact interaction of the reaction $\gamma B \to PB$

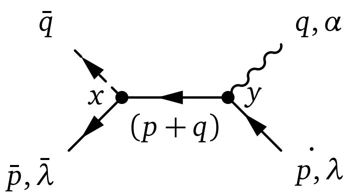
We use the parts of the chiral $SU(3)$ Lagrangian of (5.58) and derive the following on-shell scattering amplitude T_c^μ with the corresponding Clebsch-Gordan coefficient C_c:

$$
\begin{aligned}
&T_c^\mu = C_c \gamma_5 \gamma^\mu, \\
&C_c = \frac{e}{2f}\, F_A\, tr\left(\lambda_{\bar{p}}\left[\left[Q, \lambda_{\bar{q}}\right], \lambda_p\right]\right) + \frac{e}{2f}\, D_A\, tr\left(\lambda_{\bar{p}}\left\{\left[Q, \lambda_{\bar{q}}\right], \lambda_p\right\}\right).
\end{aligned}
\tag{G.2}
$$

The Dirac matrices γ_5 and γ^μ are defined in appendix A.5. The positive elementary electric charge is denoted by e while the pion decay constant f is introduced in Eq. (2.40). The free parameters F_A and D_A are dimensionless. We identify Q with the charge matrix of Eq. (2.45). The Gell-Mann matrices λ_i are specified by the momenta p and $\bar{p}$ (q and $\bar{q}$) of the initial and final fermion (boson), respectively. Our Mathematica code provides us with the complete set of invariant functions $F_n^{\pm}(\sqrt{s},t)$ as follows:

$$
\begin{aligned}
&F_1^+(\sqrt{s},t) = -i, && F_1^-(\sqrt{s},t) = -i, \\
&F_2^+(\sqrt{s},t) = -\frac{i}{\sqrt{s}}, && F_2^-(\sqrt{s},t) = \frac{i}{\sqrt{s}}, \\
&F_3^+(\sqrt{s},t) = 0, && F_3^-(\sqrt{s},t) = 0.
\end{aligned}
\tag{G.3}
$$

S-channel process with a ($J^P = \frac{1}{2}^+$)-baryon as the intermediate particle that is related to the scattering reaction $\gamma B \to PB$:



(G.4)

Figure G.2.: S-channel process of the reaction $\gamma B \to PB$

We use the parts of the chiral $SU(3)$ Lagrangian of (5.63) and derive the following on-shell scattering amplitude T_s^μ with the corresponding Clebsch-Gordan coefficient C_s:

$$
\begin{aligned}
T_s^{\mu} &= \sum_{[B]} C_s \, i\not{q}\gamma_5 \left(S_{[B]}(p+q)\right)\gamma^{\mu}, \\
C_s &= -\sum_{a=1}^{8} \frac{e}{2f} \, F_A \, tr\left(\lambda_a \left[\lambda_{\bar{q}}, \lambda_p\right]\right) \, tr\left(\lambda_{\bar{p}} \left[Q, \lambda_a\right]\right).
\end{aligned}
\tag{G.5}
$$

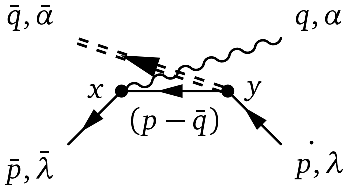
Here, we use the Feynman slash notation $\not{q} \equiv \gamma_\mu q^\mu$. The propagator $S_{[B]}(p+q)$ of the internal $(J^P = \frac{1}{2}^+)$-baryon in momentum space reads [65]:

$$
S_{[B]}(p+q) = \frac{(\not{p}+\not{q}) + m_B}{(p+q)^2 - m_B^2 + i\epsilon}.
\tag{G.6}
$$

Both kind of sums $\sum_{[B]}$ and $\sum_{a=1}^{8}$ consider the complete baryon octet states. Our Mathematica code provides us with the complete set of invariant functions $F_n^{\pm}(\sqrt{s},t)$ as follows:

$$
\begin{aligned}
F_1^{+}(\sqrt{s},t) &= \frac{\sqrt{s}m_B + s - 2w\cdot q}{s - m_B^2}, & F_1^{-}(\sqrt{s},t) &= \frac{\sqrt{s}m_B - s + 2w\cdot q}{m_B^2 - s}, \\
F_2^{+}(\sqrt{s},t) &= \frac{-\sqrt{s}m_B + s + 2w\cdot q}{(m_B^2 - s)\sqrt{s}}, & F_2^{-}(\sqrt{s},t) &= -\frac{m_B + \frac{s+2w\cdot q}{\sqrt{s}}}{m_B^2 - s}, \\
F_3^{+}(\sqrt{s},t) &= 0, & F_3^{-}(\sqrt{s},t) &= 0.
\end{aligned}
\tag{G.7}
$$

U-channel process with a $(J^P = \frac{1}{2}^+)$-baryon as the intermediate particle which is related to the reaction $\gamma B \to VB$:



