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Inequality and Financial Stability in an Agent-Based Model

Dissertation

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D 17

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Summary

The dissertation *Inequality and Financial Stability in an Agent-Based Model* considers the effect of inequality on financial stability by means of an Agent-Based model.

In section 2, an overview of the theoretical and the empirical literature of the subject is provided. In particular, the recent financial crisis is displayed. In the context of rising inequality, a bust in real estate markets emerged, which also witnessed the participation of low-income households due to deteriorating lending standards. The increased disparity of current accounts can also be partly attributed to the increased inequality. While the increased deficits in the USA and the European periphery result to a large extent from the increased import of credit for low-income individuals, the increased surpluses of China and Germany can be partly rationalized by means of a savings glut of high-income households.

Section 3 provides an overview of the theoretical literature dealing with this subject. This overview differentiates between several strands of literature ranging including literature from the field of Post-Keynesian theory. In particular, *Heterogeneous Agent Models* - an extension of classical growth models with heterogeneous agents - are contrasted with *Agent-Based Models* (ABMs). The latter category - also used in this work - differentiates especially due to the assumption of behavioral decision rules and the discussion of non-linear complex dynamics.

Section 4 provides a literature overview on the consumption and savings decision, building the theoretical foundation of our model. Besides pointing to the most important determinants, the question under which conditions debt can be sustainable is considered. In the context of inequality, the *Relative Income Hypothesis* is of utter importance as it predicts that individual level of consumption depends on the consumption level of other agents.

The underlying model is derived and discussed in section 5. The key ingredients are the consumption/savings decisions and the trading rules in a market for durables. Based on a slightly simplified model, closed-form conditions for financial stability are derived and interpreted. In particular, inequality of income contributes to increased financial instability. The latter category can be further decomposed into instability of debt and of durable markets. Moreover, we present the closed-form relationships between stock and flow variables and the resulting impacts on the interaction between personal and functional distribution of income as well as the interaction between income and wealth inequality.

The complex model dynamics can only be totally captured by means of numerical simulations (section 6). An important factor driving the aggregate dynamics is the concrete form of delveraging taken by over-indebted households being either in the form of massive consumption decreases (*Austerity*) or fire sales of durables both feeding back

to other households. The goodness of the model is evaluated by its ability to reproduce stylized facts, while robustness is tested by means of parameter variation.

Section 7 takes the developed models to discuss policy actions. In particular, different forms of redistribution are investigated. The most common form of impacting on the distribution of income is the tax and transfer system. The discussion is embedded in a literature review of the economics of taxation. The model shows that stronger redistribution by indirectly increasing the minimum consumption level can increase the indebtedness and thereby increase financial instability - finally being welfare reducing.

Section 8 provides a critical summary of the complete work.

Summary in German – Zusammenfassung

Die Dissertation *Inequality and Financial Stability in an Agent-Based Model* – zu Deutsch *Ungleichheit und Finanzstabilität in einem agentbasierten Modell* - untersucht die Wirkung von Ungleichheit auf Finanzstabilität mit Hilfe eines agentenbasierten Modells.

Nach einer knappen Einführung in die Thematik gibt Abschnitt 2 der Arbeit einen Überblick über die theoretische und empirische Literatur, die sich mit diesem Zusammenhang beschäftigt. Insbesondere wird der Verlauf der jüngsten Finanzkrise nachgezeichnet. Vor dem Hintergrund steigender Einkommensungleichheit fand in vielen Staaten ein Immobilienboom statt, an dem auch Haushalte mit niedrigen Einkommen partizipieren konnten, da Kreditvergabekriterien aufgeweicht wurden. Auch die steigende Disparität in den Leistungsbilanzsalden wird zum Teil auf die gestiegene Ungleichheit zurückgeführt. Die erhöhten Leistungsbilanzdefizite in den USA und in der Peripherie der Euro-Zone resultierten im hohen Umfang aus dem Import von Krediten, welcher von Beziehern von Niedrigeinkommen nachgefragt wurde. Auf der anderen Seite sind die erhöhten Überschüsse in China und Deutschland unter anderem aus der Sparschwemme (*savings glut*) vermögender Haushalte zu erklären.

Abschnitt 3 stellt die Fragestellung der Arbeit in den Kontext der theoretischen Literatur. Hierbei wird ein differenzierter Überblick über verschiedene Literaturstränge einschließlich der heterodoxen Literatur in der Post-Keynesianischen Tradition gegeben. Insbesondere werden *heterogene Agentenmodelle* (HAM) - eine Erweiterung des neoklassischen Wachstumsmodells mit heterogenen Individuen - den *Agenten-Basierten Modellen* (ABM) gegenübergestellt. Letztere – welche auch in dieser Arbeit verwendet werden – unterscheiden sich von HAM vor allem über die Annahme von verhaltensökonomischen Regeln und die Betrachtung von nicht-linearen komplexen Dynamiken.

In Abschnitt 4 wird ein umfassender Überblick über die Modellierung der Konsum- und Sparentscheidung gegeben, welche die theoretische Grundlage für das Modell darstellt. Neben den wichtigsten Einflussfaktoren wird auch untersucht, unter welchen Bedingungen Verschuldung nicht grenzenlos wächst und somit nachhaltig sein kann. Im Zusammenhang mit Ungleichheit ist insbesondere die Bedeutung der *Relativen Einkommenshypothese* hervorzuheben, der zufolge Individuen ihr Konsumniveau an das Konsumniveau anderer Individuen anpassen.

Das Modell wird in Abschnitt 5 aufgestellt und diskutiert. Im Zentrum stehen die Konsum- und Sparentscheidung sowie die Handelsregeln an einem Markt für ein Vermögensgut. Auf Basis einer vereinfachten Version des Modells werden Determinanten für Finanzstabilität identifiziert und interpretiert. Insbesondere trägt erhöhte Einkommensungleichheit in dem Modell zu erhöhter Finanzinstabilität bei. Instabilität wird zudem in die Unterkategorien *Stabilität von Verschuldung* und *Blasen am Markt für Vermögensgüter* aufgegliedert. Des Weiteren wird die Beziehung zwischen Strom- und Bestandsgrößen analytisch diskutiert. Auf dieser Grundlage können auch Beziehungen zwischen personeller Einkommens-Verteilung und funktioneller Verteilung (Lohn- und Kapitaleinkomen) sowie zwischen Einkommens- auf Vermögensungleichheit dargestellt werden.

Die komplexen Modelldynamiken können nur in numerischen Simulationen erfasst werden. Dies geschieht in Abschnitt 6. Entscheidend für die Dynamik ist insbesondere die Reaktion überschuldeter Haushalte, da etwa massive Konsumzurückhaltung oder Panikverkäufe von Vermögensgegenständen auch Rückkoppelungseffekte zu anderen Haushalten haben. Die Güte des Modells wird anhand seiner Fähigkeit empirische Fakten wiederzugeben untersucht, während die Robustheit des Modells durch Parametervariationen überprüft wird.

Abschnitt 7 nutzt das Modell um Politikempfehlungen zu evaluieren. Insbesondere werden verschieden Formen der Umverteilung untersucht. Die gängigste Form der Beeinflussung der Verteilung findet über das Steuer- und Transfersystem statt. Diese Systeme werden auf Basis einer breiten Literaturübersicht diskutiert. In dem Modell zeigt sich, dass verstärkte Umverteilung den Mindestkonsumstandard erhöhen kann und somit mit erhöhte Verschuldung sowie erhöhte finanzielle Instabilität auslösen kann.

Abschnitt 8 schließt die Arbeit mit einer kritischen Zusammenfassung der Ergebnisse.

Contents

List of Figures

List of Tables

List of Abbreviations

Abbreviation	Meaning
ABM	Agent-Based Model
AD	Aggregate Demand
BRIC	Brazil Russia India China
CARA	Constant Absolute Risk Aversion
CCAPM	Consumption Capital Asset Pricing Model
CDF	Compounded Density Function
CEO	Chief Executive Officer
CES	Constant Elasticity of Substitution
CRRA	Constant Relative Risk Aversion
DF	Density Function
DSGE	Dynamic Stochastic General Equilibrium
ECB	European Central Bank
EMH	Efficient Market Hypothesis
EPL	Employment Protection Legislation
EVS	Einkommens- und Verbraucherstichprobe
FOC	First Order Condition
GDP	Gross Domestic Product
GIIPS	Greece Ireland Italy Portugal Spain
HAM	Heterogeneous Agent Model
HFCS	Household Finance and Consumption Survey
IES	Intertemporal Elasticity of Substitution
IS	Investment Savings
LM	Liquidity preference - Money
MPC	Marginal Propensity to Consume
NINJA	No Income No Job No Asset
OECD	Organization for Economic Cooperation and Development
OLG	Overlapping Generation
OLS	Ordinary Least Squares
PAYG	Pay As You Go System
PSID	Panel Study of Income Dynamics
RBC	Real Business Cycle
RE	Rational Expectations
SCF	Survey of Consumer Finance
SOEP	Sozio-oekonomisches Panel
SVAR	Strucutural Vector Autoregression
SWF	Social Welfare Function
TFP	Total Factor Productivity

List of Frequently Used Symbols

Latin Letters

Symbol	Meaning	Algebraic definition
A	Total factor productivity	
a	Power-Law exponent	
AD	Aggregate demand	$AD = C + Pd$
b	Monetary rate of interest	
$\bar{c}$	Subsistence consumption	
C	Consumption	
c	Consumption ratio	$c = C/Y$
c_w	Marginal propensity to consume out of net worth	
c_y	Marginal propensity to consume out of flow income	
CoV	Coefficient of Variation	$\frac{\sigma_x}{E(x)}$
d	Demand for assets	
$\dot{D}$	Dissavings	
D	Debt	$D = -K$
E	Expectation operator	
EX	Exports	
F	Fundamental price	$F \equiv 1$
g	Aggregate growth rate	
G	Government consumption	
g_{ineq}	Growth rate of inequality	
g_K	Growth rate of capital	
GAT	Gini after taxes and subsidies	
GBT	Gini before taxes and subsidies	
H	Heritage	$H = \frac{P_0 q_0}{Y_0}$
h	Hours worked	
i	Nominal interest rate	
IM	Imports	
K	Capital	$K = -D$
k	Capital in effective labor terms	$k = \frac{K}{AN}$
k_0	Monetary assets for zero labor-income	$K(Y = 0) = k_0$
k_y	Marginal response of monetary assets to labor income	$k_y = \frac{\partial K}{\partial Y}$

Symbol	Meaning	Algebraic definition
l	Leverage	
m	Equity ratio / haircut on securities	
$MPCD$	Marginal propensity to consume durables	
N	Number of agents	
P	Price	
p	Log-price	$p = \log(P)$
q	Stock of assets	$\Delta q = d$
r	Real interest rate	$r = (1+i)/(1+\pi) \approx i - \pi$
R	Interest rate discount factor	$R = 1/(1+r)$
$Redist$	Level of redistribution	$Redist = log(GBT) - log(GAT)$
S	Savings (absolute)	$S = Y - C$
s	Savings ratio	$s = 1 - c_y$
T	Taxes	
U	Utility function operator	
W	Net worth	
w	Wage income	
X	Total income	$X = Y + rK$
Y	(Non-capital) Income (flow)	
y	Total output	$y = f(k)$
Y^d	Disposable income	
Y_{min}	Minimum income level	
Y_{TF}	Tax-free income level	
z	Capital ratio for heterogeneous agents i and j	$z_t = \frac{K_{i,t}}{K_{j,t}}$

Greek Letters

Symbol	Meaning	Algebraic definition
α	Scaling parameter of the production function	
$\bar{\alpha}$	Pseudo-capital share for CES production function	
β	Time discounting factor	$\beta = 1/(1+r)$
β_C	Aggressiveness of chartist traders	
β_F	Aggressiveness of fundamental traders	
δ	Geometric rate of depreciation	
ε	Inverse curvature of consumption function	$\frac{1}{\varepsilon} = \theta$
η	Risk aversion for CARA utility function	$\frac{U''}{U'} = \eta$
γ	Risk aversion for CRRA utility function	$\frac{U''(X)}{U'(X)} X = \gamma$
$1/\gamma$	Intertemporal rate of substitution	
Γ	Rationality (capital letter)	
κ	Capital ratio	$\kappa = k/y$
λ	Shadow price	
Λ	Memory	
μ	Market liquidity for durable market	
μ_r	Market liquidity for debt/savings market	
ν	Frisch elasticity of labor supply	
ω	Share of risky assets in portfolio	
Ω	Ratio of net worth to labor income	$\Omega = \frac{W}{Y}$
Ω_y	Marginal response of net worth to labor income	$\Omega_y = \frac{\partial W/Y}{\partial Y}$
π	Inflation	
Π	Profit income	
Π'	Profit share	$\Pi' = 1 - w/Y$
Ψ	Ratio of monetary assets to labor income	$\Psi = \frac{K}{Y}$
ρ	Rate of time preference	
σ^2	Variance	
σ_y	Standard deviation of labor income distribution	
τ	Marginal tax rate	
θ	Curvature of consumption function	$\theta = \frac{1}{\varepsilon}$

1. Introduction

> If a free society cannot help the many who are poor, it cannot save the few who are rich.
>
> John F. Kennedy (1961)

The recent financial crisis was initiated by the burst of a price bubble in the US housing market. This housing bubble was preceded by a strong rise in inequality in wealth distribution. Several popular-science books, such as Rajan (2010) and Reich (2010), see a causal relation between these two phenomena and link the topics of wealth distribution and financial stability. Their main rationale is that in the presence of low interest rate levels financial intermediaries started to expand the activities of house financing for the group of low-income households (the so-called subprime segment). This shift to risk was backed by government officials, opaque financial innovations for risk transfer, and special bankruptcy rules in the US allowing for only adhering with collateral and not with personal wealth[1]. This debt-financed price bubble in housing markets eventually burst, with major consequences for the financial sector, resulting in a strong worldwide macroeconomic shock, which in turn is now followed by a sovereign-debt crisis in many countries.

Housing business is a very special business with a widespread participation rate. In general, house purchases represent the most expensive life-cycle consumption expense of private households. Thus, they are normally backed by external financing. Houses, moreover, are the most valuable asset of households and therefore also represent their best collateral in the Geanakoplos (2003) and Kiyotaki and Moore (1997) determining the level of debt a household can incur. Hence, increasing house prices allow for higher indebtedness for low-income households. This debt-financed consumption again stimulates the economy. In a theoretic sense, a financial market transfers liquidity from high-income net-savers to low-income net-borrowers with high consumption preference and thereby can act as a measure of consumption smoothing. If, on the other hand, these effects are only backed by an unsustainable bubble, the busting of which has major macroeconomic consequences.

In the aftermath of the financial crises the widespread and strong use of leveraged finance, not only in the household sector (Mian and Sufi, 2010), but also in a general framework (Kalemli-Ozcan et al., 2012), was severely criticized. But as already pointed

[1] The latter implies a moral-hazard problem in which individuals have the incentive to default in case the value of the collateral drops below the value of the debt imposing costs of default on their lenders. In the model discussed in this work as we exclude the case of default.

out by Tobin (1980), debt only matters if economic agents are heterogeneous, otherwise it would just be a zero-sum game: the net savings of one household represent the consumption of another in excess of her current budget. In fact, debt cannot be considered in representative agent models as it always requires a counter-position. Hence, increasing inequality might also explain the growing demand for financial intermediation.

Once the asset price bubble burst, a high number of households found themselves with negative net worth and in need of deleveraging. The latter, however, can have adverse consequences for the entire macroeconomy, in particular by its effect on aggregate demand. Moreover, other desperate measures in this case - such as fire sales - also feed back with negative externalities on other agents, especially leading to a further decline of asset prices. These problems are well-known under the term *debt-deflation* coined in the Great Depression (Fisher, 1933). Similar behavior emerged in *Japan's lost decade* (Koo, 2014) and now is visible in a majority of developed economies.

While the this reasoning focuses more on the demand for debt, an alternative strand of literature blames the bubble as being the result of an extensive supply of debt from foreign countries. This literature poses a *global savings glut hypothesis* (e.g. Bernanke (2005), Obstfeld and Rogoff (2009)). Its core idea is that inequalities not only increased within countries, but also on an inter-country level. Excess savings from countries like Germany and especially China manifesting themselves in current account surpluses are also blamed for having fueled the housing price bubble in the US and other current account deficit countries.[2] As shown in Kumhof et al. (2014), the current account imbalances can also be the result of increased inequality within the country, especially from an extended supply of savings from very high-income households. The pair-relation between the US and China is reflected in a smaller scale version between Germany and virtually the entire rest of the European Monetary Union (EMU) member countries. Mutual accusation on an ex-post manner blaming the other party being responsible for building up unsustainable interlinkages, ignore that it always *takes two to tango*. While current account surplus countries boosted their export industries by low exchange rate levels, current account deficit countries were able to sustain stable consumption growth. International imbalances therefore are also always related to imbalances between consumption and investment. Current account deficits favor consumers (mostly low-income agents), while surpluses benefit savers (mostly high-income agents). Hence, inequalities in times of open financial markets are never only a problem of a single country (especially when they concern large countries such as the US), but of the world economy.

Not least after the publication of the work of Piketty (2014) and its resulting public debate, inequality is identified as one of the major problems in developed economies. As a result, the call for a higher level of redistribution gains momentum. Standard economic theory identifies a trade-off between efficiency and equity, since a higher level of redistribution distorts incentive systems and thereby might be cumbersome to growth. The issue of financial instability puts another argument in the hand of pro-redistribution

[2]For empirical evidence the reader is referred e.g. to Adam et al. (2011).

advocates. The concrete implementation is still far from trivial and has to consider certain constraints and possible unintended consequences.

Thus, last but not least, the current economic crisis may also be treated a crisis of economic theory. The major determinants of the recent crisis, inequality, financial markets, and debt levels (stock not flow) are not considered in standard macroeconomic models. Moreover, these phenomenona can be strongly linked to the effect of heterogeneity between households which is ruled out in these models by the strong assumption of a representative agent. There has been a recent countermovement mostly coming from the econophysics direction by trying to construct a model of the economy as an Agent-Based Model (ABM)[3], which according to the author is very promising to gain a better understanding of the workings of the economy. In any case, this movement is still in its infancy. The aim of this work is to built a model of heterogeneous households and thereby found a theoretical explanation not only of the current crisis but also of the impact of inequality on financial markets and the macroeconomy in general.

This dissertation is organized as follows: the following section presents long-run empirical and theoretical evidence linking the issues of inequality and financial stability. Moreover, important *stylized facts* that played a major role in the recent financial crisis are identified - not least to benchmark our theoretical model. Section 3 reviews the existing literature also covering non-mainstream strands. In particular, we classify the modeling technique of *Agent-Based Models* (ABMs). While the role of heterogeneity is also considered in so-called *Bewley*-type models - which we label as *Heterogeneous Agent Models* (HAMs) - ABMs in particular differ by their assumption of behavioral rules and the discussion of complex disequilibrium dynamics. Section 4 provides an extensive review of the literature discussing the consumption/savings decision which is the key ingredient of our model. In particular, we show conditions that lead to a stability of debt accumulation. To cover the social dimension of consumption, the role of the *Relative Consumption Theory* is furthermore emphasized. Section 5 presents our model and analytically discusses factors that impact on financial (in)stability. Moreover, we emphasize the role of stock quantities and thereby on the functional distribution of income as well as the distribution of wealth. Due to the complex nature of the model, a complete discussion of its features requires numerical simulations as presented in section 6. In this section, we not only benchmark the model to stylized facts, but also test its robustness. Section 7 uses the developed model to discuss the effect of redistribution. As the model identifies concrete channels for which inequality can increase financial instability, one might expect that the model would also favor a high degree of redistribution. We consider two forms of redistribution (1) by means of the linear tax and transfer system and a (2) *real* progressive tax system. The model, however, raises some cautious notes on the strong redistribution by means of taxation and transfers emphasizing the role of unintended consequences.

[3]A recent survey can be found in Chakraborti et al. (2011).

2. Connection between Inequality, Financial Markets, and Current Accounts

> From the ice-age to the dole-age
> There is but one concern
> I have just discovered:
> Some girls are bigger than others
>
> The Smiths - Some Girls are Bigger Than Others (1985)

In this section, we review the literature relating inequality and financial markets focusing on empirical results. Our theoretical model is to be benchmarked against these stylized facts and thereby also allows to comprehend the underlying mechanisms. In this section, we take both a long-run view (where we also give a brief review of the theoretical literature) and a more medium-run view with a particular focus on the events in the wake of the Global Financial crisis. The model at the core of this work is aimed at explaining medium-run relations for the current situation.

2.1. A Long-Run View

There is a general literature on the long-run relation between financial development (in the form of increased financial activity) and inequality, as presented in the recent survey of Demirgüç-Kunt and Levine (2009). In a nutshell, the large share of the theoretical literature emphasizes the positive impact of financial development on equality through different channels. In particular, it asserts that financial development enhancing the screening abilities will result in better credit conditions to low-income households and thereby decrease inequality. Enhanced credit markets will especially contribute to lower the cross-dynasty persistence by financing higher education for poor income households and (especially for underdeveloped countries) lowering the labor market participation of children. Extensive insurance systems will also aid households to protect against exogenous risks such as unemployment or health risks. Furthermore, better availability of credit can enhance small businesses. This, however, can eventually lead to an increase in inequality amongst a generation. Besides the increased investment opportunities in (human) capital the authors also point to the danger of destabilizing speculation and overborrowing for excess consumption rather than consumption smoothing.

In most theoretical models, capital markets are assumed to be perfect and - being only a veil - do not impact aggregate outcomes such as growth or equality. However, real credit markets differ severely from the theoretic notion of perfect credit markets. As put forward by the information economics literature, the existence of information asymmetries results in problems of adverse selection (ex-ante) (Akerlof, 1970), hidden actions (ex-interim), and moral hazard (ex-post).[1] In order to address these problems, credit is rationed (Stiglitz and Weiss, 1981), collateral is required (Kiyotaki and Moore (1997), Bernanke et al. (1998)), and screening costs are imposed on creditors (Aghion and Howitt, 2009, p. 136f.). Therefore, to make credit markets matter in theoretical models, their imperfections have to be considered. The interplay of credit constraints and wealth inequality leads to an interesting outcome on macroeconomic dynamics, in particular multiple and and even self-sustaining equilibria can exist.

Amongst the first to suggest those problems were Galor and Zeira (1993). In their seminal paper, they argue that low-wealth individuals face collateral constraints and therefore cannot borrow in order to finance education and build up human capital. Thereby, wealth inequality - by contributing to income inequality and subsequent wealth inequality - can be self-sustaining. To derive this conclusion, the only necessary assumptions are imperfect credit markets and indivisible investment in human capital. The model of Galor and Zeira (1993) - aimed at describing development traps in low-income countries - predicts a two-class society of rich and poor agents lacking a middle-class. The work of Banerjee and Newman (1993) argues in a similar way showing that the presence of collateral constraints hinders low-income agents becoming self-employed. The high supply of workers depresses wages and contributes to future income inequality. As argued in Aghion and Bolton (1997) the existence of credit markets allows the *trickle-down* of growth from high-income to low-income agents. As this mechanism is however hampered by credit constraints, (forced) government redistribution can strengthen this growth-improving behavior.

Piketty (1997) introduces the effect of credit rationing in a long-run Solow-type growth model. Rather than the standard result that interest rate equals the marginal product of capital (making wealth distribution issues irrelevant), in the credit constrained case a stationary interest rate is accompanied by a specific wealth distribution. A high interest scenario is accompanied by strong credit constraints and leads to a lower equilibrium capital and output level. High interest rates are also associated with low wealth mobility since the rich accumulate massively and poor households face a collateral constraint making this result also self-sustaining.

In their model, Aghion et al. (1999) investigate the long-run effects of the physical separation of savers and investors, which they label *dualism*, as well as the poorly functioning capital market generally hindering access to credit. Both problems tend to occur in emerging economies. Strong imperfections can not only generate a cyclical growth but also lead to a permanent slump steady state. From a policy perspective, they derive the result that the issuance of public debt during a recession can be beneficial to absorb idle savings. The effectiveness of this measure, factually presenting a redistribution from

[1] A general overview on these topics is e.g. given in Freixas and Rochet (2008).

savers to investors, is further enhanced if the proceedings are used to finance subsidies or tax cuts for investors. They admit that the higher government debt burden causes recessions to be more severe, nevertheless they conclude that a cyclical regime is superior to a permanent slump scenario.