(G.8)

Figure G.3.: U-channel process of the reaction $\gamma B \to VB$

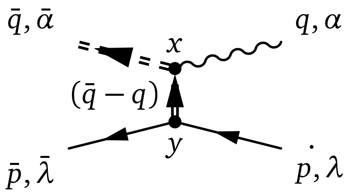
We use the parts of the chiral $SU(3)$ Lagrangian of (5.68) and derive, amongst others, the following on-shell scattering amplitude $T_{u,1}^{\bar{\mu}\mu}$ with the corresponding Clebsch-Gordan coefficient $C_{u,1}$:

$$
\begin{aligned}
T_{u,1}^{\bar{\mu}\mu} &= \sum_{[B]} \left(C_{u,1}\left(\not{\bar{q}}\bar{q}^{\bar{\mu}}\left(S_{[B]}(p-\bar{q})\right)\gamma^{\mu} - \gamma^{\bar{\mu}}(\bar{q}\cdot\bar{q})\left(S_{[B]}(p-\bar{q})\right)\gamma^{\mu}\right)\right), \\
C_{u,1} &= -\sum_{a=1}^{8}\left(\frac{eF_V}{2m_V^2} \, tr\left(\lambda_a\left[\lambda_{\bar{q}}, \lambda_p\right]\right) \, tr\left(\lambda_{\bar{p}}\left[Q, \lambda_a\right]\right)\right. \\
&\quad + \frac{eD_V}{2m_V^2} \, tr\left(\lambda_a\left\{\lambda_{\bar{q}}, \lambda_p\right\}\right) \, tr\left(\lambda_{\bar{p}}\left[Q, \lambda_a\right]\right) \\
&\quad \left. + \frac{eG_V}{2m_V^2} \, tr\left(\lambda_a\lambda_p\right) \, tr\left(\lambda_{\bar{q}}\right) \, tr\left(\lambda_{\bar{p}}\left[Q, \lambda_a\right]\right)\right).
\end{aligned}
\tag{G.9}
$$

The parameters F_V, D_V and G_V of the chiral $SU(3)$ Lagrangian are dimensionless while m_V carries the dimension of a mass. Our Mathematica code provides us with the complete set of invariant functions $F_n^{\pm}(\sqrt{s},t)$ as follows:

$$
\begin{aligned}
F_1^{+}(\sqrt{s},t) &= 0, & F_1^{-}(\sqrt{s},t) &= 0,\\
F_2^{+}(\sqrt{s},t) &= \frac{(2m_B+\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{m_B^2+2\bar{q}\cdot q-s}, & F_2^{-}(\sqrt{s},t) &= \frac{(2m_B-\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{m_B^2+2\bar{q}\cdot q-s},\\
F_3^{+}(\sqrt{s},t) &= \frac{(2m_B-\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)\sqrt{s}}, & F_3^{-}(\sqrt{s},t) &= \frac{(2m_B+\sqrt{s})(-m_B^2+s-2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)\sqrt{s}},\\
F_4^{+}(\sqrt{s},t) &= \frac{(2m_B-\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)\sqrt{s}}, & F_4^{-}(\sqrt{s},t) &= \frac{(2m_B+\sqrt{s})(-m_B^2+s-2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)\sqrt{s}},\\
F_5^{+}(\sqrt{s},t) &= 0, & F_5^{-}(\sqrt{s},t) &= 0,\\
F_6^{+}(\sqrt{s},t) &= 0, & F_6^{-}(\sqrt{s},t) &= 0,\\
F_7^{+}(\sqrt{s},t) &= 0, & F_7^{-}(\sqrt{s},t) &= 0,\\
F_8^{+}(\sqrt{s},t) &= 0, & F_8^{-}(\sqrt{s},t) &= 0,\\
F_9^{+}(\sqrt{s},t) &= \frac{(2m_B-3\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)s}, & F_9^{-}(\sqrt{s},t) &= \frac{(2m_B+3\sqrt{s})(m_B^2-s+2w\cdot\bar{q})}{(m_B^2+2\bar{q}\cdot q-s)s}.
\end{aligned}
\tag{G.10}
$$

T-channel process of $\gamma B \to VB$ with a $(J^P = 1^-)$-meson as the intermediate particle:



(G.11)

Figure G.4.: T-channel process of the reaction $\gamma B \to VB$

We use the parts of the chiral $SU(3)$ Lagrangian of (5.73) and derive, amongst others, the following on-shell scattering amplitude $T^{\bar{\mu}\mu}_{t(v),1}$ with the corresponding Clebsch-Gordan coefficient $C_{t(v),1}$:

$$
\begin{aligned}
T^{\bar{\mu}\mu}_{t(v),1} &= \sum_{[V]} \left(C_{t(v),1}\; i\,\gamma^{\alpha}(\bar{q}-q)^{\beta} \left(S^{\bar{\mu}}_{[V],\alpha\beta,\rho}(\bar{q}-q) \right) (\bar{q}-q)^{\rho}\, \bar{q}^{\mu} \right),\\
C_{t(v),1} &= -\sum_{a=0}^{8} \left(\frac{eF_V}{8m_V^2}\, tr\left(\lambda_{\bar{p}}\left[\lambda_a,\lambda_p\right]\right)\, tr\left(\lambda_a\left[Q,\lambda_{\bar{q}}\right]\right) \right.\\
&\quad + \frac{eD_V}{8m_V^2}\, tr\left(\lambda_{\bar{p}}\left\{\lambda_a,\lambda_p\right\}\right)\, tr\left(\lambda_a\left[Q,\lambda_{\bar{q}}\right]\right)\\
&\quad + \frac{eG_V}{8m_V^2}\, tr\left(\lambda_{\bar{p}}\lambda_p\right)\, tr\left(\lambda_a\right)\, tr\left(\lambda_a\left[Q,\lambda_{\bar{q}}\right]\right)\\
&\quad + \frac{eF_V}{8m_V^2}\, tr\left(\lambda_{\bar{p}}\left[\lambda_a,\lambda_p\right]\right)\, tr\left(\left[Q,\lambda_{\bar{q}}\right]\lambda_a\right)\\
&\quad + \frac{eD_V}{8m_V^2}\, tr\left(\lambda_{\bar{p}}\left\{\lambda_a,\lambda_p\right\}\right)\, tr\left(\left[Q,\lambda_{\bar{q}}\right]\lambda_a\right)
\end{aligned}
$$

$$+\frac{eG_V}{8m_V^2}\ tr\left(\lambda_{\bar{p}}\lambda_p\right)\ tr\left(\lambda_a\right)\ tr\left(\left[Q,\lambda_{\bar{q}}\right]\lambda_a\right)\Bigg). \tag{G.12}$$

Here, both kind of sums $\sum_{[V]}$ and $\sum_{a=0}^{8}$ consider the states of the complete vector meson nonet. The propagator $S_{[V],\alpha\beta,\rho\sigma}(\bar{q}-q)$ of the internal vector meson in momentum space reads [36]:

$$\begin{aligned} S_{[V],\alpha\beta,\rho\sigma}(\bar{q}-q) = -\frac{1}{m_{1^-}^2}\frac{1}{(\bar{q}-q)^2-m_{1^-}^2}\Big(&\left(m_{1^-}^2-(\bar{q}-q)^2\right)g_{\alpha\rho}g_{\beta\sigma} \\ &+g_{\alpha\rho}\left(\bar{q}-q\right)_\beta\left(\bar{q}-q\right)_\sigma - g_{\alpha\sigma}\left(\bar{q}-q\right)_\beta\left(\bar{q}-q\right)_\rho - (\alpha\leftrightarrow\beta)\Big). \end{aligned} \tag{G.13}$$

Our Mathematica code provides us with the complete set of invariant functions $F_n^\pm(\sqrt{s},t)$ as follows:

$$\begin{aligned}
&F_1^+(\sqrt{s},t)=0, && F_1^-(\sqrt{s},t)=0,\\
&F_2^+(\sqrt{s},t)=0, && F_2^-(\sqrt{s},t)=0,\\
&F_3^+(\sqrt{s},t)=-\frac{2i\ w\cdot\bar{q}(m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q)}{s(-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q))}, &&\\
&F_3^-(\sqrt{s},t)=-\frac{2i\ w\cdot\bar{q}(m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q)}{s(-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q))}, &&\\
&F_4^+(\sqrt{s},t)=0, && F_4^-(\sqrt{s},t)=0,\\
&F_5^+(\sqrt{s},t)=i\left(1-\frac{m_{1^-}^2}{-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q)}\right), &&\\
&F_5^-(\sqrt{s},t)=i\left(1-\frac{m_{1^-}^2}{-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q)}\right), &&\\
&F_6^+(\sqrt{s},t)=0, && F_6^-(\sqrt{s},t)=0,\\
&F_7^+(\sqrt{s},t)=\frac{2i}{\sqrt{s}\left(2-\frac{m_{1^-}^2}{m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q}\right)}, && F_7^-(\sqrt{s},t)=-\frac{2i}{\sqrt{s}\left(2-\frac{m_{1^-}^2}{m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q}\right)},\\
&F_8^+(\sqrt{s},t)=0, && F_8^-(\sqrt{s},t)=0,\\
&F_9^+(\sqrt{s},t)=-\frac{2i\ w\cdot\bar{q}(m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q)}{s^{\frac{3}{2}}(-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q))}, &&\\
&F_9^-(\sqrt{s},t)=\frac{2i\ w\cdot\bar{q}(m_B^2-\bar{q}\cdot q-s+w\cdot\bar{q}+w\cdot q)}{s^{\frac{3}{2}}(-2m_B^2+m_{1^-}^2+2(\bar{q}\cdot q+s-w\cdot\bar{q}-w\cdot q))}. &&
\end{aligned} \tag{G.14}$$

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