Summarizing this literature one can assert that well-functioning and poorly regulated credit markets can promote growth and equality. This also seems to be in line with the general results of the empirical literature. The positive relation between finance and growth is also suggested in various empirical studies (Levine, 2005). The key problem with empirical literature is the problem of causality. The causality could eventually run in the opposite direction as nicely put by (Robinson, 1952, p. 86): "Where enterprises leads, finance follows." for the growth-finance nexus. Modern empirical studies, however, use various sophisticated techniques to deal with these problems (Levine, 2005).

It might be argued that the relation between equality and finance is just a by-product of the relation between equality and growth. The seminal empirical contribution in this context was made by Kuznets (1955) showing that there is a inverse u-shaped relation between income level and inequality. Therefore, it is often argued that higher inequality might be accepted in the short-run since it fosters growth. Theory predicts that growth - in particular the emergence of new technologies - in the short-run increases inequality rewarding skilled labor (Quadrini, 2008). This theory argues in the same logical direction as the empirical work of Kuznets (1955). For the converse causation mechanism running from inequality to growth, the established literature certifies a negative relation, implying that higher inequality leads to lower growth. This can be rationalized from a political economy point of view arguing that higher inequality leads to more redistribution accompanied by distortionary taxes that are at odds with economic efficiency (Diamond and Mirrlees, 1971). Furthermore, wealth inequality in the presence of capital market frictions hinders the accumulation of human capital and therefore is detrimental to growth (Galor and Zeira, 1993).

Demirgüç-Kunt and Levine (2009) also survey the empirical literature on the relation between inequality and finance and make the point that - besides some strong methodological problems in these studies - the overall results suggest a negative relation between financial development and growth of the Gini coefficient (e.g. Levine et al. (2007), Beck et al. (2007)). It is important to consider that this might also be a spurious correlation since growth increases both equality[2] and finance. Recent studies using sophisticated techniques try to rule out this result and eventually also end up with different results. For instance, Gimet and Lagoarde-Segot (2011) applying a sophisticated structural autoregressive model (SVAR) for a dataset of 49 countries between 1994 and 2002, show that there is a strong causality running from financial deployment to inequality for which increased credit leads to higher inequality. This effect is amplified for high lending-deposit spreads and liquidity suggesting that banking regulation could actually lead to a more welfare enhancing use of the financial market by curtailing overborrowing as suggested in Demirgüç-Kunt and Levine (2009).

[2]This applies to developed countries on the right side of the Kuznets curve (Kuznets, 1955).

Cihak et al. (2012) report that financial depth (a pure size index), access, and efficiency unambiguously increase with the income level of an economy. Only the measurements of financial stability do not vary with income level. This is in line with the empirical studies of Kaminsky and Schmukler (2003) and Loayza and Ranciere (2006) showing that financial liberalization can have a negative impact by increasing boom-bust cycles. Therefore, the relation between finance and economic stability - measured by volatility of output - is less clear-cut. This also has implications for financial regulation. Korinek and Kreamer (2013) emphasize that financial regulation hinders the efficient functioning of the financial system and redistributes surplus away from the financial sector. By comparing it to the (technical) regulation of nuclear power plants, they, however, argue that it reduces the hazard of severe downside risks, in particular credit crunches resulting from too much ex-ante risk. As moreover, presented in the empirical investigation of Delis et al. (2013) the form of regulation also matters. While financial market regulation in the form of market discipline (private monitoring) increases inequality, banking regulation by a supervisory authority eventually decreases inequality.

The strictly positive relation between finance and growth is also put to the test in recent empirical work. Under the thought-provoking title *Too much finance?*, Berkes et al. (2012) suggest a non-monotone concave relationship between finance (measured in the form of the credit/GDP ratio) and growth. They suggest a *Goldilocks effect*, implying an optimal medium sized level (not too much, not too little) of finance amounting to an approximate level of 100% credit/GDP, which is already exceeded in many developed economies such as the US, UK, or Spain.

There are some theoretical arguments for an over-supply of credit. The over-supply could emerge for the case in which unsophisticated private households over-consume debt as predicted by behavioral theory (Zinman, 2013). The deleveraging from this high amount of debt, moreover, can have extremely undesirable macroeconomic consequences (Fisher, 1933). Too much debt might also go along with the emergence of a systemic crisis (Fischer and Riedler, 2014). Furthermore, there is a *brain drain* argument putting forward that finance *steals* human talents (in particular natural scientists or engineers) who could been of better use in other economic sectors (Tobin, 1989). In general, there is the case of *too much finance*, if finance does not support sustainable investment, but promotes asset bubbles. This argument is frequently articulated in the context of the recent financial crisis. In particular, Rajan (2010) argues that the financial crisis in the US was the result of inequality increasing (unsustainable) financial intermediation ending in the crisis.

Some empirical literature using large datasets also investigate the relation between inequality and financial crises in the long-run. In direct response to the work of Rajan (2010), Bordo and Meissner (2012), using a dataset of 14 advanced countries from 1920 to 2000, research the relation between top income growth and credit booms (preceding financial crisis), but cannot find a significant effect. The credit boom, however, can be attributed to low interest rates emphasizing an *Austrian* view for which the over-investment is a direct result of over-expansive monetary policy. The view is also in line with the study of Schularick and Taylor (2012) showing the role of credit expansion as a predictor of financial crises. Another recent empirical study by Agnello and Sousa

(2012) using data from 1980 to 2006 for 62 countries, however, shows that banking crises are preceded by growth in inequality and are followed up by lower inequality afterwards. More precisely, the increase is only significant for Non-OECD countries (as a proxy for developing countries) while the decrease is only significant for OECD countries (as a proxy for developed countries). This inversed-V-shaped behavior can be attributed to the fact that the top incomes are disproportionately affected by both the increased stock market prices leading to the bubble and its bursting (Roine et al., 2009).[3] The decrease in inequality after a banking crisis in developed countries can be attributed to the higher developed social welfare system and automatic stabilizers. The converse view (especially important for developing countries), argues that in times of financial crises low-income individuals are massively affected in their ability to smooth consumption. More generally, Maestri and Roventini (2012) investigate the relation between inequality and business cycles. While consumption inequality is cyclical, income inequality is counter-cyclical. The business cycles can explain the transitory components of inequality, yet not its general increase. Atkinson and Morelli (2011) also compile a large amount of case studies investigating the relation of changes in inequality and economic crisis, but cannot find a clear-cut relation. However, they focused on the role of changes rather than levels leaving the question unresolved whether a particular *level* of inequality can cause a crisis. A similar investigation is conducted in Aiginger and Guger (2013), presenting a cross-country analysis for the period of the Great Recession and also finding no clear patterns between inequality and subsequent economic performance.

In summary, the long-run relation between inequality, finance, economic crisis is far from being clear-cut. However, empirical work - in contrast to theory - can only predict what has already been witnessed making it subject to the *Turkey fallacy*[4] especially important in the recent financial crisis. The recent years - starting in the 1980s - however, saw some very interesting development, which we document in the following section.

2.2. Empirical Evidence for the Medium-Run

There are several studies mostly focusing on the recent development in the US linking inequality, indebtedness, and overall economic conditions (such as the level of GDP, asset prices, unemployment, bankruptcy rates). Due to the low frequency of the data

[3] The theoretical growth literature states that times of economic expansion are associated with lower income and consequential lower wealth inequality. The rationale for this is that expansion regimes are modeled as positive technological shocks that more severely affect labor income than capital income (Maliar et al., 2005). Both the Great Depression and the Great Recession, however, are associated with a positive shock to the productivity of capital only, taking the form of higher leverage, reversing the precedent argument.

[4] The idea dates back to Bertrand Russell discussing problems of inductive reasoning (Russell, 1936). A turkey being treated nicely and fed regularly may infer with increasing confidence that humans mean it no harm. A few days before Thanksgiving, however, the turkey's confidence is shattered. The latter idea had a revival under the label of *Black Swan event* in the context of the financial crisis especially put forward by Taleb (2010). Note that in his original work Russell used the image of a chicken rather than a turkey.

(especially considering data measuring inequality) this lacks the econometric rigor of the aforementioned studies. However, some *stylized facts* emerge which must be accounted for in a theoretical framework.

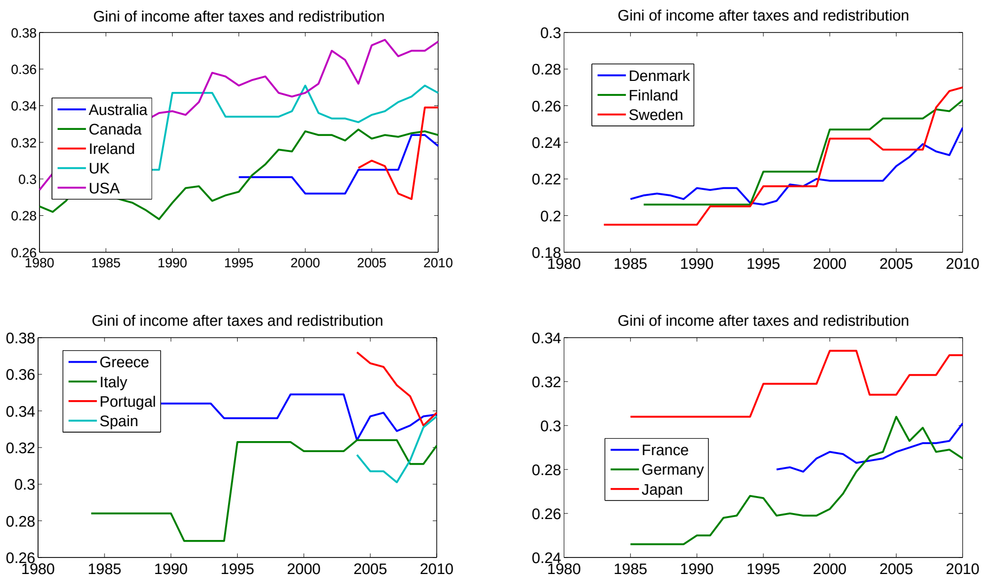


Figure 2.1.: Gini [%] after taxes and redistribution from 1980 to 2010 (Data source: OECD (2012))

The increase in inequality has been a widespread positive trend in the recent 30 years in developed economies.[5] Comparable peaks were measured at the times of the Great Depression, while the period from the end of World War II to early 1980s is associated with a leveling of top incomes leading to an overall *bathtub shaped* behavior in time (Anselmann and Krämer, 2012).[6] Figure 2.1 [7] shows that the Gini-ratio for OECD countries in general has increased since the 1980s[8] for all OECD countries. While the OECD only reports figures at a low frequency, the ambitious study of Atkinson et al.

[5] As shown in Berthold and Brunner (2011) the developing countries of the BRIC group (Brazil, Russia, India, China) eventually witnessed a decrease in income inequality.

[6] This behavior can be found for the US and Germany. The empirical assessment of Card et al. (2013) is able to track the recent increase in income inequality in Germany to an increase in both worker and plant heterogeneity, also confirming that younger firms, by not participating in collective bargaining practices and due to different management style, have a higher variance in wages.

[7] We take the index of the Gini coefficient after taxes and transfers for the working age population (18-65 years) for selected countries of the OECD database. This represents the final measurement for evaluating inequality, since it considers the value of disposable income available for consumption and savings of private households. Most countries only report this value each 5 years. Missing values are replaced by the last reported value.

[8] Notable exceptions include France (where inequality leveled) and southern European countries (where inequality eventually decreased).

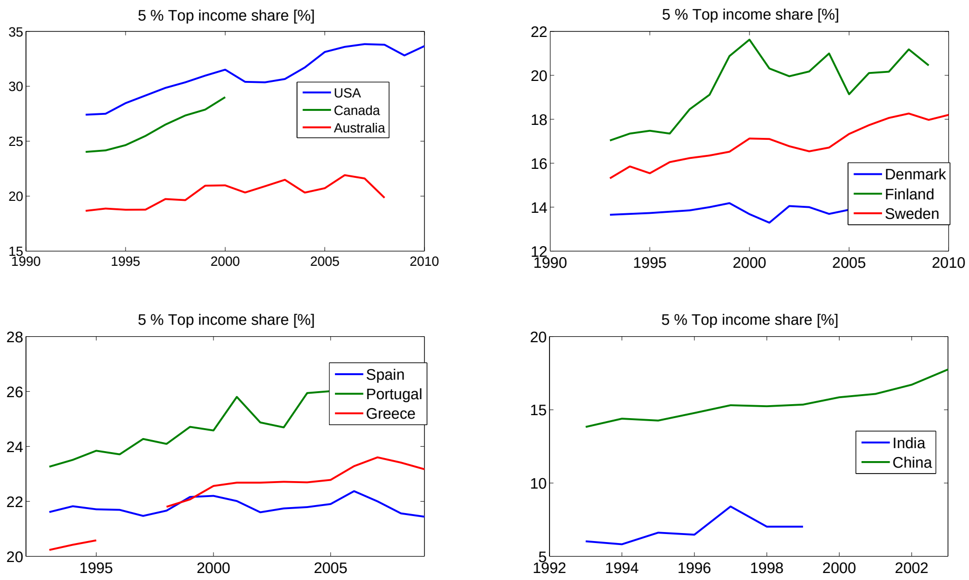


Figure 2.2.: 5% top income share [%] (Data source: Atkinson et al. (2011))

(2011)[9] using tax data to depict the share of the top incomes is able to confirm this trend with annual data. As presented in figure 2.2 [10], top income shares for various countries have an upward trend.[11] The most extreme data specification is reported the USA with the top 5% income share exceeding the 30% level. As presented in figure 2.3 both measurements are clearly related to each other. This figure also suggest that the inequality in the Anglo-Saxon countries Canada and USA (since being above the linear fit), in contrast to southern European countries Italy and Portugal (being below the fit line), is mostly driven by the higher income supporting *the rich get even richer* rather than *the poor get even poorer* narrative.

The literature identifies different sources for the increase in income inequality such as skill-biased technological change[12] (Acemoglu and Autor, 2012) as well as skill mismatch in the labor market, increased firm bargaining power through globalization and

[9] The data for the top income share is public available in *The World Top Incomes Database* `http://g-mond.parisschoolofeconomics.eu/topincomes`.

[10] This ratio presents the share of income from total income by the 5% richest households by headcount. Higher values exceeding the 5% threshold indicate inequality. We exclude capital gains presenting a lower bound for the actual inequality. If the effect of capital gains are included, this measurement of inequality furthermore increases (Atkinson et al., 2011).

[11] Important exceptions where inequality leveled were - once again - measured in southern Europe. Although having a higher frequency than the Gini data, this database suffers from a low inter-country and time coverage of data.

[12] The idea is nicely captured in the title of the work of Goldin and Katz (2009) *The Race between Education and Technology* for which the technological development as requirement for jobs outpaced the level of education creating large wage premiums for individuals with high education.

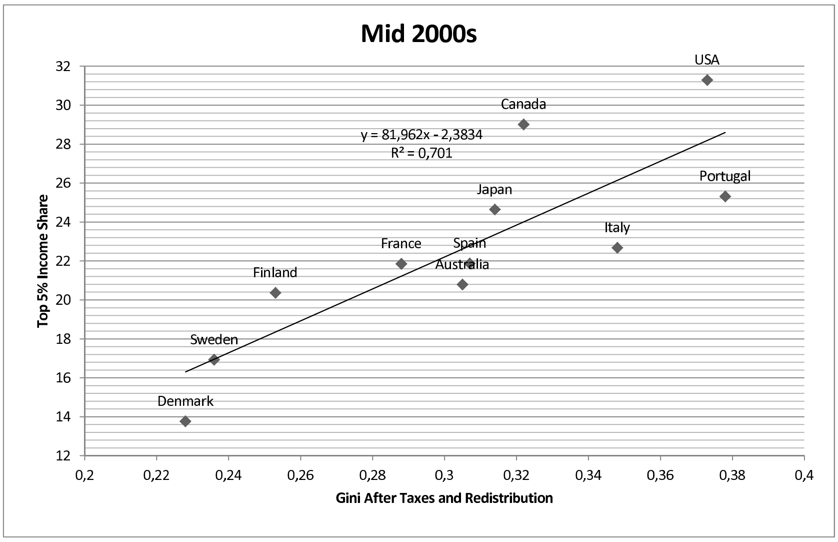


Figure 2.3.: Top income and inequality (Data source: Atkinson et al. (2011) and OECD (2012))

the threat of outsourcing (Skott, 2011), rent-seeking by top-executives[13] (especially in the financial sector) (Philippon and Reshef, 2012)[14], and assortative mating (Greenwood et al., 2014)[15]. The decrease in marginal tax rates as well as the decline of trade unions (Card et al., 2004), sometimes are even referred to as a paradigm shift to a more deregulated economic environment starting in the 1980s (Piketty et al., 2014).

A trend only sparsely covered in the literature is the convergence in redistribution as measured in the log-difference of Gini before (GBT) and after (GAT) taxes and redistribution depicted in figure 2.4 [16] especially in Europe. While high-income European

[13]As already presented, the increase in inequality in Anglo-Saxon countries is mostly driven by an increased share of the top incomes. This behavior can be rationalized by microeconomic theory (initially aimed at describing the behavior of show business as well as sports) following the theory of superstars for which productivity and wage outcome are related in a convex way (Rosen, 1981). The theory of tournaments asserts that individuals are paid according to their rank rather than their output level (Lazear and Rosen, 1981), which instead of providing an adequate compensation for the high income serves as an incentives for the low income to work harder. This behavior is expected for CEOs, for which taking an ordinal scale has lower observation costs than a cardinal scale, also implying a skewed income distribution.

[14]We treat the latter subject in a separate theoretical paper (Fischer, 2014) deriving a relation between given unequal abilities and unequal income. We show that inequality of income increases in environments with strong scale effects, low risk aversion and strong incentive mechanisms.

[15]This mechanism emphasizes that individuals with similar incomes tend to marry, increasing income inequality on an inter-household level. Or to put it more vividly, the doctor does not marry the nurse, but a lawyer. This effect is furthermore pronounced by the high female labor market participation.

[16]This is computed as follows: $Redist_t = \log(GBT_t) - \log(GAT_t) = \log(\frac{GBT_t}{GAT_t})$. The key idea of this index is that not absolute but relative changes in inequality matter. Or more vividly, a change from a before redistribution Gini of 0.9 to 0.7 represents less redistribution than a change from 0.3 to 0.1. Furthermore, this index will be of particular use in section 7.2 where we can derive closed-form solutions for this value for a given taxation-transfer system.

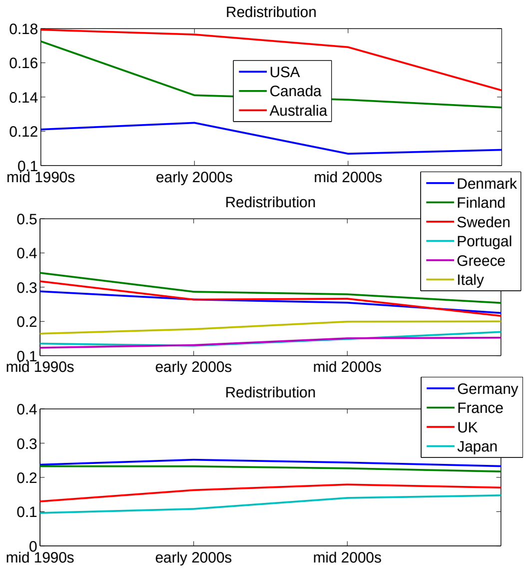


Figure 2.4.: Redistribution in various countries (Data source: OECD (2012))

countries (especially Scandinavia) decreased their redistributive measures, southern European countries increased their activities leading to a convergence to a level of roughly 0.2. We will discuss this important topic - also relating it to theoretical rationales - more thoroughly in section 7.1.

All in all, the underlying causes of income inequality are a very important subject. However, in this work we abstract from these issues and treat income inequality as exogenous.

The increase in income inequality is not accompanied by a comparable increase in consumption inequality. To cope with this effect households could react by increasing their working hours and their labor supply. This can actually be measured in the US data not only reporting longer working hours but also a higher female labor market participation[17] (Bowles and Park, 2005).

However, the most important effect for maintaining high consumption in a world with increased income inequality is the increase in consumer credit as reported in Krueger and Perri (2006). The rationale for this is that the increased saving amount by high-income households is met by an increased demand for credit of low-income households using it to smooth consumption. This result cannot only be captured by a demand perspective of credit but also from a supply perspective for which rich households accumulate massive amount of savings looking for a high yield options in which to invest. As put forward by Rajan (2010), in the USA this option was found in apparently sophisticated financial products linked to real estate.

[17] Stiglitz (2008) even makes the point that, since households work more, they have lower leisure time, which makes them seek higher consumption as a compensation mechanism. Thus, the initial problem is even aggravated.

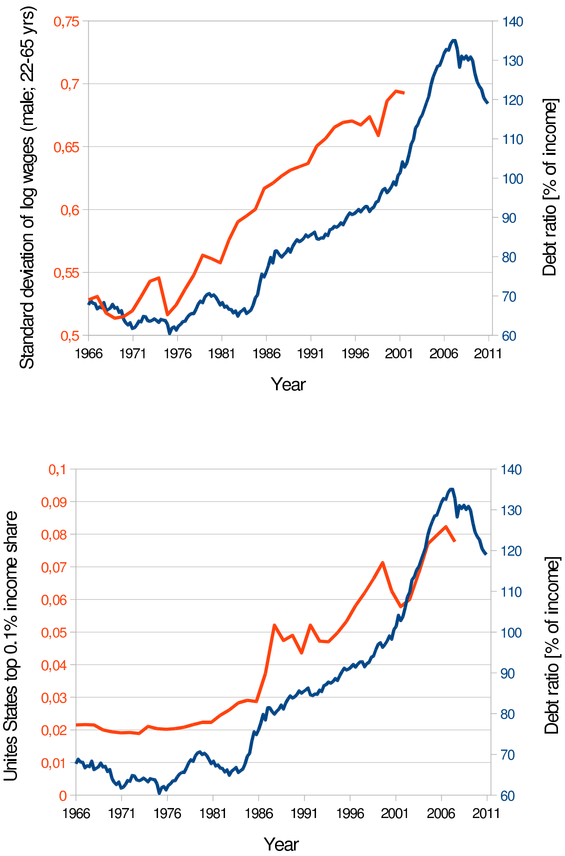


Figure 2.5.: Development of two measurements of inequality and debt in the US (Data source: Eckstein and Nagypál (2004) and Atkinson et al. (2011))

Figure 2.5 [18] shows the strong comovement of debt level and income inequality for US data from the 1960s to now. Kumhof et al. (2014) relate the very recent episode to a similar behavior leading to the Great Depression in the 1930s.

There is more sophisticated empirical literature that goes beyond the simple eyeball evidence presented in figure 2.5. Perugini et al. (2013) show a positive relationship between inequality and credit growth after controlling for conventional credit factors suggesting that credit was used by low and middle-income households to keep up with the consumption level of high-income households. Bertrand and Morse (2013) also find evidence of *trickle-down consumption* in US-data where low-income households emulate consumption of high incomes. In fact, middle-income individuals should have saved significantly more for the (counterfactual) case where their income had grown at the same pace as the income of high-income individuals.

[18] Data source for income `http://www.bea.gov/national/` and debt `http://www.federalreserve.gov/apps/fof/`. Standard deviation of log wages from Eckstein and Nagypál (2004), top income share from *The World Top Incomes Database* `http://g-mond.parisschoolofeconomics.eu/topincomes`.

One might summarize this as follows: in a world with increasing income inequality, higher net worth inequality is needed in order to sustain a given level of consumption inequality.

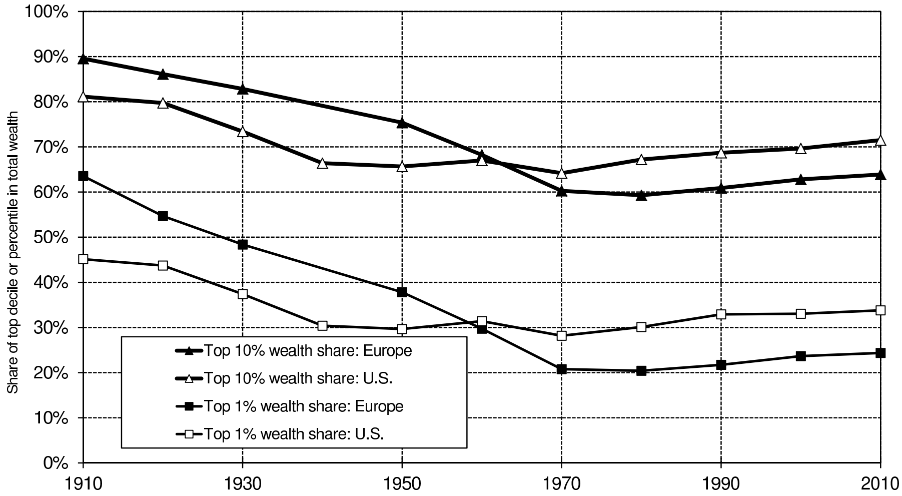


Figure 2.6.: Wealth inequality measured by the share of top 10% and top 1% (Data source: Piketty (2014))

The empirical evidence on net worth or wealth inequality is even more sparse than the evidence about income inequality.[19] One option - refraining the method of Atkinson et al. (2011) applied to the distribution of income - would be to draw on wealth or estate tax data, which, however, are seldom available to researchers. In a very ambitious study, Davies et al. (2011) survey the distribution of wealth for 39 countries. Only few countries provide accurate household balance sheet data. Other good observations are available in the form of representative panel group studies, with the most famous being the *Panel Study of Income Dynamics* (PSID)[20] conducted in the US and the *Sozio-oekonomisches Panel* (SOEP) for Germany. These studies - besides suffering from the usual problems of self-reported values[21] - moreover, deliberately exclude very high net worth individuals for privacy reasons. There are, however, lists compiled by private organizations aimed at estimating the wealth of extremely high net worth individuals - such as the *Forbes 400 list* for the USA. The sampling technique of these studies, however, is highly opaque and the data quality is also rather dubious. A recent attempt at conducting a standardized estimation for Euro area countries under the supervision of the European Central Bank

[19]Note that - albeit imprecise - we will use these terms interchangeably in this section, implying that whenever we speak of wealth inequality we refer to values net of debt. This is only for readability purposes.

[20]For the US, furthermore, data from the *Survey of Consumer Finance* (SCF) is available even providing more detailed data. In this dataset, moreover, the rich population is oversampled, yet the data lacks the panel structure of the PSID (Cagetti and De Nardi, 2008).

[21]Amongst the most important ones in our context are non-response of high net worth individuals leading to a biased sample and non-truthful response resulting from the delicate subject.

is made under the label of the so-called *Household Finance and Consumption Survey* (HFCS). First results were recently presented (Bover et al., 2014).

In the long-run, wealth inequality decreased until the 1980s, as reported in the survey of Davies and Shorrocks (2000). The authors rationalize this result by the emergence of *popular assets* - in particular real estate - with a widespread participation rate amongst different households. Piketty (2014), moreover, emphasizes the role of the world war destruction in Europe that not only contributed to a lower stock level of wealth but also to a more egalitarian distribution of it. However, Piketty (2014) also emphasizes that this trend reversed in the 1980s (see figure 2.6 [22]). A natural explanation for this would be the higher inequality of the flow quantity of income (as e.g. reported in figure 2.2) that cumulated over time.[23] Another recent attempt for measuring wealth inequality was presented in Saez and Zucman (2014) in which the authors use data from the capitalized income tax to estimate the distribution of wealth for the USA. They confirm the results of Piketty (2014) indicating an increase of wealth inequality - as measured by the share of top-wealth holders - setting in in the 1980s after a long period for which wealth inequality declined. Moreover, they report even more extreme values than identified by Piketty (2014) and depicted in figure 2.6. Saez and Zucman (2014) report a current share of the top 10% roughly amounting to 77% and top 1% share of approx. 42%. As underlying factors they identify the - already reported - increase in the flow quantity of labor income inequality and savings ratios that increase with the level of wealth. In fact, they show that the middle-class savings rates were negative in the 2000s. They also report the members of the top wealth holder group have become younger in time. The middle-class (defined as the bottom 90%) exhibited an inverse u-shaped pattern in time. The increased holding of the popular asset real estate was counteracted by an increase in mortgage debt and other forms of debt (student loans, credit card debt) eventually lowering the net worth of the middle-class. Using the result of Shorrocks et al. (2011) respectively Shorrocks et al. (2013), we take a more short-run view in the following.

Table 2.1 reports values of the study of Shorrocks et al. (2011) and an updated version (Shorrocks et al., 2013).[24] The table reports a strong increase in wealth inequality between the outbreak of the financial crisis and the topic date 2013 for most countries. Most notable is the value from Denmark where the Gini of net-worth inequality exceeds the 100% level.[25] The second and third highest Gini ratios of wealth for 2013 are re-

[22]The underlying data is available online at `piketty.pse.ens.fr/capital21c`.

[23]In his work Piketty (2014), moreover, emphasizes other theoretical factors - especially the role of inheritance and the relation between the growth rate and the rate of interest. We will analyze these (theoretical) arguments more precisely in section 5.5.2.

[24]The results are based on the methodology presented in Davies et al. (2011). The authors update these results annually and publish it under the label of *Credit Suisse - Global Wealth Databook*. In the table, we only report values for countries with a very good data quality - which are in the minority.

[25]This result seems very odd at first sight. Note, however, that the ordering of the agents according to level only satisfies the condition that the Lorenz-curve is convex. If the input-values are negative (implying negative net-worth individuals) the slope is negative. If the amount of negative net worth as well as the share of those individuals holding it is high enough, the Gini coefficient can exceed the 100% threshold.

	Gini of Net Worth Inequality [%]			
Country	2010	2013	Change	Gini of income after taxes (average 1981-2011)
Australia		63.6		32.4
Canada	68.3	72.7	6%	32.8
Denmark	84	107.7	28%	24.3
Finland	57.8	66.4	15%	25.8
France	75.8	69	-9%	29.2
Germany	68.4	77.1	13%	30
Italy	62.6	65	4%	28.9
Japan		63.5		32.3
Netherlands	64.3	73.2	14%	29.7
Spain	56.5	66.1	17%	31.3
Switzerland	88	80.6	-9%	29
United Kingdom	71.7	67.7	-6%	34.5
USA	80.9	85.1	5%	37
World	88.1	90.5	3%	

Table 2.1.: Wealth inequality (Data source: Shorrocks et al. (2011), Shorrocks et al. (2013) and OECD (2012))

ported for Russia and the Ukraine (93.1 respectively 90), whereas the lowest values is documented for Slovenia (53.5). Concerning very high-income individuals the USA and Switzerland stick out. According to Shorrocks et al. (2013) 10% of the Swiss population have a net worth of at least one million US-dollars being accompanied by a mean wealth of approx. 500,000 US-dollars. The absolute number of persons owning (at least) one million US-dollars, however, is highest in the USA, amounting to 1.3 million, and thereby contributing to a share of approx. 40% of world-wide millionaires. Table 2.1 also contrasts the wealth inequality with the income inequality. The stock size of wealth is always more unequally distributed than the flow size income. Nevertheless, some very egalitarian countries with respect to income - especially Germany and Scandinavia - exhibit a high degree of wealth inequality.[26] Davies et al. (2011) partly attribute this result to the existence of a well-functioning public pension system that weakens the private savings and wealth accumulation motive. As put forward in Davies et al. (2011), a high level of income is usually accompanied by with a high level of wealth. Notable exceptions

[26]This fact also sparked a public debate in the Euro zone after the publication of the first results of the HCFS study showing that the median wealth in the net transfer paying country Germany is lower than in net transfer receiving countries such as Italy or Greece. As shown by DeGrauwe and Ji (2013) the ratio between mean and median is very high for Germany suggesting high wealth inequality. Other explanations for this counter-intuitive result include the low real-estate wealth of Germans (due to well-evolved rental markets) and the long-lasting effects of war destruction in Germany.

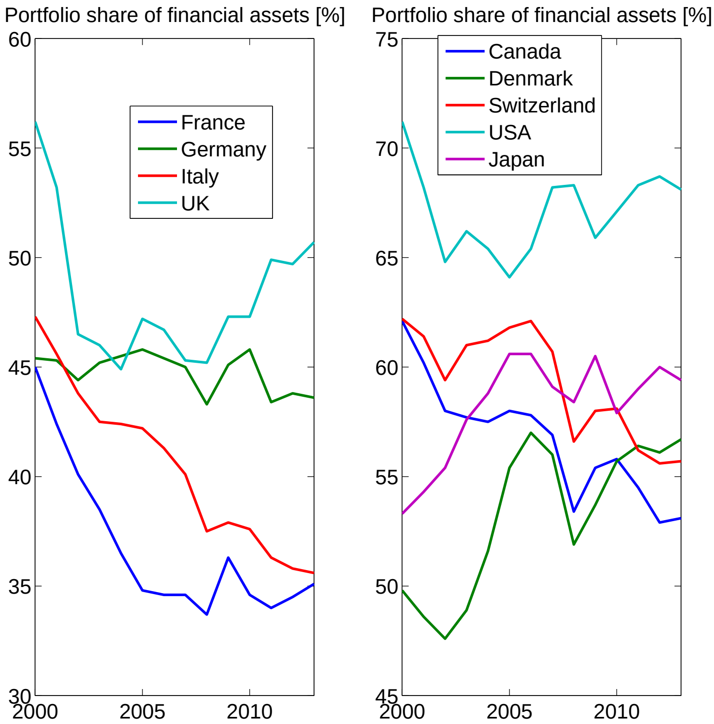


Figure 2.7.: Average portfolio share [%] of financial assets for private households (Data source: Shorrocks et al. (2013))

where a low level of income is accompanied by a high level of wealth are reported for Hong Kong or Ireland.

In an international comparison, there are also substantial cross-country differences in the composition of the wealth portfolio. In particular in the USA, the (average) portfolio mainly consists of financial assets rather than other assets such as real estate (see figure 2.7). We can also investigate the passive side of the private household balance sheet and discuss the share of debt as depicted in figure 2.8. Germany exhibits a long-run downward trend. Moreover, there is considerable time variation. In particular, the UK, USA, and Denmark all peak in the year 2008. Once again, the special role of Denmark with its massive levels of debt is notable. As Denmark also exhibited an extreme case of wealth inequality debt seems to be a good predictor for wealth inequality. We investigate this hypothesis more formally in a simple empirical model.

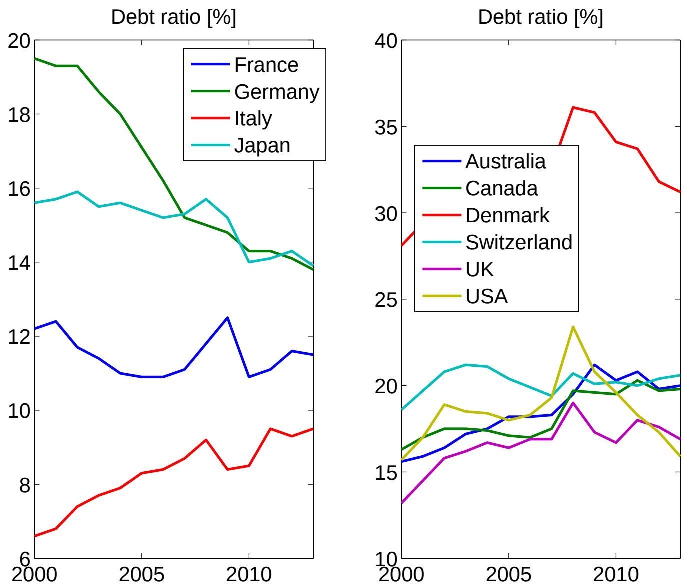


Figure 2.8.: Average debt ratio [%] for private households (Data source: Shorrocks et al. (2013))

In this model we estimate the transformed Gini-ratio of wealth[27] [28]. As explanatory variables, we choose the Gini of income after taxes ($Gini(y)$), the log of the absolute value of wealth (W), the average share of financial assets (w_{fin}), the average share of debt (D), and the real interest rate (r).[29] The expected signs of the regressions are presented in the square brackets:

$$Gini(W) = f(W[+], w_{fin}[+], D[+], r[+], Gini(y)[+]). \tag{2.1}$$

A higher level of mean wealth (W) indicates that there are some very high net worth individuals contributing to inequality. As high-wealth individuals have a high share of financial assets (Wolff, 2013), a high average share of financial assets (w_{fin}) is expected to coincide with high wealth inequality. The second most important asset, (self-used)

[27] For the transformation, we follow Galli and von der Hoeven (2001) transforming the Gini (which for the considered year 2010 is bounded within $0 < Gini < 1$) using $y = \log\left(\frac{Gini}{1-Gini}\right)$ with $-\infty < y < \infty$ (the logit-transfomration).

[28] The sample only includes 13 countries (Australia, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Spain, Switzerland, USA, and UK). These countries are chosen since they are the only countries with high data quality as reported in Shorrocks et al. (2011). Thereby, we only have developed countries in our sample. In theses cases, wealth inequality does not have to be estimated based on regressions, but is reported with concrete values. If this were not be the case, our regression would only regress on a regression.

[29] The wealth inequality, the level of wealth, the share of financial assets and debt all originate from the study of Shorrocks et al. (2011). As the variables real interest rate and income inequality impact in the long-run, we take the long-run average for the last 30 years. This data originates from the World Bank (`http://data.worldbank.org/`).

real estate is more of a middle-class phenomenon. A high share of debt (D) implies that there exists some very low net worth individuals contributing to high net worth inequality. The real interest rate, on the one hand, presents the income out of capital for high net worth individuals and, on the other hand, the costs of debt for low net worth individuals. A high real rate of interest thereby contributes to high net worth inequality. The flow income is the essential source of stock inequality. If flow income is already distributed very unequally, the stock of net worth should also be unequally distributed.

Transformed Gini of wealth (2010)	All Controls	No Gini	No fin. assets	No Gini & fin. assets
Intercept	-26.535881 *	-23.83884 **	-26.79344 **	-23.10065 **
	(4.946557)	(4.19037)	(4.11406)	(4.38304)
Log-wealth	2.058987 *	1.92308 **	2.07374 **	1.90382 **
	(0.357606)	(0.33296)	(0.30266)	(0.35137)
Share of debt	0.051975	0.03301 .	0.05556 **	0.04575 **
	(0.023598)	(00.01449)	(0.01042)	(0.01093)
Real interest rate	0.122851 .	0.10297 .	0.12361 *	0.086 .
	(0.043668)	(0.03919)	(0.03783)	(0.03887)
Share of financial assets	0.002606 (0.014779)	0.01321 (0.01051)		
Gini of income after taxes	3.773958 (3.715010)		4.23658 (2.29004)	
R^2	0.9316	0.908	0.9309	0.8717
Adjusted R^2	0.8175	0.8161	0.8617	0.7947
p-value	0.05708	0.02382	0.01368	0.01145

Table 2.2.: OLS regression of wealth inequality (standard error in parentheses) (.,*,** significance at 10%, 5%, 1% level)

Spearman-correlation	w_{fin}	W	D	r	$Gini(y)$
w_{fin}					
W	-0.2545				
D	0.6818*	-0.2091			
r	-0.4364	0.2198	-0.1000		
$Gini(y)$	0.2273	0.1099	-0.0273	-0.4615	

Table 2.3.: Spearman correlation matrix of regression controls (* significance at 5% level)

The results of a simple Ordinary Least Squares (OLS) regression are reported in table 2.2. All coefficients show the expected signs. For the complete classification, however, only the level of wealth is significant at a reasonable level. The Spearman correlation matrix of the controls is presented in table 2.3. Only the correlation between financial assets and debt is significantly different from zero, while both indices are positively correlated. This emphasizes the view that the debt of one (low-income) agent is the financial asset of another (high-income) agent.[30] Debt, however, has a slightly stronger explanatory power for wealth inequality than the share of financial assets. Excluding financial assets from the explanatory variables increases the explanatory power of the regression and the significance of the factors. In particular, debt - besides the level of wealth - turns out to be the most important determinant for net worth inequality. Meanwhile, the Gini of income never came up significant in any specification. However, its inclusion increases the explanatory power of the real interest rate being negatively correlated with debt and especially financial assets (albeit not at a level that ensures significant difference from zero, see table 2.3). A theoretical argument for the latter could be that income inequality increases the supply of debt and thereby lowers the real interest rate. Thereby, the wealth inequality is self-destroying, implying that higher wealth inequality leads to a higher supply of financial assets, lower interest rates and finally to lower wealth inequality. This is an important subject we treat more extensively from a theoretic perspective in the section 5.5.2. The key finding - yet not surprising result - is that debt massively increases net worth inequality.

Albeit the current debate about excessive public debt, the level of private debt[31] [32] far exceeds the sovereign debt for developed countries. Figure 2.9 [33] reports the ratio of private to public debt. Apart from the case of Japan with its enormous sovereign debt, this ratio exceeds 1 indicating higher private than public debt. The trend-reversal in the very recent years can be attributed to the massive debt-financed government spending trying to compensate for public deleveraging. Schularick (2014), using a long-run empirical sample of developed countries and controlling for war times, shows that financial stability risks have come from private debt rather than public debt. Moreover, he shows that private debt evolution and public debt are negatively correlated indicating the former thesis that public debt steps in times of private deleveraging and vice versa.

[30] A more positive view - frequently emphasized in the literature - is that both are proxies for high financial development (see also section 2.1).

[31] In this case - and in contrast to the results presented in figure 2.8 - we present the debt of both - private households and firms. This certainly subsumes two very different forms of debt, as debt for private households has little growth-payoff in contrast to private business debt (Levine et al., 2007). The values are in nominal terms, not accounting for inflation.

[32] As reported in Kalemli-Ozcan et al. (2012) there was a general increase in leverage for both private households and firms. These debt positions present claim positions to other entities. Ultimately, they can be attributed to private households holding the firms. The switch of firms from equity to debt, on the converse side presents a switch from shares traded in the stock exchange to bonds. As pointed out in Merton (1974), the underlying contract structure implies a change from call to put options, which are senior to call options whose gain, however, is leveled.

[33] The data originates from the study of Cihak et al. (2012) benchmarking financial systems around the globe drawing on World Bank data.

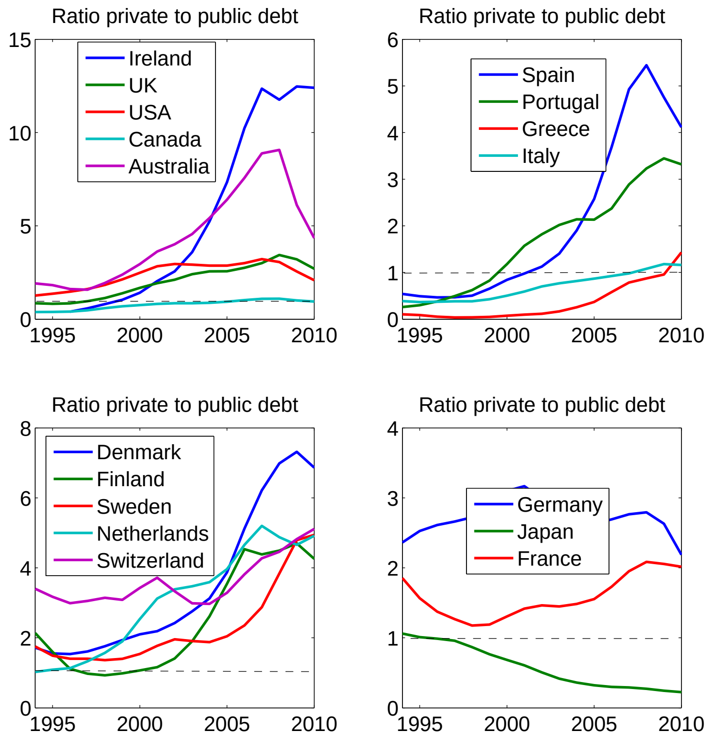


Figure 2.9.: Ratio of private to public debt (Data source: Cihak et al. (2012))

Iacoviello (2008) states that both inequality and debt levels exhibit long-term growth. The average growth rate of the private debt/GDP level[34] has been positive for all countries but Germany and Japan (see figure 2.10) and sometimes even excessive (especially for Ireland). As furthermore emphasized by Iacoviello (2008) debt growth itself, however, is cyclical and clearly related to overall economic conditions especially collateral prices. Due to information asymmetries in the financial market, lenders require collateral, resulting in the fact that in times of ascending collateral value the volume of credit also increases. The latter feature is nicely illustrated by Taylor (2012) (presented in figure 2.11) showing a strong comovement of housing prices and debt level for the US. Both factors increased until 2006. After that, both declined, with house prices leading and debt levels lagging as indicated by the clockwise spiral.

[34] We measure growth rate as log-difference of debt levels.

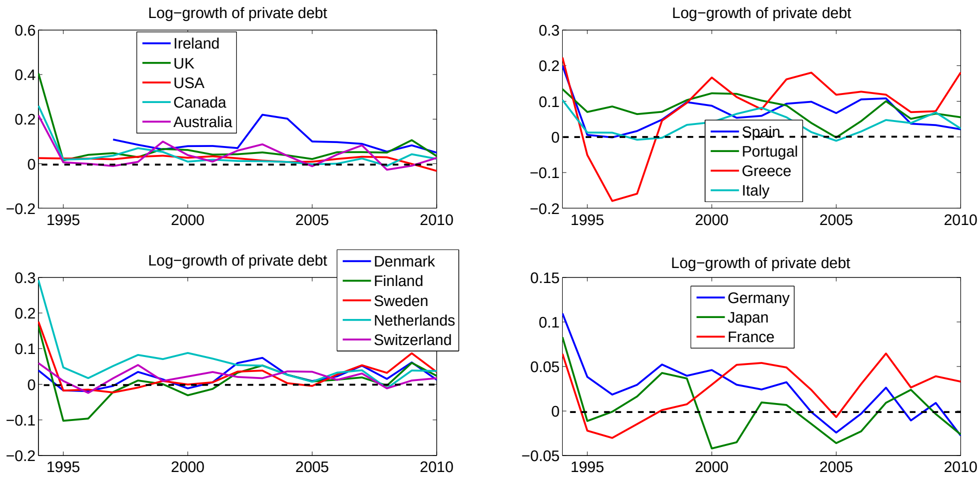


Figure 2.10.: Log-growth of private debt (Data source: Cihak et al. (2012))

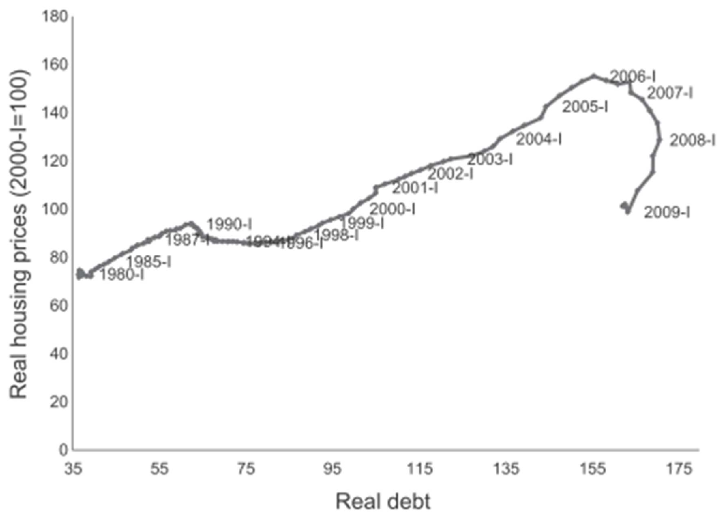


Figure 2.11.: Real US housing prices and real consumer debt (Taylor, 2012, p. 59)

Kumhof et al. (2014) report that value added GDP of the financial sector increased from approximately 4.5% to over 7.5% from 1982 to 2007.[35] Mah-Hui and Ee (2010) even state that the financial sector is the largest contributor to US GDP, even surpassing wholesale/retail trade as well as manufacturing. This growth not only contributed to the increased inequality - as stressed in Philippon and Reshef (2012), arguing with excessive compensation schemes in financial industry - but also emerges as a result (and thereby lags) inequality. Increased debt levels signal increased activity of financial intermediation. The debt positions are mostly held by low-income households. Kumhof et al. (2014) show that the debt to income ratio severely increased for the bottom 95% of the wealth distribution (crossing the 100% level roughly in 2006, implying that households have negative net worth) financing their consumption increase, whilst being at a significant lower and at a rather constant level for the top 5% of wealth distribution.

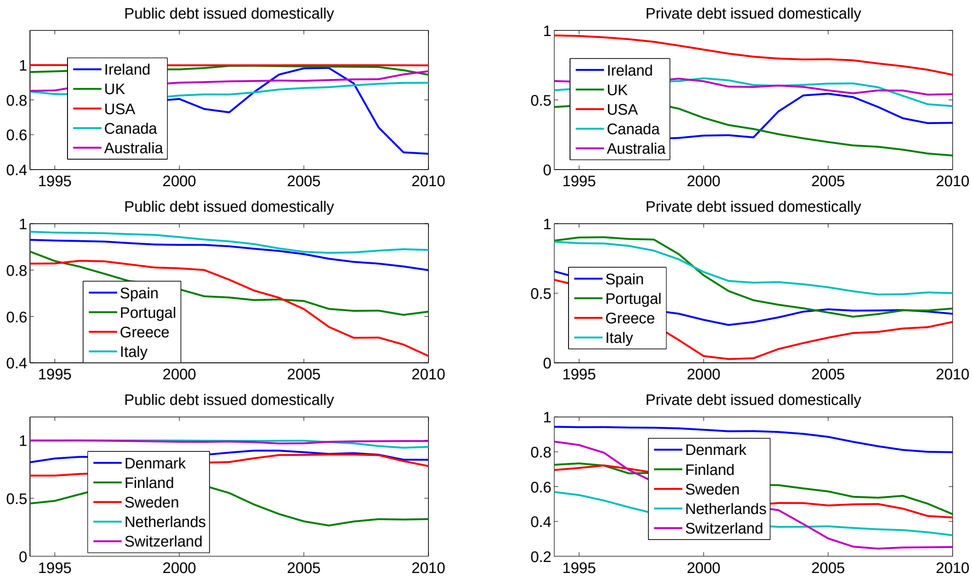


Figure 2.12.: Ratio of domestic to total debt for public (left panel) and private debt (right panel) for Anglo-Saxon, northern and southern European countries in time (Data source: Cihak et al. (2012))

The private debt run-up is also related to the following recession. In a study using micro data of several US counties, Mian and Sufi (2010) also show that the massive employment of debt, as measured by the level of leverage, has predictive behavior for severity of a recession, as measured by consumer default on debt or unemployment.

[35] A rationale for the large share of the financial sector is that - as e.g. documented in Piketty (2014) - the wealth/GDP ratio increased massively in the last decade. The role of the financial sector is to manage this stock of wealth. Gennaioli et al. (2014) arguing in a theoretical framework, show that the rising share of financial services in total GDP even sustains when management fees decline as the volume effect prevails.

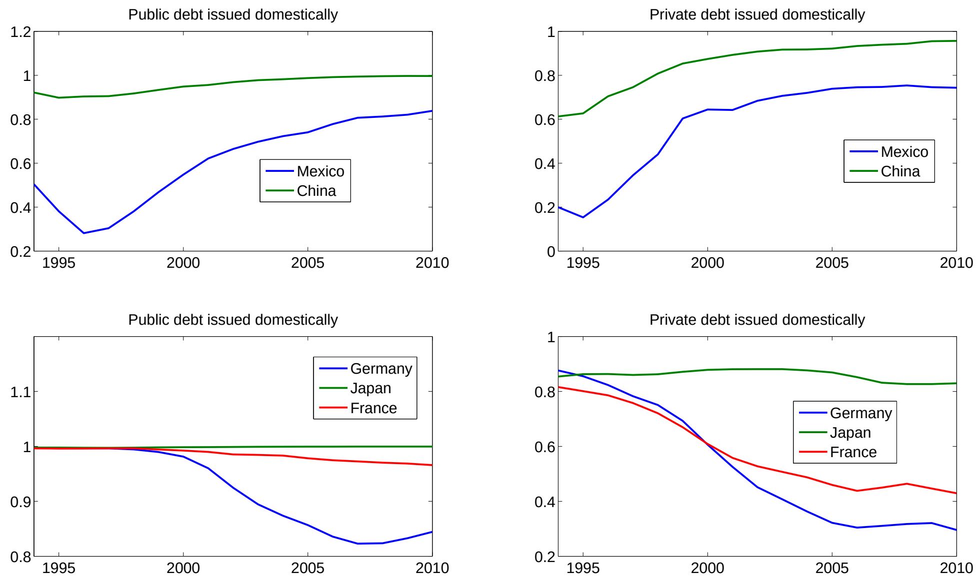


Figure 2.13.: Ratio of domestic to total debt for public (left panel) and private debt (right panel) for various countries in time (Data source: Cihak et al. (2012)

They show statistically significant correlations between the piling up of household debt until 2006 and the default rate after 2006, indicating that counties with high increases in debt level (they especially name counties in California and Florida) were subject to higher default rates afterwards. On the other hand, an increase in the debt level until the peak of 2006 is also accompanied by more severe housing price slumps. This can be justified by the classic Fisher (1933) debt-deflation argument reasoning that fire sales aimed at reducing debt levels go along with higher real indebtedness through asset price deflation. Similar behavior (even though lacking the strong statistical significance of the house market) can be found when analyzing car sales (Mian and Sufi, 2010). Comparable to houses (even though not in such an important manner) cars represent durable goods that frequently require debt financing and can be used as collateral.

Glick and Lansing (2010) - closely related to the approach of Mian and Sufi (2010) - find the same positive correlations between leverage run-up until 2007 and house price increase afterwards in a cross-country international study. Furthermore, they report a negative correlation between leverage increase and consumption growth on an international country level. The strongest absolute values in leverage growth, consumption decline and house price level change are recorded for Ireland. In contrast to the other sample countries, Germany and Japan actually had a decline in private leverage levels.

As opposed to public debt, which is subject to a strong home bias[36] [37], private debt is held to a large extent by foreigners (see figure 2.12 and 2.13). The financial globalization eventually enforces international indebtedness as indicated by downward trends of domestic debt issuance. However, this makes private debt also more sensitive to global macroeconomic conditions. While in Anglo-Saxon and northern European countries there is still a shift from domestic to international issuance of debt, in central and and especially southern Europe this trend broke down or even reversed. In fact, the trend to domestic debt is the case of a *sudden stop* (Calvo, 1996). The classic example is the so-called *Tequila Crisis* in Mexico in the mid 1990s clearly visible in the data (see figure 2.13). On the other side, developing countries China and Mexico exhibit a strong trend to domestic debt issuance for both public and private debt. The reported ratio which can also be interpreted as an inverse measurement of financial openness is especially high for Asian countries China and Japan implying low financial openness.

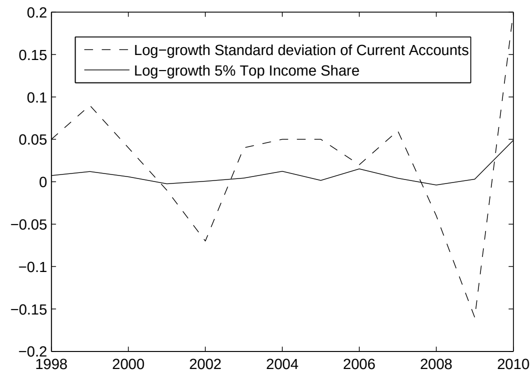


Figure 2.14.: Growth rate of top incomes and the standard deviation of current accounts (Data source: Atkinson et al. (2011) and OECD (2012))

Similar to the increase in world wide income inequality, current accounts imbalances have increased in recent years. Lane and Milesi-Ferretti (2012) reports increasing standard deviations of current accounts in the years leading to the crisis, showing that inequality prevailed not only on an inter-country but also an international level. If we relate the growth rate of the standard deviation of current accounts to the growth of top incomes (as an index of inequality), we find a strong comovement[38] (see figure 2.14). While the current accounts show a higher inter-time variance, both peak at sim-

[36]This strong home bias eventually increases pressure on politicians not to default on government debt, as the default would hurt the domestic population and thereby potential voters. Yet, as the example of Argentina also tells, default on foreign debt can block access to urgently needed international capital and therefore also is accompanied by high costs.

[37]Note that the data only reports the issuing location of the debt, but not the location where the debt is held. In particular, the US - as the world's largest financial market issues most debt domestically. The buyers of these bonds, however, frequently come from abroad.

[38]The Spearman correlation coefficient amounts to 0.628 and is statistically different from zero at a 5% level. The data set includes China, Denmark, Finland, Sweden, Spain, Portugal, Italy, Japan,

ilar points. Especially the negative peaks coinciding with the bursting of the dot-com bubble and the outbreak of the financial crisis are worth mentioning. The collapse of asset prices and the deterioration of financing conditions not only lessened inter-country trade and financing, but also inequality on an inter-agent level.

Lane and Milesi-Ferretti (2012) interpret the dispersion of current accounts as the outcome of asset (and especially oil) price run-ups as well as low risk aversion being accompanied by a loosening of credit conditions. The crisis, however, reversed these trends also accompanied by a decrease in *excessive* current account imbalances which were primarily stemmed by deficit countries conducted through the channel of demand reduction. Meanwhile, the surplus countries remain at a high level of current account surplus.

Kollmann et al. (2014) analyze the extreme current account surplus of the German economy for the European Commission estimating a three country Dynamic Stochastic General Equilibrium (DSGE) model with Germany, the rest of the Euro zone and the rest of the world. They come to the conclusion that the Germany's current account can be explained by the existence of positive shocks to the German savings rate (without further presenting an underlying rationale), an increased demand for German export goods by the rest of the world, and positive aggregate supply shock - in particular labor market reforms in Germany. Hale and Obstfeld (2014) meanwhile focus on the peripheral countries and their deficits. Their empirical analysis suggests that credit booms going along with current account deficits in the peripheral countries were supported by low interest rates as a result of greater financial integration. The core countries of the European Monetary Union increased their borrowing from the rest of the world while lending to European periphery (the GIIPS countries; Greece Ireland, Italy, Portugal, Spain) thereby increasing their own fragility.

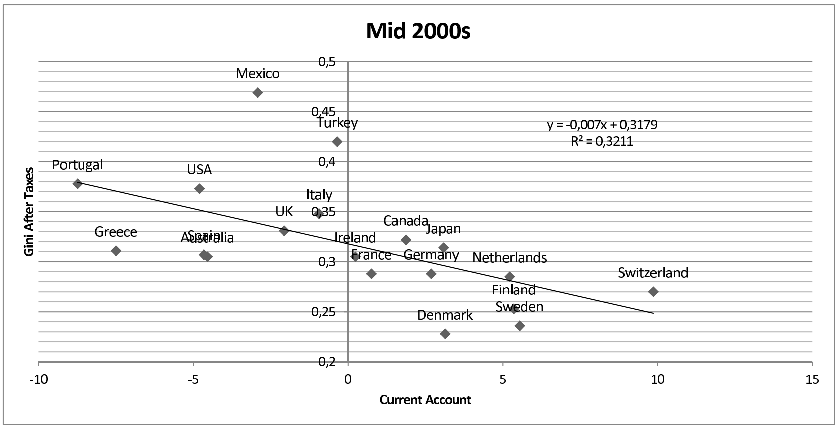


Figure 2.15.: Current Account Balance [% of GDP] and Inequality (Data source: OECD (2012))

France, USA, and Australia. The choice of countries was made according to maximize data availability of top incomes.

	Current account	Current account
Intercept	14.557 (5.198)	37.02 (32.166)
Gini of income after taxes	-45.656** (16.102)	-62.867* (29.295)
log(income)		-1.69 (2.388)
R^2 adjusted R^2	0.3211 0.2811	0.3417 0.2594
p-value	0.01142	0.03527

Table 2.4.: OLS-regression between inequality, income, and current accounts (standard error in parentheses) (*,** significance at 5%, 1% level)

	CA	$Gini(y)$	y
CA			
$Gini(y)$	-0.726 ***		
y	0.546 *	-0.693 **	

Table 2.5.: Spearman correlation matrix of regression controls (*,**,*** significance at 5%, 1%, 0.1% level)

As the comovement presented in figure 2.14 suggests, there seems to be a positive relation between current account imbalances and income inequality. The result, however, does not make a statement about the sign of the relation between growth of inequality and current account growth. To delve deeper into the subject, we relate the level of current accounts and inequality for the mid 2000's in the scatter-plot in figure 2.15. As shown in table 2.4, in a static and simple univariate analysis current account deficits go along with high inequality suggesting that both problems come as twins. This result was also presented in Kumhof et al. (2012).[39] As the authors, however, emphasize this result only holds if emerging economies are excluded. As with any univariate regression omitted-variable bias is a key concern. A natural candidate for the latter would be income.

As stated in the seminal result of Kuznets (1955), for developed countries (being on the right hand side of the *Kuznets curve*), a higher level of development (as measured by the income level) is accompanied by higher equality. As reported in the Spearman correlation matrix (see table 2.5 [40]) the factors income and inequality are negatively correlated. Meanwhile, theory suggests that - for a common technological progress - high-income countries already being at a high level of development only offer a small rate of return, should have a current account surplus by exporting capital abroad in line

[39]In their study, they take top incomes as measurement of inequality and also discuss changes rather than levels, which we will also do in the latter.

[40]In this case, $Gini(y)$ is income inequality, y is income level, and CA signifies the current account.

with neoclassical growth theory (Solow (1956), Swan (1956)). The latter is confirmed by the sign of the correlation in table 2.5 as well as by a more sophisticated recent study for the Euro area (Herrmann and Kleinert, 2014). This also explains the breakdown of the finding of Kumhof et al. (2012) by including developing countries. Developing countries are expected to be on the left side of the Kuznets curve, for which a positive correlation between income and inequality is measured. Together with the positive relation between current accounts and income, this results in an overall positive relation between current accounts and inequality. It is also important to point out that the two developing countries in the sample (Mexico and Turkey) have the strongest deviation from the regression curve as shown in the scatter plot in figure 2.15, furthermore confirming their different behavior.

By including income as a control the negative relation loses its significance, yet remains existing as reported in table 2.4. Kumhof et al. (2012) argue that a theoretical explanation for the latter behavior can be found in the *Relative Income Hypothesis* (which will be treated more rigorously in section 4.3). In order to keep up consumption levels for low-income individuals, developed economies with highly developed financial markets - in particular the USA and the UK - imported capital from abroad to finance consumption of low-income households. The increased level of debt in this case is explained from a demand perspective. As the relative income hypothesis is not a theory of levels but of changes, we discuss changes in the times of the crisis.

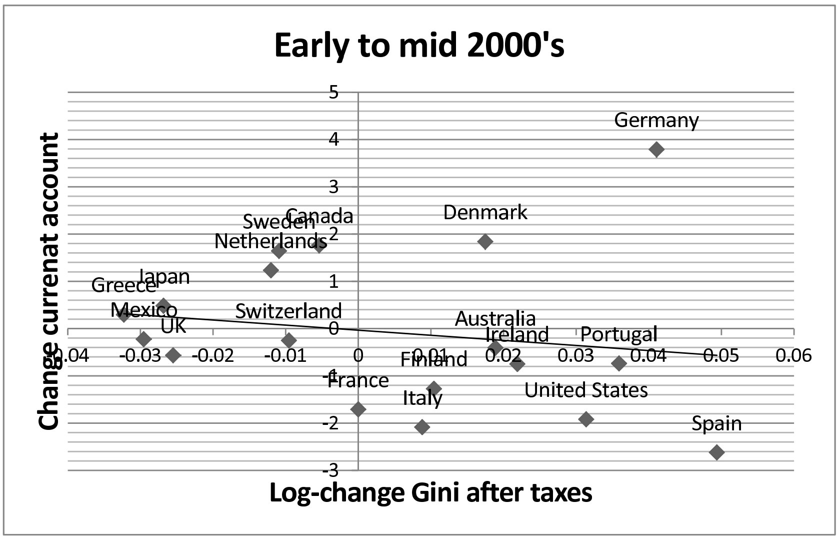
Figure 2.16(a) shows a scatter plot of the change in inequality compared to the change in current accounts. The related regressions are reported in table 2.6. As before, the coefficient of inequality shows the expected negative sign. The overall regression, however, is not significant at all. If we take a closer look at the scatter plot, this can be related to some countries grouped in the first quadrant of the coordinate system. These countries - in particular Denmark and Germany - exhibit both increased income inequality as well as increased current accounts. Thus, the thesis of Kumhof et al. (2012), arguing with the relative income hypothesis is far from universal.[41] As also argued by Al-Hussami and Álvaro Martín Remesal (2012), up to the crisis inequality - rather than influencing the change in current accounts - influenced the *absolute* change in current accounts (see figure 2.16(b) as well as the regression results in table 2.6). Once again the inclusion of the (change) of income level, slightly weakens the results, but does not destroy them.[42]

[41] In a related paper, Schmidt-Hebbel and Serven (2000) showed that there is no clear-cut evidence between inequality and the aggregate savings ratio. However, the authors also show that on a cross-country basis higher inequality is accompanied by higher aggregate savings - standing in diametrical result to the hypothesis of Kumhof et al. (2012).

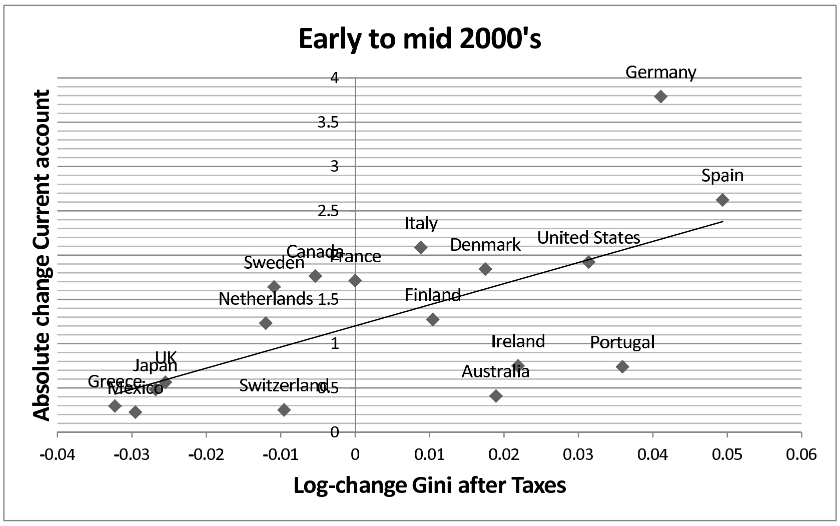
[42] Note that this time we compare two differences. This *difference-in-differences* approach is able to control for omitted-variable bias for non time-varying variables. As the income level, however, is time varying it should be included.

	Δ CA	Δ CA	$\lvert\Delta$ CA$\rvert$	$\lvert\Delta$ CA$\rvert$
Intercept	-0.03384 (0.40285)	0.01705 (0.63806)	1.2002 *** (0.1834)	1.4366 *** (0.2793)
$\Delta Gini(y)$	-4.66814 (6.95271)	-5.0893 (8.21948)	10.3624 ** (3.1658)	8.4054 * (3.5974)
$\Delta \log(y)$		-0.86468 (8.22132)		-4.0178 (3.5982)
R^2 adjusted R^2	0.0274 -0.03338	0.02812 -0.1015	0.4011 0.3636	0.447 0.3733
p-value	0.5115	0.8074	0.004781	0.01176

Table 2.6.: OLS-regression for changes (2000's to 2005's) in inequality, income, and current accounts (standard error in parentheses) (*,**,*** significance at 5%, 1%, 0.1% level)



(a)



(b)

Figure 2.16.: Change inequality and change (left) respectively absolute change (right) of current account early to mid 2000s (Data source: OECD (2012))

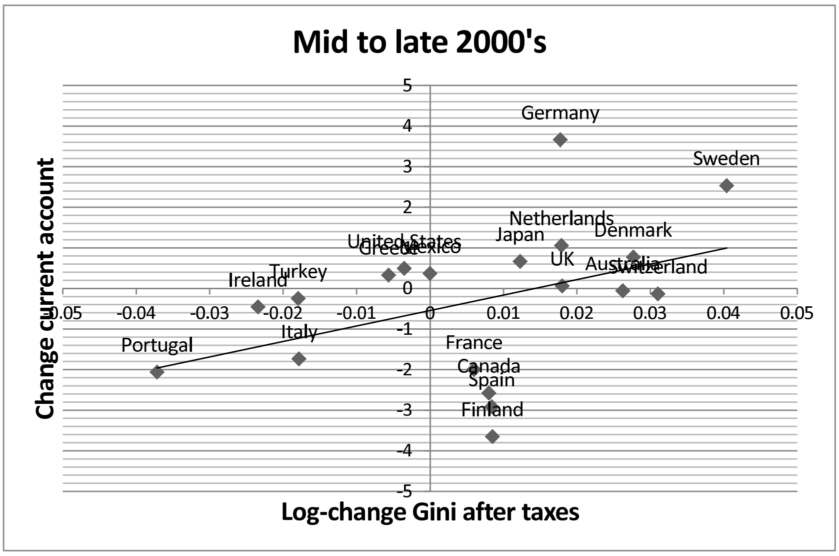


Figure 2.17.: Change of inequality and change of current account mid to late 2000s (Data source: OECD (2012))

The run-up to the crisis (early 2000's to mid 2000's) is well described by this behavior. As also pointed out by Al-Hussami and Álvaro Martín Remesal (2012), the behavior, however, is not stable in time. The follow-up of the crisis shows a different picture (see figure 2.17 and table 2.7). In contrast to the run-up period, the econometric analysis shows no evidence of a relation between change in inequality and the absolute change in current accounts. However, there is some weak evidence that after the crisis higher inequality led to higher levels of current account.

The post-crisis results and the behavior of countries like Germany are at odds with the theory of relative consumption. Opposed to that, one could argue that inequality not only influences the demand for debt and financial assets but also the supply of it.[43] Theorizing the latter would imply that rich households - in particular in countries with underdeveloped domestic markets - export their capital to foreign countries. This argument bares close resemblance to the *global savings glut hypothesis* first articulated by Bernanke (2005).

While debt is a stock, savings are a flow. When looking at global savings[44], there is a trend of decreasing savings (see figure 2.18) until and especially in the peak of the crisis. Yet, there is a strong cross-country heterogeneity. Decrease in savings can be especially found for the USA, UK, and southern Europe. Increased savings are measured for Scandinavia, Canada, and Germany but especially for the case of China. China not

[43]Note that our econometric evidence only makes a statement about correlations. The depiction of the results suggests a causation running from inequality to current accounts. For a more sophisticated econometric approach supporting the latter notion the reader is referred to Al-Hussami and Álvaro Martín Remesal (2012). A rationale for a reverse mechanism (which we, however, feel to be a bit far-fetched) could be that positive current accounts - indicating a strong export sector - disproportionally benefit entrepreneurs in contrast to workers creating domestic income inequality.

[44]Once again the data originates from the study of Cihak et al. (2012) drawing from World Bank data.

	Δ CA	Δ CA	$\|\Delta$ CA$\|$	$\|\Delta$ CA$\|$
Intercept	-0.5443	-0.4028	1.3349 ***	1.5751 ***
	(0.4126)	(0.5222)	(0.3054)	(0.3754)
$\Delta Gini(y)$	16.5587 .	17.0453 .	1.5535	2.3796
	(8.7251)	(8.9966)	(6.4582)	(6.4680)
$\Delta \log(y)$		-2.2999		-3.9045
		(4.9885)		(3.5864)
R^2	0.1748	0.1856	0.003392	0.07213
adjusted R^2	0.1263	0.08385	-0.05523	-0.04385
p-value	0.07484	0.1934	0.8128	0.5494

Table 2.7.: OLS-regression for changes (2005's to 2010's) in inequality, income, and current accounts (standard error in parentheses) (., *,**,*** significance at 10%, 5%, 1%, 0.1% level)

only exhibits an upward trend but also an anomalous high level which also stands in stark contrast to its GDP per capita since high savings ratios are usually associated with high living standards. Bosworth and Chodorow-Reich (2007) argue that the increased Chinese savings emerge due to lower fertility combined with lower mortality in the presence of underdeveloped public retirement systems. Rather than relying on the intergenerational transfer individuals now have to *take care of themselves*.

Gu et al. (2014) argue that the relation between income inequality and aggregate savings depends on the countries characteristics. For the group of OECD countries they find a negative correlation between inequality and the aggregate savings rate, whereas for Asian countries a positive correlation emerges. The confirm this behavior by means of a sophisticated econometric analysis also controlling for other candidate explanations such as demographic factors. They even argue that while the mechanism of a current account deficit e.g. witnessed in the OECD countries implies a finance-led growth strategy, the Asian countries and Germany follow a export-led growth strategy to counteract inequality.

In another empirical assessment, Al-Hussami and Álvaro Martín Remesal (2012) report that high financial development contributes to a current account deficit. The empirical analysis of Chinn and Ito (2007) further points out that, rather than financial development, the openness of the financial markets (refer also to figure 2.12 and 2.13 for a simple (inverse) proxy of financial openness) and the development of the financial markets contribute to a current account deficit. This explains the effect that financial development for Asian countries is associated with increased savings and depressed investment.

On the other side, the savings ratios in the developed countries also increased after the onset of the crisis. Carroll et al. (2012) argue that this deleveraging effect can be rationalized in a theoretical framework by the presence of higher unemployment risk resulting in a higher steady state savings ratio accompanied by temporary overshooting of savings. Empirical analysis also supports the notion that lower credit availability

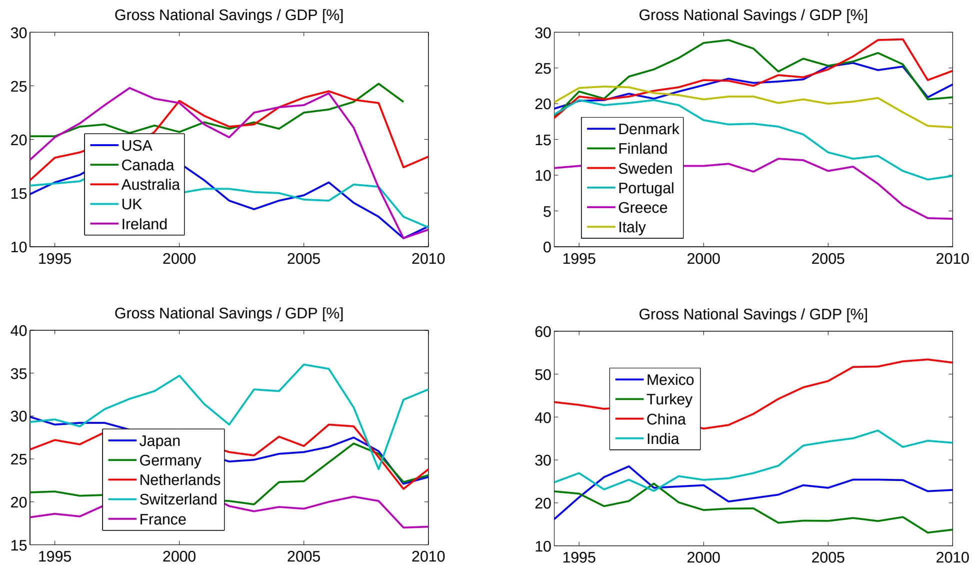


Figure 2.18.: Gross National Savings/GDP [%] in various countries (Data source: Cihak et al. (2012))

(therefore arguing from a supply rather than a demand side) and negative shocks to household wealth contributed to the result. According to Gropp et al. (2014), debt was mostly driven by supply. In their econometric analysis based on a micro data-set, they show that the decrease in debt in counties with strong real estate price slumps also affected renters, who are not subject to an adverse wealth shock. Koo (2014) argues that this private deleveraging only comes to a halt, once households reached a minimum consumption level. In the tradition of Fisher (1933), he emphasizes its adverse macroeconomic effect in the form of a *Balance Sheet Recession* - arguing from a stock rather than a flow perspective. He relates this to both *Japan's lost decade* and the Great Depression and argues that this behavior - despite expansive monetary policy - led to a decrease in total money supply and thereby to deflation. Therefore, he favors the increase in public debt during the times it takes for private households to *repair* their balance sheets.

As already pointed out, the big current account imbalances currently primarily exists amongst the pair USA (deficit) and China (surplus) and within the Euro area, with the southern European countries having large deficits, while Germany especially takes the role of the surplus country. The US-Chinese interconnection is discussed under the label of the *global savings glut hypothesis*.

The *global savings glut hypothesis* was introduced into the academic discourse by Bernanke (2005), stating that the US current account deficit was the result of an oversupply of savings of surplus countries, in particular China. The different savings ratio cannot only be rationalized by heterogeneous time preference but also by different abilities to produce in the present and the future. Countries having a competitive advantage in producing in the present are expected to exhibit a higher savings ratio. Usually these countries are developed countries with a high capital stock. However, the empirical evidence contradicts the results of standard textbook theory, making the effect of *money flowing upwards* for which money flows from developing countries (i.e. China) to developed countries (i.e. USA), a puzzle (Gourinchas and Jeanne, 2007). There are several theories trying to solve this puzzle arguing both from the perspective of developed countries and developing countries.

Taking the perspective of the developed countries - in particular the USA - it is often argued that over expansive monetary policy fueled the unsustainable credit boom (Taylor, 2009). Laibson and Mollerstrom (2010) in particular point out that the savings inflow was not used for investment but for consumption which in turn promoted an asset bubble in the real estate market. Lastly, the special role of the US-dollar still maintaining an *exorbitant privilege* amongst the world currencies is emphasized (Gourinchas et al., 2010). The USA - rather than having a competitive advantage in producing goods - has a competitive advantage in producing *safe* financial assets making it attractive to foreign investors (Caballero et al., 2008). This, moreover, makes the USA the *consumer of last resort*.

On the other hand, we can take the perspective of the developing countries. As already mentioned, their underdeveloped financial markets together with the demographic change in their society make it necessary to increase savings in foreign currency (Mendoza et al., 2009). Furthermore, the savings can also be thought of as hedge against commodity price volatility (especially for oil exporting countries) leading to a surplus in commodity booms and vice versa. The massive holdings can also be rationalized against the background of the Asian crisis in the 1990s. The Asian surplus countries hold foreign assets in order to hedge against *sudden stops* of foreign capital inflow. Some authors go even further, arguing that China actively devaluates its exchange rate in order to stimulate their export industry (Jeanne, 2012) coining the term *New Mercantilism* (Durdu et al., 2009).

We can compare this behavior to the Euro area. In contrast to the pair US / China, the flow of capital is in line with neo-classical theory by flowing from highly developed countries to those with lower development (Herrmann and Kleinert, 2014).[45] The persistence of these imbalances is due to the fixed exchange rate. In the opposite scenario, a floating exchange rate is self-stabilizing: a surplus would increase the price of the domestic currency thereby raising the prices of export goods and finally lower the surplus. While the fixed exchange rate in the Euro zone is *de jure* by means of a common currency area, the exchange rate for the case of US / China is only *de facto* fixed by the peg of the Chinese Renminbi to the US dollar. Similar to the former case, we discuss both the case of the deficit and the surplus economy.

The deficit economies in the Euro area seem to exhibit a preference for consumption. Similar to the US case, the flow of savings was primarily channeled into the real estate sector, fueling an unsustainable bubble (especially in Ireland and Spain). Germany -

[45]Or put negatively, this study disconfirms the Lucas paradox (Lucas, 1990) showing that underdeveloped countries tend to have a current account deficit.

being the large surplus country - in this relation mostly argues with the competitiveness of its export products as a result of moderate wage growth. However, the competitiveness is also a result of the fixed exchange rate regime. The fear of a *sudden stop* or the effect of commodity price booms does not apply for Germany. To use the same nomenclature as before, the southern European countries are *consumers of last resort*, whereas Germany ends up being a *producer of last resort*.

We, however, also think that inequality is important in explaining the results. If the supply of savings of high-income households cannot be channeled into investment (either due to lack of high yield investment opportunities or due to underdeveloped financial markets), these savings are exported to foreigners. There is a recent debate, which in Germany is led by von Weizsäcker (2014), arguing that the excess supply of savings has suppressed the real rate of interest to a negative level. von Weizsäcker (2014) following a Austrian tradition - argues that the negative level of real interest is the result of the *Second Law of Thermodynamics*, for which increasing entropy in dynamics systems reduces the productivity of investments. In a more traditional economic rationale, one could argue that the return on capital - decreasing with the level of capital [46] [47]- is lower than a constant rate of depreciation. These idle savings could flow to the factor land being in constant supply. This would result in an increase in the price of real estate, as eventually witnessed in countries such as the US or Spain.

The large savings of China are frequently explained by the demographic structure - especially an increase in longevity - which surely is an important factor for Germany too. In theoretical models, this can be accounted for in the framework of Overlapping Generations (OLG) (Samuelson (1958), Diamond (1965)) models. However, the social pensions systems are highly developed in Germany.

The high supply of savings can also be rationalized in the so-called *Bewley*-type models (Huggett (1993), Aiyagari (1994)) for which the high level of savings results from an uninsurable income risk. Yet, the latter case could also be excluded in Germany due to highly developed unemployment insurance systems.

Eggertsson and Mehrotra (2014) present a simple New-Keynesian model with overlapping generations that allows for closed-form solutions to tackle the issue of secular stagnation reoccurring in the current debate. In their model a negative real-interest rate can emerge as a log-run phenomenon - in contrast to other models such as Eggertsson and Krugman (2012) - in which it is only of a temporary nature due to a deleveraging shock. The low real interest rate emerges due to deleveraging, a slowdown in population growth, a decrease in the price of investment goods due to advances in information technology (Karabarbounis and Neiman, 2013), and strong income inequality increasing the supply of savings by wealthy households. As a result, the aggregate demand curve has a kink and a positive slope for low values of inflation allowing for the existence of multiple equilibria. Thereby, diverse *Keynesian paradoxes* emerge. Besides the clas-

[46] This is for output y being a function of capital in effective labor terms k; $y = f(k)$, we have $\frac{\partial^2 f(k)}{\partial k^2} < 0$ as in the standard Cobb-Douglas specification ($f(k) = k^\alpha$) with $\alpha < 1$.

[47] It is also important to point out that this is a point made by Karl Marx. Yet, he eventually argues that this effect would result in the long-run breakdown of the capitalist system.

sic *paradox of thrift* (higher saving leads to less aggregate demand in turn resulting in lower income), the *paradox of toil* (Eggertsson, 2010) (more labor supply leading to lower wages lowering income), and a *paradox of flexibility* (Eggertsson and Krugman, 2012) for which higher flexibility in wages and prices lowers the real interest rate which, however, cannot be followed by interest rate cuts due to the binding zero lower bound on nominal interest rates. As a policy conclusion they recommend a substantial increase in the inflation target, an increase in government spending (directly counteracting the adverse effects of private delevearging), or redistribution of income.

It is notable to point out that not only in the latter model but also in the general OLG as well as the *Bewley*-type model literature, public debt can eventually be welfare enhancing.[48] This stands in stark contrast to the *Ricardian equivalence*. In his seminal paper, Barro (1974) asked whether *government bonds are private wealth* to answer in the negative. The basic idea is that increased government expenditures financed by means of a deficit are completely outdone by increased private savings of the same amount. As the net wealth remains unchanged, there is also no aggregate consumption effect to a government expenditure increase in line with the permanent income hypothesis (Friedman, 1953). The basic underlying notion is that representative forward-looking rational agents anticipate that an increase in government debt today is followed by an increase in taxes in the future posing the so-called *Ricardian equivalence*. In a framework with heterogeneous agents that are subject to idiosyncratic uninsurable income risk, Aiyagari and McGrattan (1998) show that government debt can provide utility to private households by allowing to conduct precautionary savings. This relaxes borrowing constraints and allows to smooth and thereby (temporary) increase private consumption in times of negative income shocks. In the framework of Aiyagari and McGrattan (1998), these effects highly outweigh adverse effects of public debt such as crowding out of investment (by means of increasing the interest rate) and future taxes. Empirical evidence also shows that the distribution of income matters when quantifying the impact of a government spending shock. Anderson et al. (2012) show that wealthy households eventually behave like *Ricardian* households as assumed in Real Business Cycle (RBC) models. Meanwhile, low net worth households are *Non-Ricardian* and increase their consumption due to binding credit constraints. This is eventually in line with standard textbook IS-LM analysis emphasizing the role of disposable income rather than wealth. To put it even more bold, RBC-models assume that the representative consumer is wealthy, whereas standard Keynesian model assume that the representative consumer is rather poor in terms of wealth. Anderson et al. (2012) also show that expansionary fiscal policy is accompanied by a reduction in consumption inequality.

Bhandari et al. (2013) refresh the discussion regarding the *Ricardian equivalence*. In contrast to Barro (1974) arguing in a representative agent framework, they discuss the model for the case of heterogeneous agents in a incomplete market setup and aggregate shocks. Bhandari et al. (2013) echo the famous result of Barro (1974) showing that the absolute level of debt does not matter, but its distribution is of importance. As

[48] This is especially interesting since the latter type of models mainly differentiate from standard models by the assumption of heterogeneous agents.

government debt is an asset for high-income individuals this would increase the level of inequality. As the Ramsey planner is assumed to also exhibit an equity concern he reacts by also increasing taxes and transfers. They show that during recessions (negative aggregate shocks) it is advisable to issue debt as well as increase taxes *and* transfers. If the social planner would not care about inequality, it would react to a recession (modeled by a negative Total Factor Productivity (TFP) shock) by increasing debt and taxes but decreasing transfers in order to maintain a balanced budget.

Note that - at this point of time - we do not consider the role of the government debt. Note that our model does not claim to give a full representation of a full macroeconomy but focuses on important agents, in particular private households. Nevertheless, this is probably the first natural candidate for a model extension. In contrast to private households, the government finds it easier to maintain a high level of debt not least owing to the fact that government can create income by means of sovereign power by increasing taxes (for high-income individuals). However, we introduce a taxation system in section 7.2. In contrast to government bonds, tax payers (especially high income) *must* pay taxes, while they *can* buy government bonds. Bonds being a debt contract, however, also reassure them a return.

In our theoretical framework, we focus on the effect of inequality of income rather than age as in the OLG-type literature. We, moreover, feel that the *Bewley*-type literature fails to address this problem properly. The following section gives an overview about theoretical models trying to explain the recent crisis, categorizes them, presents their results as well as shortcomings.

3. Literature Overview of Theoretical Models in this Area

> The difficulty lies,
> not in the new ideas,
> but in escaping from the old ones [...].
>
> (Keynes, 1936, p. xxiii (preface))

Several models try to account for the presented facts. We contrast the mainstream approach (DSGE, New-Keynesian etc.) with a heterodox approach - both presenting stylized models. On the other hand, we discuss complex models with heterogeneous agents both from a more traditional economic background (in particular the so called *Bewley*-type models) and the more heterodox approach of Agent-Based Modeling coming from the econophysics field. For the sake of distinguishing them we will label the former as *Heterogeneous Agent Models* (HAMs) and the latter as *Agent-Based Models* (ABMs).[1] Both models are closely related to the discipline of computational economics as they are hard or virtually impossible to solve in a closed-form analytical manner. We follow the ABM-type paradigm and will discuss advantages as well as shortcomings of this approach.

3.1. Dynamic Stochastic General Equilibrium Models

The models related to more standard literature mostly only slightly soften the representative agent assumption by introducing different forms of consumers. A highly stylized model is presented in King (1994). A two-type agent economy differing in initial endowment (high endowment agents being creditors and vice versa for debtors) is modeled. Downturns result in changes of net worth from debtors to creditors, who have a lower marginal propensity to consume. Upon this assumption, a non-monotonous aggregate demand curve is modeled resulting in one unstable and two stable (one at low-level and one at a high-level) equilibria accounting for financial instability.

Another representation of this approach can be found in Eggertsson and Krugman (2012), introducing two types of consumers, patient and impatient, in a standard New-Keynesian framework. In the model agents differ only by their time discount rate. The authors interpret the model as a representation of the behavior in Japan's lost decade and the Great Depression as well as the current situation. They show the emergence

[1] Note, however, that the distinction is not always that explicit in the literature.

of a debt-deflation spiral triggering a negative equilibrium interest rate and thereby is accompanied by the problem of zero lower bound of the nominal interest rate. This *topsy-turvy* (Eggertsson and Krugman, 2012) situation is accompanied by the Keynesian *paradox of thrift* as well as another destabilizing mechanism coined *paradox of toil*, for which in the presence of the lower interest rate bound increased labor demand lowers real wage level leading to lower aggregate demand resulting in lower labor demand. In the framework of a New-Keynesian model, they derive the policy conclusion that this situation has to be met by a higher inflation target and especially by higher government expenditure.

Kumhof et al. (2014) try to explain the stylized facts of inequality and debt level growth in a model with utility maximizing workers and investors. Workers in their modeling framework only derive utility from consumption, whilst investors also derive utility from physical capital and deposits. Consistent with empirical data, they assume a positive relation between leverage level and default probability. By applying a negative shock in worker's bargaining power (resulting in increased wealth inequality) and a exogenous crisis event (leading to higher loan rates due to default as well as a drop of wages), the stylized facts are being rebuilt. If subsistence consumption is assumed to be variable, workers reduce consumption significantly in downturns and thereby reduce debt levels as well as crisis probability. The authors derive the policy conclusion that orderly debt restructuring tapers the severeness of the downturn. Meanwhile, an increase in worker's bargaining power can reduce the leverage build-up and thereby the crisis probability.

Kumhof et al. (2012) extend the model of Kumhof et al. (2014) by considering an open economy set-up. In this case, the loans for workers originate from abroad resulting in a current account deficit. To derive this result in their model they not only require higher inequality (as modeled once again in the form of a negative bargaining shock of workers) but also financial liberalization (modeled by a lower banking spread). The model is able to represent the behavior of the US and UK (eventually being calibrated to UK data), for which increased inequality resulted in a current account deficit. However, as already elaborated on in section 2.2 this is far from a universal result. Kumhof et al. (2012) at least explain the inverse behavior of China - where increased inequality was accompanied by a current account surplus - via underdeveloped financial markets. To account for this result in the model they assume the presence of binding credit constraints.

Midrigan and Philippon (2011) build a theoretical model upon the empirical result that a higher initial debt increase results in stronger slumps (as measured by unemployment) found in Mian and Sufi (2010). They model households with durable and non-durable consumption and a collateral constraint depending on the price of durables interpreted as houses. They assume heterogeneity in borrowing constraints as well as shocks to borrowing constraints. Simulations prove that heterogeneity decreases the effect of shocks due to trade amongst agents. The negative shock in credit can be met by a central bank's increase in high powered money, since - in the given model - both assets are perfect substitutes.

Iacoviello (2005) examines the effect of housing prices on monetary policy in a framework with entrepreneurs, patient and impatient households with different time preference rates owning houses and using them as collateral. He shows that monetary shocks are stronger if the collateral effect as well as debt-deflation is assumed (by supposing that debt is not indexed). If debt were indexed, the price adjustment process would dampen these shocks. This can also explain the sluggish reaction to inflation shocks found in empirical data. On the other side, supply side shocks are stabilized by nominal debt that is not indexed. Therefore, the author argues against indexed debt, since the demand shocks - counteracted by nominal debt - are under the control of monetary policy in any case. In fact, the presence of debt amplifies the policy instrument of the central bank. As a result, Iacoviello (2005) shows that reacting to asset price inflation only increases welfare marginally. On the other hand, the demand shocks amplified by debt-deflation can run out of control if the central bank faces other constraints such as the zero lower bound of nominal interest.

Following a similar modeling strategy, the author addresses the current situation of income inequality and household debt in Iacoviello (2008). In this model, agents differ in their patience, derive utility from consumption and the stock of houses, and are subject to a collateral constraint. Shocks in the collateral margin (due to changes in risk perception) as well as household income shocks are estimated from empirical time series and imposed on the model. The constrained households behave like *hand to mouth* consumers and engage in debt-financed consumption and durable acquisition (in the form of houses). The model is able to capture many of the stylized facts found in the recent crisis including growth in debt-level, short-term cyclical behavior of debt, and a small rise in consumption inequality compared to a strong rise in wealth inequality. The correlation between debt and inequality highly depends on the amount of constrained agents. Iacoviello and Pavan (2013) further extend this model of the housing market with a life cycle profile of housing consumption. The model is able to capture realistic patterns of wealth inequality and further is able to replicate the recent macroeconomic development. The *Great Moderation* - lasting from the early 1980s to the onset of the financial crisis (Stock and Watson, 2003) - in this model can be (partly) attributed to higher individual income risk - implying higher income inequality - and lower downpayment requirements on loans making the model more stable in the face of small shocks. The same mechanisms in the Great Recession, characterized by a tightening of credit conditions and modeled as a combination of a negative financial and technological shock, make the economy more unstable to large shocks. In a short extension, moreover, the important effect of default is brought up.

Chadha et al. (2013) present a model with savers and borrowers that are linked through a commercial bank giving loans at interest rate with a premium relative to the central bank rate of interest. The equilibrium interest rate in this case results from an optimizing behavior of the bank. The consumption of the lenders and the borrowers are mirror images of each other. Lower interest rates and a lower loan to value ratio on debt easing the credit conditions for lenders increase their consumption while decreasing consumption of borrowers and vice versa. For the given calibration, the consumption effect of borrowers is stronger than for lenders. Chadha et al. (2013), moreover, show that consumption growth and asset prices are clearly correlated. This is not due to a wealth channel but the result of a credit channel, as higher collateral value allows for higher borrowing for consumption purposes. In contrast to Iacoviello (2005), they argue that monetary policy should target asset price inflation due to its impact on consumption.

Menno and Oliviero (2013) remodel the Great Recession as the case of heterogeneous households in the presence of collateral constraints. The crisis itself is simulated by assuming a negative shock to financial intermediation and income (a technological shock). The adverse price shock to house prices is a negative welfare shock to borrowers (the constrained agents). Meanwhile, the savers (unconstrained agents) eventually use the times of low asset prices to stock up on real estate assets. Similar to the results of Chadha et al. (2013), the intermediation shock accompanied by higher credit spreads - by redistributing from borrowers to lenders - increases consumption of lenders while decreasing consumption of borrowers in a mirror-image manner. On the other hand, the negative technology shock affects both types of agents.

In summary, the mainstream literature describes the Great Recession as a debt-deflation problem similar to the Great Depression. As the presence of debt, however, requires the existence of heterogeneous agents - lenders and borrowers - the role of inequality is also discussed in some of the more standard literature.

3.2. Heterodox Literature

In contrast to mainstream macroeconomics, heterodox economics has traditionally considered the distribution of income. But rather than focusing on the personal distribution of income, it focused on the functional distribution of income.

The literature focusing on the recent behavior does not find clear-cut evidence on the evolution of the profit share. On the one hand, it is argued that the functional distribution lost its importance in the last 50 years as compared to the personal distribution. It is argued that the current situation therefore cannot be attributed to surplus redistribution from the factor labor to capital but rather to a stronger dispersion of wage income (Anselmann and Krämer, 2012). In the classic Marxist framework, inequality persists because capitalists exploit the working class. However, in the current situation, for which the capitalists are *working rich*, capitalists tend to exploit *themselves* in a competitive rat race creating inequality. The latter is not only measured by a higher dispersion of outcomes (i.e. labor compensation), but also by a higher dispersion of provided effort (e.g. overtime hours) (Landers et al., 1996). Moreover, as put forward in Parker and Vissing-Jorgensen (2009), since high-income agents have become workers rather than rentiers, the source of these fluctuations has shifted from stock market risk to income risk that for this particular group is substantially connected to the economic situation of the firm in the form of performance-based payments.

On the other hand, Karabarbounis and Neiman (2013) document a decline in the labor share during the last 40 years. In a calibrated theoretical model, they argue that this is the case due to the decline in the price of investment goods leading to a substitution of labor for capital. The latter effect is related to the advances in information technology. Moreover, Piketty (2014) argues that we have been recently witnessing a "return of capital" and "patrimonial capitalism" for which inherited wealth gains momentum as opposed to self-earned wealth. Piketty (2014) acknowledges that the recent increase in income inequality was due to the increased dispersion of wages. The years from the 1970s to the 2010s saw *working rich* rather than *coupon clipping rich*, as in the times of Marx. The strong wage inequality, however, allowed some agents to accumulate high amounts of wealth replacing wage income with capital income as main source of their income.[2] In the further course of this work, in particular in section 5.5.1, we will relate personal and functional distribution of income in a formal way.

There is some theoretical literature from the heterodox perspective discussing the recent crisis. Kim (2012) presents a stock-flow consistent neo-Kaleckian model with workers and capitalists and the effect of consumption emulation.[3] In this framework, lower bargaining power of workers increases the profit share. As workers try to emulate the consumption of capitalists, aggregate demand increases resulting in higher growth. The accumulation of debt by means of the establishment of lender-borrower links between workers and capitalists, however, also results in potential instabilities.

An interesting approach is presented in Kapeller and Schütz (2012). By introducing conspicuous consumption effects in a Post-Keynesian model the authors are able to create what they label as *Minsky-Veblen-cycles*.[4] They put forward that theoretical literature deduces that the increase in the profit share in the US should go along with increased investments. Or - to use standard Post-Keynesian terminology - growth should be profit-led rather than wage-led. In fact, the increased profit share was accompanied by an increased consumption level. Kapeller and Schütz (2012) therefore coin the term *debt-led* growth, since the increased consumption was financed via consumer debt. The authors also point to similar behavior leading to the Great Depression.

By introducing the dichotomy between capitalists and workers the Post-Keynesian models argue based upon two agents and allow for closed-form solutions. The next section - discussing Heterogeneous Agent Models - present a more complex approach with N agents allowing for inequality in the personal distribution of income.

3.3. Heterogeneous Agent Models

Heterogeneous Agent Models (HAMs) drop the assumption of a representative agent in favor for a group of heterogeneous agents. Compared to the more stylized approaches presented in the previous sections assuming two or three types of agents, these models involve *real* heterogeneity with N agents. Eventually, this literature already has a rather long tradition dating back to seminal work of Huggett (1993) and Aiyagari (1994). Most of the literature does not distinguish between Heterogeneous Agent Models and Agent-

[2] The causality eventually could also run in the opposed direction. An increase in the profit share, increases wealth inequality, since wealth (as source of capital income) is more unequally distributed than wage income (Milanovic, 2014).

[3] It is interesting to point out that this model is very close to the model of Kumhof et al. (2014). One might even go as far as labeling the model of Kumhof et al. (2014) a Post-Keynesian model disguised in a DSGE framework.

[4] This expression is owed to Hyman Minsky as the creator of the Financial Instability Hypothesis and Thorstein Veblen as the creator of the theory of relative consumption.

Based Modeling. We, however, make a clear distinction in the nomenclature. The former are rather standard growth models that introduce the notion of heterogeneous agents, whereas the latter are more heterodox. In particular, for the latter the behavior of the agents results from behavioral rules. We will treat Agent-Based Models in the subsequent section.

The assumption of a representative agent driving the behavior of the complete macroeconomy is prevalent in standard economic theory. Sometimes this assumption is concealed under some very technical terms. As elaborated in Keen (2011), to derive a standard well-behaved[5] aggregate demand function, some very strong assumptions are needed. As put forward in the so-called *Sonnenschein-Mantel-Debreu* theorem (Sonnenschein, 1972), (1) all goods must be neutral or homothetic (implying a linear slope of the Engel-curve linking income level and consumption demand) and (2) the Engel-curves of individual agents must be parallel. The first assumption - concerning the heterogeneous goods - rules out important cases of luxury or commodity goods, implying that the level of income does not impact on demand. The second assumption - aimed at heterogeneous consumers - is even more important in the context of our model. To make it hold, the distribution of income should be independent of the level of prices. Taking both assumptions together implies that the Engel curves and thereby the marginal propensities to consume a specific good i of a an agent j are identical for all agents. If this condition hold, it is convenient to assume a representative agent (or an army of identical clones). We elaborate more thoroughly on this topic in A.2 under the label *fallacy of composition*. There are some macroeconomic models with heterogeneous agents for which - by construction - inequality does not impact on aggregate results allowing for an independent discussion of aggregate dynamics (e.g. growth) and distributional issues. In Chatterjee (1994), wealth inequality is given exogenously and shown to converge or diverge depending on the relation between average savings propensity and wealth level.[6] In Ventura and Caselli (2000), furthermore, inequality in tastes and skills are introduced. The authors show that despite identical aggregate dynamics a wide array of distributions in consumption, assets, and income can emerge.

The existing literature treating heterogeneity in a DSGE framework is often referred to as *Bewley*-type models.[7] These models assume that the income heterogeneity - resulting in subsequent wealth heterogeneity - is a result of insurable idiosyncratic income risks. The seminal contributions are Aiyagari (1994) and Huggett (1993), assuming heterogeneous agents with precautionary savings subject to liquidity constraints. By assuming an idiosyncratic uninsurable income risk, income inequality resulting in subsequent wealth inequality is created. This - slightly obscure assumption - however also implies that, since the idiosyncratic risk has the same functional form for all agents, the

[5]By this, we mean that the aggregate demand for a good i depends negatively on its price $x_i(p_i)$. Moreover, a specific level of price goes along with a specific level of aggregate demand. Or to use a term from mathematics, the demand function is injective.

[6]He assumes a homothetic CRRA function, however, also incorporates a minimum consumption level (cf. section 4.3). As discussed more thoroughly in section 5.5.2, this leads to a divergence of wealth inequality.

[7]Named after US-economist Truman F. Bewley sparking the initial idea to this form of modeling.

distribution of income amongst agents at a particular point of time equals the distribution of a particular agent during his life time. Or - as put in Castaneda et al. (2003) - all agents are essentially identical and only subject to specific circumstances of (bad) luck. In the aggregate, the equilibrium interest rate is lower than the rate of time preference. If higher borrowing limits are introduced (weakening the imperfection resulting from the credit constraints), aggregate capital is reduced and the interest rate increases coming closer to the level of time preference. Meanwhile, higher idiosyncratic risk increases inequality and savings and thereby lowers the equilibrium interest rate indicating that inequality is accompanied by a low level of the equilibrium interest rate (Aiyagari, 1994). It is interesting to point out that the *oversavings* results are also derived in Overlapping Generations (OLG) models (e.g. Diamond (1965)), for which the heterogeneity of agents results from the heterogeneous age.

In essence, the inequality emerges due to non-existing insurance options. Thereby - based upon these models - it is often argued in favor of financial liberalization creating assets that allow for insurance against any unfavorable state of the world. This is in particular the case since different sources of negative income shocks - such as (un)employment, health and martial status (i.e. divorce) - are positively correlated (Heathcote et al., 2009). On the other hand, there are other options to insure against these idiosyncratic shocks and thereby counteracting inequality. Government can provide public goods such as health and education as well as public social security systems (Piketty, 2014).[8] Moreover, agents can adjust their labor supply according to the shocks - given sufficiently flexible labor markets (Heathcote et al., 2009). On a more individual level family also provides a measure of insurance (Becker and Tomes, 1986). Concretely, children are supported by their parents, the elderly by their offspring, and non-working individuals by their partners (Heathcote et al., 2009). In the latter case - and in the contrast to the government system and the financial market - the risk sharing ability is, however, very limited.

Krusell and Smith (1998) can probably be considered the seminal contribution in the *Bewley*-type literature. They present a simple stochastic growth model for which households maximize intertemporal utility from consumption subject to a budget constraint and a Cobb-Douglas type production technology. As the households, however, are subject to an uninsurable idiosyncratic income risk, income heterogeneity emerges. Moreover, the authors assume heterogeneous time preferences. Yet, the authors conclude that income and wealth heterogeneity does not have a strong impact on macroeconomic dynamics and the behavior of the macroeconomy is well described by the behavior of a representative agent multiplied by the number of agents. However, they also admit that in this type of model - due to the incompleteness of insurance markets - consumption is more closely linked to income (rather than wealth) as emphasized in the Keynesian literature. Moreover, the existence of borrowing constraints leads to the fact that income distribution can not be completely neglected. There is still a large amount of literature

[8]We discuss this idea intensively in section 7.

stemming from the original work of Krusell and Smith (1998). Yet, this literature mostly focuses on computational issues in solving the sophisticated models.[9]

The presented models, however, also suffer from some deficiencies. First and foremost, they underestimate the degree of inequality in both income and especially wealth. An important factor is that they cannot rationalize the high amount of savings by very rich individuals. A very good representation of US earnings and wealth inequality is achieved in the model of Castaneda et al. (2003) by incorporating aspects of dynastic models with bequests to rationalize the high savings of rich individuals. They, however, fail to address the low savings and attribute this to the fact that they disregard public social security. Benhabib et al. (2011) try to address these problems in a continuous time OLG-model with a finite life of agents and intergenerational transmission of wealth. Besides being subject to labor income risk, they add capital market risk in their model. By doing so they are able to replicate the power-law behavior for the right tail of wealth distribution measured in empirical data. While both strong capital income taxation and estate taxes reduce wealth inequality in this framework, strong bequest motives lead to a higher degree of wealth inequality. In a similar model following the tradition of Blanchard (1985) and Yaari (1965), Benhabib et al. (2014) are able to replicate a double Pareto-distribution of wealth featuring power-law behavior at both the left and the right tail of the distribution.

A very interesting *Bewley*-type model inspired by the recent situation is presented in Guerrieri and Lorenzoni (2011). The authors discuss a model for which a credit crunch (modeled as a shock to the debt limit) leads to a deleveraging of indebted households and an increase in precautionary savings by high-income households both promoting a fall in interest rate. The resulting consumption decrease and consequential output decrease is further promoted in the case of a zero lower bound. If, moreover, durable assets are introduced in the model the credit crunch leads to a accumulation of durables for high-income (as a form of precautionary holding) and respective sales for constrained households. In effect, this eventually leads to an output level stabilization as a result of increased durable consumption. An increase in banking intermediation costs (modeled as a positive shock of the credit spread) affecting all households, however, results in strong output contraction. This result is further intensified by nominal rigidity of prices.

3.4. Agent-Based Modeling

In this section, we will present the paradigm of Agent-Based Modeling (ABM). The basic modeling idea is presented and contrasted with the well-established approach of DSGE modeling. The most important shortcoming can be subsumed under the label of the *Lucas critique*. We give an overview of the (aspiring) literature of Agent-Based

[9] A survey is e.g. found in Heer and Maussner (2005). The underlying problem is that no closed-form solution can be found as the model includes a distribution which is an object of an infinite dimension (Heathcote et al., 2009). The solution strategy is usually to take a finite number of measurements (e.g. the mean and variance in the case of Krusell and Smith (1998)) to compute a numerical solution.

Modeling and qualify our own contribution within this area. We also point out other arguments brought forward against ABMs and our attempt to address them.

Agent-Based Models give up an explicit microfoundation with a representative agent in favor of a behavioral foundation with heterogeneous agents. The heterogeneous agents in this framework rather than being utility maximizing individuals with rational expectations exhibit bounded rationality in the Simon (1955) sense (due to limited resources time and money) and therefore employ heuristics in expectation formation. In contrast to this, in the standard established theory perfectly rational and forward-looking agents continuously solve optimal control problems. The ABM-approach is nicely summarized by Bruun (2008) in the way that these models "rather than assuming complex behavior in a simple world [...] model simple rule-based behavior in a complex world". Thus, they are closely linked to experimental and psychological literature, but also to rule-of-thumb recommendations from the management literature (Raberto et al., 2012). The modeling paradigm is *bottom up* rather than *top down* as in the established representative agent macro literature. In contrast to the DSGE and Real Business Cycle literature, economic cycles in this context do not arise due to exogenous shocks but endogenously via the process of expectation formation. Furthermore, ABMs can account for instability, whilst this aspect is ruled out by assumption in DSGE models that only measure impulse propagation with long-run convergence to equilibrium. ABMs, on the other hand, can produce long-run growth with cyclical behavior around this growth trend (Dosi et al., 2006) as measured in time series of GDP, but also of debt levels. The emergence of these models is also closely linked to the recent developments in computer technology and software engineering.[10]

Driscoll and Holden (2014) discuss the role of behavioral economics in macroeconomic models in an extensive manner starting out from the standard New-Keynesian model. Rule-of-thumb consumption can alter the IS-curve which will also be the main concern of our work. Behavioral assumption also matter for aggregate supply in the form of the expectation process especially regarding the future level of inflation allowing for multiple equilibria and complex dynamics. The concrete form of the expectation process is also of importance for discussing financial markets in particular to explain asset price bubbles. Driscoll and Holden (2014), moreover, emphasizes that seemingly irrational behavior is frequently result of an underlying principal-agent problem for which the principal (e.g. reacting to short-term incentives) violates the rules of optimality at the cost of the agent.

The advantages of the ABMs as put forward in the previous paragraph, however, can also be converted into shortcomings following the strong *Lucas critique*. In this vein, one could argue that while there is only one single optimal decision rule, there exist an indefinite number of behavioral *rules of thumb*. Or to paraphrase Tolstoy: all rational agents are rational in the same way, whereas all irrational agents are irrational in their very own way.[11] As a result, the role chosen by the modeler of the ABMs is an

[10] An interesting trivia is that the the publication of the *General Theory* (Keynes, 1936) and the development of the first computer Z1 (1935-1938) by Konrad Zuse coincide.

[11] This is sometimes referred to as the *Anna Karenina Principle* owing to the first stanza of the famous novel saying "Happy families are all alike; every unhappy family is unhappy in its own way." (Diamond, 1997).

arbitrary decision only to derive a desired model outcome - as e.g. to promote a certain political recommendation.[12] The well-known debate surrounding e.g. the Phillips-curve makes the point that economic agents are not subject to fallacies such as money illusion. Therefore, money is neutral in the long-run. As money is only a veil, the established literature assumes money to be absent and thereby (implicitly) models pure transaction or barter economies. In their ABM, Ashraf et al. (2012) (partly) address the *Lucas critique* by showing that their model results are robust to changes in the behavioral parameters.

It is not surprising that the debate in macroeconomics and especially the *Rational Expectations Revolution* resembles the debate in finance in the context of the Efficient Market Hypothesis (EMH). On the one hand, we have a very simple and powerful theory basically relying on a non-arbitrage argument arguing that all variations in economic variables are the result of exogenous shocks. Its predictions - at least in the short-run - however are counter-factual. As the finance domain is dominated by business practitioners only interested in exploiting profit-making opportunities it is, however, more open to heterodox approaches such as Agent-Based Modeling.[13] In contrast to that, macroeconomists from all sides - ranging from neo-classical to Marxist approaches - feel obliged to their school of thought and try to defend it against contrasting approaches.

ABMs are intensively used and widely accepted for modeling the behavior of stock markets (as presented for instance in the surveys of Chiarella et al. (2009), Lux (2009) and Hommes and Wagener (2009)). Their are several attempts to adopt this modeling framework, also heavily used in other scientific fields such as biology, to a broader macroeconomic scope. Amongst the most famous supporter of these models was the former chairman of the European Central Bank (ECB) Jean-Claude Trichet (Trichet, 2010). One major shortcoming of these models - also resulting from its infancy - is that in the contrast to the New-Keynesian (Woodford, 2011) or Real Business Cycle (RBC) (Kydland and Prescott, 1977) literature, there is no canonical model for discussing macroeconomic problems in an ABM setting.[14] [15] Yet, there are different coexisting and competing attempts. A very ambitious attempt is undertaken in the so-called EURACE project (Deissenberg et al., 2008). This project involves a large number of re-

[12]As we will, however, also elaborate for our particular setting in section 4.3, the microfounded models rely on an arbitrary utility and production functions to derive their results.

[13]Due to this *whatever works*-premise, ABMs are now widely used in trading companies. The official authorities, therefore, are also interested in implementing them to counteract possible resulting instabilities for the financial system (Bookstaber, 2012).

[14]Moreover, one practical shortcoming that - in contrast to DSGE models running on the Matlab based tool Dynare - there is no canonical simulation software. A widely used software package coming from the fields of biology is Repast - a open-source software package based on the object-oriented (being highly suitable for the agent-based paradigm as every agent represents an object) programing language Java. The simulation work presented in this paper is done in Matlab also reflecting the more traditional approach chosen in this work.

[15]Another practical shortcoming in the ABM-area is the problem of finding a common language. Standard models are presented by means of mathematical equations. As the decision rules in the ABM-framework, however, derive from an *if-then-else* logic, they are frequently represented in the form of pseudo-code. As the models, moreover, are highly complex it is hard to represent them completely in an journal article, leading to a high degree of opaqueness (see e.g. Geanakoplos et al. (2012)).

searchers focusing on different key aspects of the model such as the labor market (Dawid et al., 2008) or the financial market (e.g. Raberto et al. (2012)). Beside this large scale project there are bold attempts by individual researchers in modeling the macroeconomy in ABMs. Lengnick (2013) presents a rather simplistic ABM that is, however, capable of reproducing a wide array of interesting macroeconomic phenomena such as the Beveridge curve or boom-bust cycles endogenously. In contrast to this simple approach, there are also very complicated and ambitious approaches by individual researchers or small groups. While Haber (2008) focuses on a realistic representation of central bank activity and Ashraf et al. (2011) on the role of banks, Ashraf et al. (2012) concentrate on the effect of inflation, whereas Oeffner (2008) presents a very wide approach.

The latter attempt, on the other hand, also reveals the major shortcoming of these models. The model consists of a large amount of behavioral variables not directly measurable in empirical data. Therefore, researchers should keep in mind the famous Joan Robinson saying that a map on the scale 1:1 is useless (Robinson, 1962). The highly non-linear behavior of the model also makes it difficult for either analytical discussion or econometric calibration.[16] Bringing complexity into economics is therefore not only an achievement of ABMs, but a shortcoming at the same time. Resulting from that, discussion of ABMs mostly rely on simulation studies or description of representation of stylized facts. In the case of financial time series this mostly concerns the higher moments of return distribution indicating non-Gaussian behavior (Lux, 2009). Similar results can be found for the case of firm size distributions which exhibit fat tails that can be described by power-law behavior (Gatti et al., 2010) as well right skewness (Dosi et al., 2006). The heterogeneity in the financial markets models mostly comes from different expectation formation (e.g. Hommes and Wagener (2009)). Attempts like Gatti et al. (2010) or Raberto et al. (2012) meanwhile focus on the heterogeneous characteristics of firms and banks and thereby follow the rationale by Schumpeter, considering the entrepreneur as the key economic agent. We take a more Keynesian approach in building the model around the private household or more precisely the consumer. In most of the existing models, the consumer is treated in a very simple manner, especially not allowing for consumption credits (Raberto et al., 2012).

Chakraborti et al. (2011) survey the econophysics attempts to account for wealth inequality in an ABM-framework. The exchange models closely linked to the kinetic theory of gases from physics (even though being very simplistic in their formulation) can account for power-law behavior in distribution as initially observed by Pareto (1896). Recently, this model type has been advanced by a simple microfoundation (Chakrabarti and Chakrabarti, 2009).

In an extension of the Dosi et al. (2010) model, Dosi et al. (2013) investigate the effect of income inequality in an Agent-Based setup. In this case, the functional distribution between profits and labor (rather than the personal distribution) - as modeled by high price mark-up ratios - is taken into account. The general result is that high inequality can amplify business cycles implying periods of high unemployment and a high crisis

[16] A very bold attempt at calibrating ABMs is conducted by Jeleskovic (2011). For a more precise overview on the subject also refer to Sornette (2014).

probability. In this setting, a redistributive fiscal policy can, however, be very effective. Meanwhile, monetary policy is not effective below a certain interest rate threshold. Above this threshold increasing interest rates are counterproductive since firms prefer financing via volatile internal funds leading to lower growth rates and larger output volatility.

As we consider inequality by means of an ABM, our model is close to Dosi et al. (2013). Another very comparable paper is Geanakoplos et al. (2012). The authors use the ABM-framework to discuss systemic risk effects arising from the housing market motivated by the recent financial crisis. This very rich model uses micro data from the Washington area to calibrate the model. The results suggest that instability is driven by high leverage of private households as well as low interest rates. Another very recent and comparable paper is König and Größl (2014) in which the authors use the ABM-paradigm to discuss the role of the interaction of relative consumption effects, borrowing constraints, and private debt. By means of simulation they show that if poor households overstretch their resources by excessive borrowing, the resulting defaults can have adverse effects on the aggregate macroeconomy putting forward an argument for tight borrowing constraints.

Our model is a partial model of the economy abstracting from important factors, in particular labor markets and behavior of firms. The chosen simplicity, however, allows to derive some closed-form solutions.[17] By this means we also hope to address the problem of arbitrary model tunings that are also relevant for the DSGE models. Basically - and thereby similar to e.g. Krusell and Smith (1998) - the key component of the model is the intertemporal decision how much to consume and to save.

Bruun (2001) discusses the effect of a wealth tax on inequality in an ABM-framework. The model predicts a strong correlation between wealth inequality and business cycle, resulting in high values for the Gini coefficient (as an index of inequality) in times of low asset prices and vice versa. One interesting key assumption made in this model is that agents base their consumption on the consumption of their neighbors as proposed in Duesenberry (1949). A theory of consumer inequality therefore firstly requires a theory of consumption. The next section therefore will present different forms of consumption theories and evaluate them.

[17]The paramount share of section 5 is devoted to the analytical analysis of the model.

4. Theoretical Consumption Functions - Assumptions and Implications

> Rather go to bed without dinner than to rise in debt.
>
> Attributed to Benjamin Franklin (1706-1790)

> I still owe money to the money to the money I owe
>
> The National - Bloodbuzz Ohio (2010)

The theory of consumption asks the question how much households consume today and will consume in the future and - as a mirror image - save today. There are several competing theories which build on different fundamental assumptions. Moreover, the different theories also have mixed empirical validity. Consumption decisions require knowledge about future behavior of different variables such as income, inflation and interest rates. The most popular are the opposing theories ad-hoc theory in the spirit of Keynes (1936) and the utility maximizing rational expectations approach (Lucas, 1972) as theoretical foundation for the permanent income (Friedman, 1957) and life-cycle hypothesis (Brumberg and Modigliani, 1954). In this section, we follow a similar approach as in the literature overview of section 3 in comparing (more) standard theories with behavioral and heterodox approaches. Moreover, we focus on the sustainability of debt, which will be of major importance in our model, and also benchmark the theories with some empirical data.

4.1. Keynesian and (Neo)Classical Consumption Theory

One of the first major attempts at modeling consumer behavior was made by Keynes (1936) in particular elaborating on the marginal propensity to consume.[1] Here, we present a simple text book treatment (e.g. Carlin and Soskice (2005)) and - for the sake of convenience - in the further course will refer to it as the *Keynesian consumption function*. The main determinant of the consumption level in the work of Keynes is the disposable income Y_t^d [2] - representing a flow variable. Furthermore, the textbook literature assumes that there is a subsistence level of consumption $\bar{c}$ independent of actual disposable income. The second important assumption he makes is that the *marginal propensity to consume* (MPC) is positive but less than 1 ($0 < \frac{dC_t}{dY_t^d} = c_y < 1$), implying that consumption is less volatile than income.[3] These assumptions can be presented in a simple linear equation:

$$C_t = \bar{c} + c_y Y_t^d. \tag{4.1}$$

The assumption of a subsistence level of consumption yields the result that the average propensity to consume $\frac{Y_t^d}{C_t} = \frac{\bar{c}}{Y_t^d} + c_y$ is lower for higher incomes. This theory furthermore implies that consumers who have a disposable income lower than a threshold level $Y^* < \frac{\bar{c}}{1-c_y}$ engage in dissavings, while consumers above this level represent the net savers (also see figure 4.1). Dissavings can be conducted by means of selling already accumulated wealth or - and this is the case emphasized in this work - by aggregation of debt. Moreover, the combination of the assumption $\bar{c} > 0$ and $0 < c_y < 1$ guarantees a well-defined stable steady state of income and consumption Y^*.

The Keynesian ideas are still at the core of modern consumption theory. Thus, the function itself is based upon rather *ad-hoc assumptions* without proper micro- or behavioral foundation and lacks empirical evidence (Sørensen and Whitta-Jacobsen, 2010). Macroeconomic time series data suggest that the subsistence level of consumption $\bar{c}$ is zero, whilst confirming a MPC lower than one[4] and at a very low level.

Following the argumentation of the Friedman (1953) permanent income hypothesis, households only respond to permanent changes in income and not to transitory variations. Friedman (1953) emphasizes that the stock level of wealth rather than the flow of current income determines consumption behavior. For those individuals born without physical capital or land (allowing them to have factor income in the form of profits or rents), their wealth is represented by the time discounted value of their labor income - their human capital. While physical wealth can be (with some specific liquidity con-

[1] Note that he argues from a top-down macroeconomic perspective. In contrast, the (neo)classical literature argues from a bottom-up microeconomic perspective.

[2] This equals income from wages, rents and capital gains less taxes.

[3] The latter is easy to show given the relation presented in equation 4.1: $\frac{Var(C_t)}{Var(Y_t^d)} = c_y^2 < 1$.

[4] If consumption is plotted as a function of disposable income, this implies that the line is always below the 45-degree line.

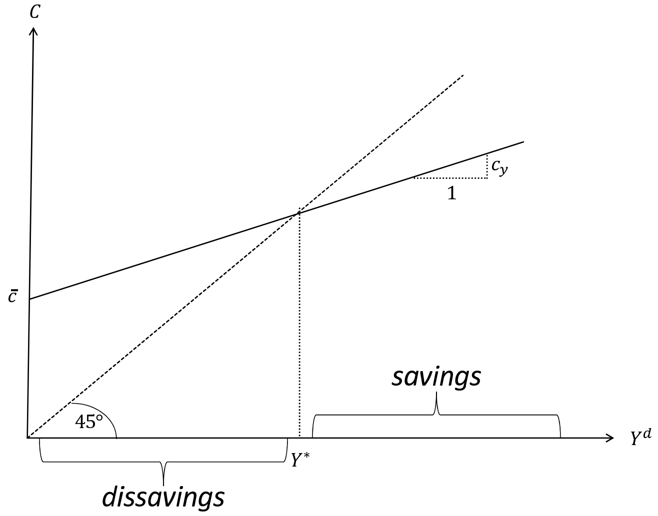


Figure 4.1.: The basic Keynesian consumption function with consumption C being a function of disposable income Y^d

straints) be traded on markets as a bulk, the *human* capital can not be sold entirely as a stock (in the absence of slavery).[5]

Microfounded models present the consumption/savings decision as an intertemporal decision of consumption in time. They assume that economic agents optimize a utility function $U(C_0)$ [6] and exhibit an individual time discount factor $0 < \beta = \frac{1}{1+\rho} < 1$ with $0 < \rho$ being the rate of time preference. They therefore maximize the following target function:

$$U(C) = \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{4.2}$$

In the presence of a perfect capital market[7] agents with infinite lifetime, initial wealth level W_0, and a disposable income Y_t [8] in every period t are subject to an intertemporal budget constraint:

$$\sum_{t=1}^{\infty} R^t C_t = W_0 + \sum_{t=1}^{\infty} R^t Y_t. \tag{4.3}$$

[5]Note following a similar argument the absence of slavery in modern societies is the key rationale why Piketty (2014) does not include human capital as the *capital* in his work.

[6]This follows the standard assumptions of positive ($\frac{dU}{dC} = U' > 0$) but decreasing ($\frac{d^2U}{dC^2} = U'' < 0$) marginal utility.

[7]Major assumptions of a perfect capital market are equal market rates $R < 1$ for borrowing and lending and no credit constraints (especially collateral).

[8]Note that for sake of readability we exclude the index d in the further course.

The factor $R = \frac{1}{1+r}$ represents the discount factor determined by the market interest rate r.[9] This leads to the following optimality condition:[10]

$$\frac{U'(C_t)}{U'(C_{t+T})} = \left(\frac{\beta}{R}\right)^T. \tag{4.4}$$

This condition implies that individuals with stronger future discounting than market rate ($\beta < R$) prefer current consumption and vice versa. The difference in individual time preference has been a key rationale in many models describing the current heterogeneous savings-consumption patterns witnessed in between nations and individuals (see section 3.1).

A recent large-scale international survey based on experiments by Wang et al. (2011) revealed strong variation in discount rates between international regions. They measure patience (and thereby high values for β) for Germanic/Nordic as well as Anglo-Saxon countries and high time preference for Latin American, Latin Europe as well as African countries. The authors attribute this effect to cultural differences and are able to disentangle it from a wealth effect. Especially, the strong difference between between Germanic/Nordic countries and Latin European countries is a good candidate for explaining the current Euro crisis.

If, for sake of convenience, we assume $T = 1$ and a simple utility function of the Constant Relative Risk Aversion (CRRA) type ($U(C_t) = \ln(C_t)$)[11] the so-called Ramsey equation can be derived (Ramsey, 1928):

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{C_{t+1}}{C_t} = \frac{\beta}{R} \Rightarrow \frac{C_{t+1} - C_t}{C_t} = \frac{\beta - R}{R}. \tag{4.5}$$

If we transform the difference equation into a differential equation a closed-form solution of the consumption path can be derived:

$$\frac{dC(t)}{C(t)} = \frac{\beta - R}{R} \Rightarrow C(t) = C_0 \cdot \exp\left(t \cdot \frac{\beta - R}{R}\right). \tag{4.6}$$

In the case of $\beta > R$ this results in an expanding consumption path over life time and vice versa for the opposite case (see figure 4.2). The market interest rate R is determined by the aggregate savings and demand for capital. Since by the no-arbitrage-law no excess gains can be made, the market interest rate R should equal the time preference rate β. Accordingly, in the representative agent economy for all agents $\beta = R$. In the Ramsey-sense this leads to the result $C_t = C_0$, implying that agents consume an equal amount in every period. This effect is called consumption smoothing as argued by Friedman (1953). Similar to the results of the Efficient Market Hypothesis (EMH) the no arbitrage condition, representative agent supposition as well as the rational expectations

[9] Note that - in contrast to a large amount of literature - we define $R = \frac{1}{1+r}$ rather than $R = 1 + r$, since we feel that it aids readability especially when comparing β and R.

[10] A formal derivation of the result is presented in the appendix A.1.

[11] In this case the relative risk aversion equals 1: $-\frac{U''}{U'} \cdot C = 1$.

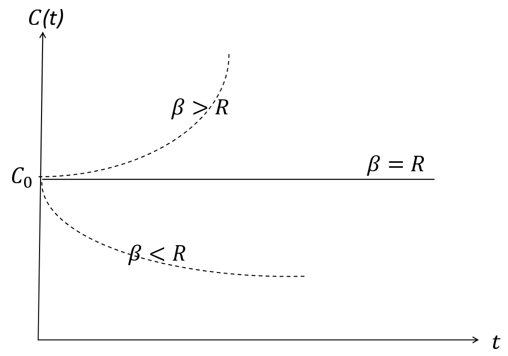


Figure 4.2.: Consumption $C(t)$ as a function of time t for different values of time preference β

assumptions result in the fact that consumption should behave like a random walk triggered only by new information about future income (Hall, 1978).

To account more precisely for risk aversion, we now investigate a more general CRRA utility function for which γ captures risk aversion:[12]

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{4.7}$$

This yields the following optimality condition:

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{C_{t+1}^{\gamma}}{C_t^{\gamma}} = \frac{\beta}{R} \Rightarrow \frac{C_{t+1} - C_t}{C_t} = \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}} - 1. \tag{4.8}$$

If this difference equation is transformed into a differential equation, there is a closed-form solution to the Ramsey equation:

$$C(t) = C_0 \exp\left(t \cdot \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}} - 1\right). \tag{4.9}$$

The rationale of this equation is that with increasing risk aversion (higher values for γ) the slope of the consumption function decreases:[13] This implies that more risk averse individuals engage more severely in consumption smoothing.

Cagetti (2003) estimates the effect of education on the parameters of the Euler-equation using Panel Study of Income Dynamics (PSID) as well as SCF-data (Survey of Consumer Finance) both reporting data for the US. A key finding is that higher education contributes to higher risk aversion γ and a higher discount factor β. As education

[12]The beforehand presented utility function $U(C) = \ln(C)$ represent the special case of this one with $\gamma \to 1$. The relative risk aversion in this case is defined as follows: $-\frac{U''}{U'} \cdot C = \gamma$.

[13]This is for the case without perfect consumption smoothing ($\beta \neq R$).

is correlated with income (Mincer, 1958), this implies that high-income individuals are more patient and hold less risky assets. Furthermore, the consumption path peaks earlier for lower education. In fact, for college education there is no peak but consumption is ever increasing (Cagetti, 2003).

Another interpretation is given when considering the reciprocal value $\frac{1}{\gamma}$ that is referred to as the elasticity of substitution.[14] The intertemporal elasticity of substitution (IES) also plays an important role in qualifying the consumption effect of an interest rate change. If we log-linearize[15] the first order condition, the following result emerges:

$$\begin{aligned} \log\left(\frac{C_{t+1}}{C_t}\right) = \log(C_{t+1}) - \log(C_t) \equiv \Delta C = \log\left(\frac{\beta^{\frac{1}{\gamma}}}{R}\right) \\ = \frac{1}{\gamma}\left(\log(\beta) - \log(R)\right) \approx \frac{r-\rho}{\gamma}. \end{aligned} \tag{4.10}$$

This variable is an important feature of macroeconomic values by accessing the effect of an (exogenous) interest rate change on consumption also impacting on savings and thereby investment. A recent meta-analysis of empirical studies compares different empirical estimates and points to a wide diversity of results (Havranek et al., 2013). The authors put forward that in countries with low income, low asset market participation, and strong liquidity constraints - all characterizing underdeveloped financial markets - the IES is low in magnitude, implying that monetary policy has little impact on consumption.

Guvenen (2006), moreover, emphasizes that heterogeneity of agents is important when estimating the IES. Estimation from consumption data point to a level of the IES close to zero. This, eventually, mainly captures the IES of low-income individuals contributing to the major share of aggregate consumption. In contrast to that, calibrated models that try to derive realistic interest premia usually assume a value of the IES close to one (the isoelastic case). This eventually captures the behavior of the wealthy agent being the investors agitating at asset markets. In essence, this implies that high-income agents have a higher IES and thereby a lower risk aversion implying that they hold a riskier portfolio yielding a higher return.[16] Guvenen (2006) emphasizes that this is also a candidate explanation for the low risk-free rate of interest and the high equity premium. Moreover, it implies that low-income agents eventually smooth consumption[17] in a stronger manner than high-income agents.[18]

[14] Formally, it is defined as $\frac{1}{\gamma} = \frac{d\ln\frac{C_t}{C_{t+1}}}{d\ln(MRS)}$, in which the marginal rate of substitution MRS is given as follows $MRS_t = -\frac{\frac{\partial U}{\partial C_t}}{\frac{\partial U}{\partial C_{t+1}}}$.

[15] In the process of linearization the linear Taylor-approximation for the logarithm is assumed: $\log(1+x) \approx x$ for small values of x.

[16] This issue will also be of importance when discussing the stability of the wealth distribution, since it implies that wealth inequality converges to a Gini of 1 (see section 5.5.2).

[17] The latter can also be rationalized by public transfer systems - that are especially important for low income individuals - smoothing their income and thereby their consumption.

[18] Note, however, that these results (indirectly) stand in contrast to the results of Cagetti (2003) presented earlier. The results of Guvenen (2006) imply that wealthier households have a lower risk

In line with human capital theory and also confirmed by empirical results, it is reasonable to assume that income grows with age (Mincer, 1958). Consider the case for which the rate of time preference equals the market rate $\beta \equiv R = \frac{1}{1+r}$ resulting in a constant level of consumption in life time (also refer to figure 4.2). Furthermore, consumption is assumed to grow with a constant rate $g > 0$ following a simple exponential growth process ($Y_t = (1+g)^t Y_0$). Using the constraint 4.3, the following consumption level - being constant in time - emerges for the infinite living agent with no initial wealth $W_0 = 0$:[19] [20]

$$C_t = C_{t+1} = C = Y_0 \frac{1+g}{r-g}. \tag{4.11}$$

A higher growth of income g increases life-time wealth and consumption. In this case, in the early years of life time consumption exceeds current income requiring individuals to accumulate debt.[21] In the latter years, agents consumer less than their current income and use the residual amount to pay interest on accumulated debt. In this scenario emphasized by Friedman (1953) consumption only depends on the stock quantity wealth.

The time-varying income is also accounted for in the life-cycle hypothesis of Brumberg and Modigliani (1954). Life time income follows a hump-shaped pattern, being low (or even zero) in early years of education, steadily growing during employment and falling back to zero at retirement. Thus, agents engage in consumption smoothing, implying high saving rates during working times and dissavings for students and pensioners. The classical theory and especially the permanent income hypothesis thereby predict a roughly constant level of consumption. Empirical examinations, especially for least developed countries, on the other hand show that consumption is closely linked to current income (Deaton, 1992). This result might be rationalized by the underdevelopment of financial markets in these regions imposing severe liquidity constraints on households.

Rather than assuming infinitely living agents we assume a finite life-time of only two periods. For this simple two-period case using the first-order condition (equation 4.8: $C_2 = C_1 \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}}$) as well as the budget constraint (equation 4.3), the following result can be derived:

$$C_1 + RC_2 = W_0 + Y_1 + RY_2 = C_1(1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}) \Rightarrow C_1 = \frac{W_0 + Y_1 + RY_2}{1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}}. \tag{4.12}$$

In fact, this can be considered the representation of a simple Overlapping Generations (OLG) model with only two periods; the period when agents are young ($t = 1$) and when they are old and retired ($t = 2$). This result shows that households with higher wealth

aversion. As wealthier households also have a higher income which goes a long with a higher level of education, a higher level of education should also go along with higher risk aversion. However, Cagetti (2003) shows the contrary result.

[19] The life time level of income is computed using the rules for geometric series and requires the assumption $g < r$.

[20] A more formal argument using the technique of optimal control is made in section 4.2.

[21] More precisely for the case with no initial wealth ($W_0 = 0$), this level equals $t < \frac{\ln(r) - \ln(r-g)}{\ln(1+g)}$.

W_0 and present value of work income consume more.[22] In fact, for the OLG-case it is often assumed that the work income of retirees is zero ($Y_2 = 0$). Moreover, the marginal propensity to consume out of the current disposable income Y_1 can be defined:

$$\frac{\partial C_1}{\partial Y_1} = \frac{1}{1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}} \equiv c_y. \tag{4.13}$$

Considering the definition of the market rate R and the time preference rate β this model yields Keynesian features ($0 < c_y < 1$). If time preference rate equals market interest rate ($\beta = R$) the MPC only depends on the interest rate ($MPC = c_1 = \frac{1}{1+R} \equiv \frac{1}{1+\beta}$). The same result holds true for infinite risk aversion ($\gamma \to \infty$) as consumption is smoothed out perfectly.

We can also define the marginal propensity to consume out of total wealth. The latter is defined as the sum of initial capital and (time-discounted) human capital ($W \equiv W_0 + Y_1 + RY_2$):

$$\frac{\partial C_1}{\partial W} = \frac{1}{1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}} \equiv c_1 \equiv \frac{\partial C_1}{\partial Y_1}. \tag{4.14}$$

It is important to point out that the marginal propensity to consume out of total wealth (stock) is identical to the marginal propensity to consume out of current income (flow) for the young agents ($t = 1$).

As emphasized in Dynan et al. (2004), the assumed CRRA type utility function implies homothetic preferences, implying that the savings ratio does not depend on the level of income or wealth.[23] Thus, consumption is proportional to income. Thereby, for the case in which all agents have the identical risk aversion γ and time preference β the total savings (respectively consumption) can easily be derived from a representative agent.[24]

Starting from equation 4.12, the effect of an interest rate change can be further examined:

$$\frac{\partial C_1}{\partial R} = \frac{(1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}})Y_2 - (1 - \frac{1}{\gamma})(\frac{\beta}{R}^{\frac{1}{\gamma}})(W_0 + Y_1 + RY_2)}{(1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}})^2}. \tag{4.15}$$

The case of low risk aversion ($\gamma < 1$) leads to a positive partial derivative. Recalling the relation between the discount factor R and the interest rate r ($R = \frac{1}{1+r}$) an increase in the real interest rate leads to lower consumption or higher savings today. Vice versa, lower real interest rates leads to lower saving rates. This effect is referred to as the *substitution effect*. The idea is that in times of high interest rates, agents decide to postpone more consumption into the future. It also implies that in time of lower interest rates agents consume more in the present. The opposite can be observed for a strong *income* effect. This is the case for very high risk aversion ($\gamma >> 1$). In this case, the very risk averse agents react to the decrease in interest rates by also decreasing their current consumption and vice versa for a higher interest rate. Oeffner (2008) surveys

[22] In this case present value of income is $PVY = Y_1 + RY_2$. This results in the following relations $\frac{\partial C_1}{\partial W_0} > 0$ and $\frac{\partial C_1}{\partial PVY} > 0$ justifying the preceding statement.

[23] Or put differently the Engel curve of consumption is linear in income and wealth.

[24] A more formal proof for the latter is given in appendix A.2.

15 empirical studies devoted to the dominating effect and finds a stronger result for the substitution effect and thereby for increased saving in time of high interest rates. The latter is highly relevant for macroeconomic models as it implies that a decrease in the real rate of interest - e.g. by means of an expansionary monetary policy - results in higher current consumption.

We can also discuss the special case of $\gamma = 1$ leading to logarithmic utility. This case is also referred to as the isoelastic case since income and substitution effect exactly cancel out each other (Bertola, 2000):

$$\frac{\partial C_1}{\partial W} \equiv \frac{\partial C_1}{\partial Y_1} = \frac{1}{1+\beta} = \frac{1+\rho}{2+\rho}. \tag{4.16}$$

For a given level of wealth W the consumption in the first period increases with the rate of time preference ρ without being affected by the rate of interest r. Resulting from the intertemporal budget constraint (equation 4.3) the marginal consumption out of wealth in the second period is given as follows:

$$\frac{\partial C_2}{\partial W} = \frac{\beta}{(1+\beta)R} = \frac{1+r}{2+\rho}. \tag{4.17}$$

This result shows that the interest rate r counteracts the rate of time preference ρ. If both are identical, consumption is smoothed out and identical in both periods (also refer to figure 4.2). In a scenario without interest rates $r = 0$, the rate of time preference $\rho > 0$ guarantees higher current consumption ($C_1 > C_2$). The converse holds true for a case without a rate of time preference ($\rho = 0$ and $r > 0$ leading to $C_1 < C_2$). For the case in which neither interest rates nor time preferences exists ($\rho = r = 0$) consumption is distributed equally among the life span of the individual, which for the simple case of $T = 2$ implies a 50/50-distribution.

We have shown so far that the level of interest r does not impact first-period consumption C_1 for the log-case ($\gamma = 1$). We can reconsider equation 4.15 for the log-case resulting in:

$$\frac{\partial C_1}{\partial R} = \frac{Y_2}{1+\beta} = Y_2\frac{1+\rho}{2+\rho} > 0. \tag{4.18}$$

Given this partial derivative a decrease in interest rate r leads to an increase in first period consumption C_1. This can be attributed to a pure wealth effect, since with lower interest rates the present value of future wages Y_2 is higher (Bertola, 2000). Note that this is only the case for agents receiving income in period $t = 2$. For the case in which agents are retired in the second period and only receive capital income ($Y_2 = 0$), changes in the interest rate do not impact on the consumption in the first period.

In result, the individual reaction on an (exogenous) interest rate change crucially depends on the shape of the utility function; in particular the value of γ. As the latter is also of importance to discuss the effect of a capital income tax, we summarize these effects in table 7.2 in section 7.2.1. The consumption function is at the heart of macroeconomic models. In fact, the Euler-equation in combination with a negative scale production technology[25] constitutes the IS-curve - representing the equilibrium of savings and investments linking money and good markets - in the literature of New-Keynesian models (Gertler et al., 1999). Combining this IS-curve with a mechanical Taylor-rule (Taylor, 1993) - which itself computes interest rates as a function of inflation, thereby also linking money and good markets - an aggregate demand curve (AD-curve) can be constructed. The Phillips-curve introduces the relation between labor and good markets and constitutes the aggregate supply curve finalizing the macroeconomic equilibrium.

Now, we relax the assumption of perfect foresight in disposable income. Most employers do not have well defined career-paths and face the risk of unemployment. Furthermore, agents receive income from holding fixed income and company shares, thereby being subject to interest rate and price risk.

The role of uncertainty in income can be discussed in a model assuming a Constant Absolute Risk Aversion[26] (CARA) utility function in which agents live for $T = 2$ periods (Menz, 2010). If we take the optimality condition and the budget constraint[27] for the two-period case and further assume that future income is normally distributed ($\tilde{Y}_2 \sim N(\mu_Y, \sigma_Y^2)$), the following result can be derived:

$$\frac{U'(C_1)}{U'(C_2)} = \frac{\beta}{R} = \frac{e^{-\eta C_1}}{e^{-\eta C_2}} \Rightarrow -\eta C_1 = \ln\left(\frac{\beta}{R}\right) - \frac{\eta}{R}(W_0 + Y_1 - C_1) + \ln(E_1[e^{-\eta \tilde{Y}_2}]). \quad (4.19)$$

After short manipulation the following result for current consumption C_1 can be presented:

$$C_1 = -\frac{R}{\eta(1+R)} \ln\left(\frac{\beta}{R}\right) + \frac{R}{1+R}\left(\frac{W_0 + Y_1}{R} + \mu_Y\right) - \frac{R}{1+R}\frac{1}{2}\eta\sigma_y^2. \quad (4.20)$$

Firstly, this result confirms the previous results that stronger future discounting than market rate ($\beta < R$), higher wealth W_0, higher current income Y_1, and expected income μ_Y, increase current consumption C_1. Secondly, higher uncertainty of future income (as measured in the variance in disposable income σ_Y^2) together with a higher risk aversion η lead to lower current consumption C_1. This introduces a savings motive out of income uncertainty.

In focusing on consumption out of disposable income, a large amount of literature neglects the role of wealth as a source of consumption. US-American households hold their wealth in home equity rather than stock or bond market equity (Belsky, 2004).[28] As emphasized by Bostic et al. (2009) the housing wealth is not only a consumption good but also an instrument of savings. There is also a strong relation between the age

[25] The latter implies that marginal returns of investment decrease with the level of investment implying a curve with a negative slope.

[26] We assume the function $U(C_t) = -\frac{1}{\eta}e^{-\eta C_t}$. This function implies a constant absolute risk aversion ($-\frac{U''}{U'} = \eta$).

[27] This condition can be solved for future consumption yielding the following result: $C_2 = \frac{W_0+Y_1-C_1}{R} + \tilde{Y}_2$.

[28] As the authors also point out a notable exception to this regularity occurred during the dot-com bubble period ranging roughly from 1996 to 2000.

of the household and the wealth holding, in which old households have a high propensity to consume out of housing wealth[29] while young households face problems in financing this lumpy illiquid asset due to a low amount of accumulated life-time wealth serving as collateral (Campbell, 2006). The marginal propensity to consume out of wealth in different empirical estimations is estimated at a level between 0.05 and 0.3 (Bostic et al., 2009). This result, however, relies on the strong notion of a representative households. As pointed out in Carroll (2012) the level can be far higher for low-income households that pose a non-neglectable amount of the wealth distribution.[30] The authors derive the MPC out of wealth using both US data and the recent Household Finance and Consumption Survey (HFCS) of the Euro area allowing to show cross-country differences (Carroll et al., 2014). In Europe the MPC is lower than in the US due to the fact that wealth is more equally distributed in Europe and Europeans also hold a higher absolute amount of wealth.[31] A theory of consumption therefore cannot disregard the effects of wealth, however, also is a theory of wealth distribution by describing the accumulation of wealth.

As already emphasized at the beginning of this section consumption and savings are two sides of the same coin. While classic theory emphasizes the role of time preferences for the consumption path, the seminal work of Deaton (1991) and Carroll (1997) labeled the *buffer stock theory of savings* stretch the role of savings. In their model framework savings (flow) are accumulated in order to protect against income shocks. Formally, agents solve the following forward-looking problem:

$$\max_{C_\tau} E\left\{\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_\tau)\right\}, \tag{4.21}$$

with E being the expectation operator. The agent is subject to a cash-on-hand constraint restricting current consumption to the sum of current income and wealth:

$$C_t \leq W_t + Y_t. \tag{4.22}$$

Without restricting assumptions about the evolution of the income process no closed-form solution can be derived. In this model, agents target a specific level of wealth. Another testable implication of the model is that the growth of aggregate consumption will equal the growth of aggregate income (Carroll, 2004). Moreover, this model is able to produce decreasing marginal propensity to consume for higher incomes. In contrast to the Keynesian representation that required a subsistence level of consumption $\bar{c} > 0$, this is solely due to a risk aversion motive requiring higher rates of savings for high-income individuals. We will compare both theories more precisely in section 4.4, where we estimate a consumption function for Germany.

The detailed HFCS dataset allows to test the theories for European households and point to other factors not incorporated by standard theory. In the sample, for approx. 70

[29] Practically, this result of the empirical macroeconomic literature implies that older households sell their real estate assets when getting older to finance current consumption.

[30] This can be accounted for by a concave consumption function.

[31] The latter is the case since the concave consumption function implies a lower MPC for higher values.

percent of households consumption equals current income, making them hand-to-mouth consumer in terms of economic theory. Le Blanc et al. (2014) show that wealthier households have a higher probability of saving. The inverse case is true for households whose head is female, divorced, low-educated, or young. Motives for savings not only include old-age provision (in line with the classic buffer stock theory), but also are made in order to provide support and education for children and grandchildren. The third most important motive for saving is financing expensive goods such as real estate or vehicles.

4.2. Stability of Debt Aggregation

In the previous section, we summarized the established approaches to classify the determinants of consumption and related it to some empirical evidence. In all of the presented theories[32], consumption and current income can diverge. For the simple Keynesian case, consumption can exceed current disposable income if income is below a certain threshold. For the permanent income hypothesis of Friedman (1953) income is only given by the level of wealth and constant in time and may exceed current income or cash-on-hand. In order to smooth consumption, financial markets allow for borrowing. A household that borrows accumulates debt. The life-cycle hypothesis in the sense of Brumberg and Modigliani (1954) furthermore argues that while young households smooth consumption by incurring debt ($D = -K > 0$), old and retired households do so by consuming accumulated capital ($K > 0$).

The HFCS dataset also provides some interesting empirical evidence for the ways of financing (Le Blanc et al., 2014). For rich individuals negative savings are mostly covered by decumulating existing stock of wealth. Low-wealth households meanwhile do so by means of credit cards or overdraft facilities. Moreover, informal credit from relatives and friends is also a highly popular instrument for financing. Another key result is that households in Mediterranean European countries are more likely to be subject to credit constraints. The empirical evidence also points to the fact that credit constraints are reduced if personal bankruptcy laws exist also indicating a moral hazard problem. This result, however, is highly overshadowed by the fact that the data were collected in 2010/2011 at the height of the economic crisis in the peripheral EU. Moreover, the cross country comparison shows that the presence of highly evolved public pension system lowers private savings implying that private savings are substituted by public savings.

In this section, we present approaches which consider the stability of debt accumulation. In principle, we restrict ourselves to the two most common approaches. On the one hand, we present the debt sustainability approach in the tradition of Domar (1944), which is widely used for classifying the sustainability of government debt and can also be easily transfered to private households. It also helps us to relate flow and stock. This model - close to the Keynesian approach - has a more *ad hoc* nature. We contrast it

[32] A notable exception is the *buffer stock* theory which by introducing the cash-on-hand constraint - especially designed to explain the behavior in countries with underdeveloped financial markets - assumes that there is no way of financing consumption with debt.

with a more sophisticated approach from the theory of optimal control (Kamien and Schwartz, 1991).

The approach of Domar (1944) is (implicitly) assumed in the influential work of Piketty (2014). Moreover, it is close to the neo-classical growth model. Moreover, this approach. In the latter model (Swan (1956), Solow (1956)) capital $k = \frac{K}{AN}$ (in effective labor terms) evolves according to the following equation:

$$\dot{k} = sf(k) - (\delta + g)k, \tag{4.23}$$

for which s represents a constant savings ratio out of income, δ the geometric rate of depreciation of capital and $g = g_A + g_N$ the growth of total factor productivity and population. Assume the standard Cobb-Douglas type technology:

$$y = K^\alpha (AN)^{1-\alpha} \equiv k^\alpha = f(k). \tag{4.24}$$

Inserting this result in the flow equation and solving for the steady state leads to:[33] [34]

$$\frac{k}{y} = \frac{sk^\alpha}{(\delta + g)k^\alpha} = \frac{s}{\delta + g} \equiv \kappa. \tag{4.25}$$

This ratio presents a steady state value between the stock value of capital k and the flow value of income y, which increases with the savings ratio s and decreases with depreciation of physical capital δ and growth of income g. We will refer to this result as the result of the *capital model*. In a recent empirical study, Piketty and Zucman (2014) compute this value, showing that for countries covered in their study (USA, UK, Germany, and France) this value rose from 2-3 in the 1970s to the value of 4-6 in the 2010s and thereby comparable to values seen in Europe in the 18th and 19th centuries. Given equation 4.25, they explain this behavior by the increase in asset prices and a slowdown in productivity (a fall in δ and g). Rather than being extraordinary high at the moment, the authors argue that the capital-income ratio was extraordinary low in the 1970s and now returns back to its long-run averages. They attribute the low rates in the middle of the last century to war destructions and high inflation as well as a general more labor friendly atmosphere. They also decompose wealth and show that rather than agricultural land (dominating in the 18th and 19th century) housing wealth

[33] This result does not rely on the restricting assumption of the Cobb-Douglas technology, but can be derived for any production function $f(k)$.

[34] As put forward by Krusell and Smith (2014), in his book Piketty (2014) presents the ratio $\tilde{\kappa} = \frac{k}{\tilde{y}} = \frac{\tilde{s}}{g}$ with $\tilde{y} = y - \delta k$ and $\tilde{s} = s\left(\frac{g}{g+\delta}\right)\frac{\tilde{\kappa}}{\kappa}$ being net of depreciation. In his argument, a convergence of the growth rate to zero thereby would result in explosion of the capital ratio κ. This is, however, a fallacy as even in a zero growth environment, the capital ratio would converge as the gross savings rate $\tilde{s} \equiv s\frac{g}{g+\delta(1-s)}$ also converges to zero for a given linear savings rate s. As they show, we have: $\tilde{\kappa} = \frac{s}{g+\delta(1-s)}$ which for $g = 0$ implies $\tilde{\kappa} = \frac{s}{1-s}\frac{1}{\delta} < \infty$.

plays an important role nowadays.[35] [36] Piketty (2014) argues that the profit share can be directly related to the presented variable by a simple accounting equality:[37]

$$\alpha \equiv \frac{rk}{y} = r\kappa. \tag{4.26}$$

The latter, however, tacitly assumes that the interest rate and the rate of capital are independently determined. The latter is e.g. the case for a linear production function $r = f'(k) = const$ (since $f''(k) = 0$). As we will show however in section 6.2.1, a positive relation between the level of capital k and the capital share α can be constructed in a (neo)classical setup if a Constant Elasticity of Substitution (CES) production function that is *capital-biased* is assumed.

Piketty (2014) - following a rationale close to Domar (1944) - argues that the decrease in the capital share was a period in which $r < g$, whereas currently this inequality has changed to its long-run average $r > g$.

In the approach of Domar (1944) debt rather than capital is used. Formally, this only changes the sign of the equation ($D = -K$) leading to the following flow equation:

$$D_t = (1 + r)D_{t-1} - Y_t + C_t. \tag{4.27}$$

Once again, assuming a constant savings ratio ($s = \frac{Y_t - C_t}{Y_t}$) and income following a geometric growth process $Y_t = (1 + g)Y_{t-1}$, this results in the following equation:

$$\frac{D_t}{Y_t} = \frac{(1 + r)D_{t-1}}{Y_t} - s = \frac{(1 + r)D_{t-1}}{(1 + g)Y_{t-1}} - s, \tag{4.28}$$

implying the following long-run steady-state:[38]

$$\frac{D}{Y} = \frac{-s(1 + g)}{g - r} = \frac{s(1 + g)}{r - g}, \tag{4.29}$$

which for small values of g [39] result in:

$$\frac{D}{Y} = \frac{s}{r - g} \equiv d. \tag{4.30}$$

A steady state of the debt-income ratio d is only reached if $r < g$. For negative savings ratio ($s < 0$) this results in a steady state with positive debt ($d > 0$). For positive savings ratios ($s > 0$) satisfying the condition $r < g$ a positive steady state of capital

[35] Another interesting result of their study is that slaves represented the major assets in the Confederate US states during 1770 and 1810.

[36] Jones (2014), however, emphasizes that the trend of increasing capital ratios strongly diminishes once real estate is excluded from *capital* as defined in Piketty and Zucman (2014).

[37] The left-side of this equation holds if we - as frequently done in neo-classical economics - assume that factors are paid according to their marginal productivity leading to $r = \frac{\partial f(k)}{\partial k} = \frac{\partial y}{\partial k} = \alpha k^{\alpha-1} = \alpha \frac{y}{k}$.

[38] Note that for indicating the steady-state we spare the time indexes.

[39] Implying that the product $s \cdot g \approx 0$ is negligible.

emerges ($\kappa \equiv -d > 0$) as emphasized in the work of Piketty (2014). For the inverse case $r > g$ the situation is unstable - also referred to as the the *Ponzi case*, implying that interest on debt is served by accumulating more debt.[40] The equation (without simplification) can also presented in the following manner:

$$\Delta d = \frac{r-g}{1+g} d - s. \tag{4.31}$$

We depict this equation in a phase diagram of figure 4.3 showing the four possible cases. This model follows the rationale of Domar (1944) and is widely used nowadays for evaluating the sustainability of government debt. In particular, the Maastricht criteria of the European Monetary Union - starting from the simplified version of the model - setting the maximum budget deficit ($-s$ in the terms of our model) to 3% and the maximum debt to GDP ratio to 60%. The latter implies an excess growth rate ($g - r$) of 5% which seems highly unrealistic and points to an arbitrariness of the concrete values.

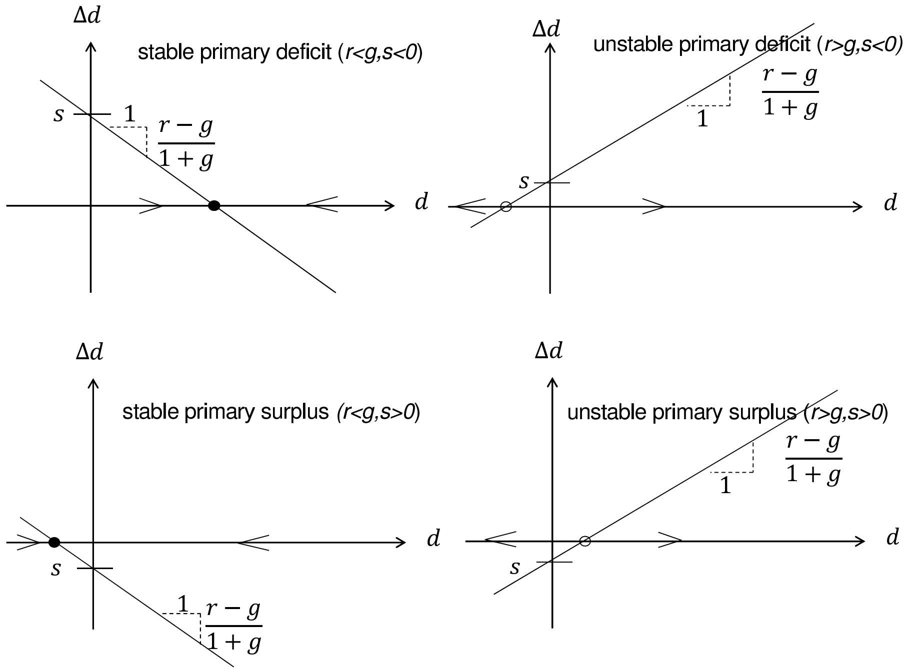


Figure 4.3.: Phase diagram for the debt-income ratio based on (Carlin and Soskice, 2005, p. 184f.)

In our model, we will refer to the sustainability of private household debt. Regarding the fact that low-income households are frequently charged a high rate of interest on credits (resulting from a risk compensation consideration of the lender) and that their

[40]The name refers to one of the most notorious pyramid schemes conducted by Charles Ponzi in the early 1900s.

income grows at a slow pace, point to the fact that the *No-Ponzi condition* ($r < g$) is frequently not satisfied for private households.

One can also compare the results of the capital ratio κ with the debt ratio d. The *capital model* assumes capital to be a physical good that is subject to technical depreciation for which the return is given by its marginal productivity. In the debt model of Domar (1944), the rate of interest is given exogenously. The key difference, however, is that in the Solow type model, capital income is reinvested, contributing to an increase in income, whereas in the Domar type argumentation interest on debt is transferred to an exogenous sink and thereby reduces disposable income. Furthermore, there is no depreciation of capital also not in the form of consumption out of wealth.

We contrast this simple approach that assumes a simplistic and ad-hoc consumption / savings function with a more complex approach using the technique of optimal control allowing for deriving a sustainable debt and consumption path. While the aforementioned example is also rather easy to grasp for non-economists (such as politicians), this approach is more sophisticated and therefore hard to explain to someone without a graduate degree in economics - in particular for private households that face debt problems. As we, however, will also show it has some very intuitive results close to the *Keynesian* consumption function.

We consider the problem in continuous rather than in discrete time. The problem itself is identical to the discrete time problem presented in the preceding section, yet allows us to make a precise statement about sustainability of debt. The following objective function shall be maximized (compare with equation 4.2 [41]):

$$\int_0^\infty \exp(-\rho t) U(C(t)) dt, \tag{4.32}$$

subject to the following flow equation:[42]

$$\dot{D} = rD + C - Y. \tag{4.33}$$

Note that - once again - we consider an infinite horizon problem for which the upper bound satisfies $T \to \infty$. This implies the unrealistic assumption that economic agents live infinitely. In the strong form of the life-cycle hypothesis of Brumberg and Modigliani (1954), agents die with zero debt. This, however, implies that agents precisely know the time of their death. To use the technical terms of optimal control theory, this transforms the infinite horizon problem into a fixed end-point problem. There are optimal control formulations that incorporate the risk of not knowing when exactly dying - the *longevity* risk - which acts like an increase in the discount rate r contributing to higher savings (Kamien and Schwartz, 1991, p. 62f.).

Blanchard (1985) present such a model with a death probability p (respectively an expected life-time of $\frac{1}{p}$).[43] This parameter can easily model the effect of longevity as seen

[41] Note that by log-linearizing $\exp(-\rho t) \approx \beta^t = \left(\frac{1}{1+\rho}\right)^t \leftrightarrow -\rho t \approx -t\ln(1+\rho)$.

[42] Note that this approach does not incorporate uninsurable idiosyncratic income risk as done in the *Bewley*-type literature.

[43] The survival probability in a certain time t is given as $1 - F(t) = p\exp(-pt)$. In fact the probability of dying in a particular period t subject to survival until then is $p = \frac{dF(t)/dt}{1-F(t)}$. The expected life time

in recent years as a result of progress in medicine. Longer horizons, respectively longevity (lower values of p), lead to a lower interest rate and a higher steady-state level of capital. However, the presence of a finite horizon per se decreases capital accumulation. Finite lives also lead to the fact that aggregate consumption is cyclical and not completely smoothed out.

OLG-models take into account that agents know the precise time of their dead; nevertheless they die with a capital larger than zero. This is the case since a bequest motive is introduced in which parent generations have an incentive to transfer wealth to their offspring. Furthermore, this altruistic *warm glow* motive introduces wealth into the utility function. The infinite horizon problem - discussed in this section - can be therefore seen as the case of agents optimizing the results for their whole dynasty.

To solve the problem, the Hamiltonian $\mathcal{H}$ has to be constructed:

$$\mathcal{H} = \exp(-\rho t)U(C) - \lambda(\dot{D} - rD + C - Y). \tag{4.34}$$

We have to compute the first-order conditions with respect to the control (here the level of consumption C) and the state (here the level of debt D). The optimality conditions are as follows:

$$\frac{\partial \mathcal{H}}{\partial C} \overset{!}{=} 0 \rightarrow \exp(-\rho t)U'(C) = \lambda, \tag{4.35}$$

and

$$\frac{\partial \mathcal{H}}{\partial D} = -\lambda r \overset{!}{=} \dot{\lambda}. \tag{4.36}$$

Differentiating equation 4.35 and equating it with equation 4.36 yields:

$$-\lambda r = exp(-\rho t)U'(C)r \overset{!}{=} \dot{\lambda} = -\rho \exp(-\rho t)U'(C) + \exp(-\rho t)U''(C)C', \tag{4.37}$$

which can be solved leading to:

$$-\frac{U''(C)C'}{U'(C)} = r - \rho. \tag{4.38}$$

Once again, let us assume the simple CRRA utility with a constant relative risk aversion of γ resulting in the well-known Ramsey equation:

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\gamma}, \tag{4.39}$$

with the closed-form solution (compare equation 4.9:[44])

$$C(t) = C_0 \exp\left(\frac{r - \rho}{\gamma} t\right). \tag{4.40}$$

is computed by $\int_0^\infty (1 - F(t))tdt = \frac{1}{p}$. In fact $p = 0$ is the special case of infinitely living agents as discussed in the model presented in the following.

[44]Note that the solution in equation 4.9 required no simplifications, for which the growth rate of consumption amounts to: $\frac{\beta}{R} - 1 = \frac{1+r}{1+\rho} - 1 = \frac{r-\rho}{1+\rho}$ which approximates $r - \rho$ for small values of ρ.

Reinserting this result in the flow-constraint (equation 4.33) results in a first-order linear differential equation:

$$\dot{D} - rD = C_0 \exp\left(\frac{r-\rho}{\gamma}t\right) - Y, \tag{4.41}$$

that solves as follows:

$$D(t) = C_1 \exp(rt) - \frac{C_0\gamma}{\rho - r(1-\gamma)} \exp\left(\frac{r-\rho}{\gamma}t\right) + \frac{Y}{r}. \tag{4.42}$$

To completely solve this model we have to define initial and end conditions. One restricting condition is the so-called *Transversality Condition*. This is frequently shifted to a footnote and treated as a mere technicality, yet is of utter importance for achieving long-run stability. For a finite end-point problem ending at time T it is required that:

$$\lambda(T)D(T) = 0. \tag{4.43}$$

For the infinite horizon case this condition reads as follows:

$$\lim_{T\to\infty} \lambda(T)D(T) = 0, \tag{4.44}$$

and is often referred to as the *No-Ponzi condition* indicating its role as leading to stability of debt. Note that this condition is weaker than a condition requiring debt to be zero in the long-run ($\lim_{T\to\infty} D(T) = 0$). Moreover, it does not even demand debt to be finite in the long-run. It only requires the value of debt multiplied by the Lagrange-multiplier λ at time T to be zero in the long-run. Resulting from equation 4.35, this multiplier can be interpreted as the shadow price of consumption. Inserting equation 4.35 into the No-Ponzi condition yields:

$$\lim_{T\to\infty} \exp(-\rho T)U'(C(T))D(T) \stackrel{!}{=} 0. \tag{4.45}$$

Note that for any positive time preference rate $\rho > 0$ the time discounting factor converges to zero in the long-run ($\lim_{T\to\infty} \exp(-\rho T) = 0$). If, however, in the long-run consumption converges to zero ($\lim_{T\to\infty} C(T) = 0$) the marginal utility of consumption diverges ($\lim_{C\to 0} U'(C) = \infty$)[45] leading to no clear-cut result.

We can also use the result of equation 4.36. Solving this linear differential equation leads to:

$$\lambda(t) = \lambda(0)\exp(-rt) = U'(C(0))\exp(-rt), \tag{4.46}$$

which can be reinserted into the No-Ponzi condition resulting in:

$$\lim_{T\to\infty} U'(C(0))\exp(-rT)D(T). \tag{4.47}$$

Assume that debt grows in a geometric process with a constant rate g_D ($\frac{\dot{D}}{D} = g_D \leftrightarrow D(t) = D_0 exp(g_D t)$). To ensure stability[46] the growth rate of debt should be smaller than the rate of interest going along with the following necessary condition:

$$r > g_D. \tag{4.48}$$

[45] This can be considered an Inada condition for the utility function.

[46] This is the case since the marginal utility of initial consumption $U'(C(0))$ is finite.

For the case of a time varying interest rate $r(t)$, the constant interest rate r has to be replaced by the average interest rate $\bar{r} = \frac{1}{t}\int_0^t r(\tau)d\tau$.

Now we insert the result of equation 4.42 into the transversality condition:

$$\lim_{T\to\infty} U'(C(0))\exp(-rT)\left[C_1\exp(rT) - \frac{C_0\gamma}{\rho - r(1-\gamma)}\exp\left(\frac{r-\rho}{\gamma}T\right) + \frac{Y}{r}\right] = \\ U'(C(0))C_1 \stackrel{!}{=} 0 \Rightarrow C_1 = 0. \tag{4.49}$$

The other necessary conditions are the existence of a positive time preference $\rho > 0$ and an inelastic utility function ($\gamma > 1$) for any positive interest rate $r > 0$. To completely solve the system we have to set a second boundary condition. Consider the special case for which the agents start with initial debt (or wealth) ($D(0) = D_0 = -K_0$):

$$D(0) = -\frac{C_0\gamma}{\rho - r(1-\gamma)} + \frac{Y}{r} \stackrel{!}{=} D_0 \to C_0 = \frac{\rho - r(1-\gamma)}{\gamma}\left(-D_0 + \frac{Y}{r}\right), \tag{4.50}$$

leading to the closed-form solutions:

$$D(t) = \left(-\frac{Y}{r} + D_0\right)\exp\left(\frac{r-\rho}{\gamma}t\right) + \frac{Y}{r}, \tag{4.51}$$

and

$$C(t) = \frac{\rho - r(1-\gamma)}{\gamma}\left(\frac{Y}{r} - D_0\right)\exp\left(\frac{r-\rho}{\gamma}t\right). \tag{4.52}$$

As required in equation 4.48 the growth rate of debt is strictly smaller than the interest on debt ($g_D = \frac{r-\rho}{\gamma} < r \leftrightarrow r(1-\gamma) < 0 < \rho$ for all $\gamma > 1$ [47]).

Moreover, debt and consumption grow at the same rate:

$$\frac{\dot{C}}{C} \equiv \frac{\dot{D}}{D} = \frac{r-\rho}{\gamma}. \tag{4.53}$$

Furthermore, they evolve in the inverse direction:

$$\frac{\dot{C}}{\dot{D}} = -\frac{\rho - r(1-\gamma)}{\gamma} < 0. \tag{4.54}$$

The isoelastic case ($\gamma = 1$) is nested in this result with debt evolving according to:

$$D(t) = \left(-\frac{Y}{r} + D_0\right)\exp\left([r-\rho]t\right) + \frac{Y}{r} \tag{4.55}$$

and consumption following:

$$C(t) = \rho\left(\frac{Y}{r} - D_0\right)\exp\left([r-\rho]t\right). \tag{4.56}$$

For sake of simplicity, we restrict our further analysis to this case.

[47] The condition can also hold for $0 < \gamma < 1$ if the interest rate is sufficiently low.

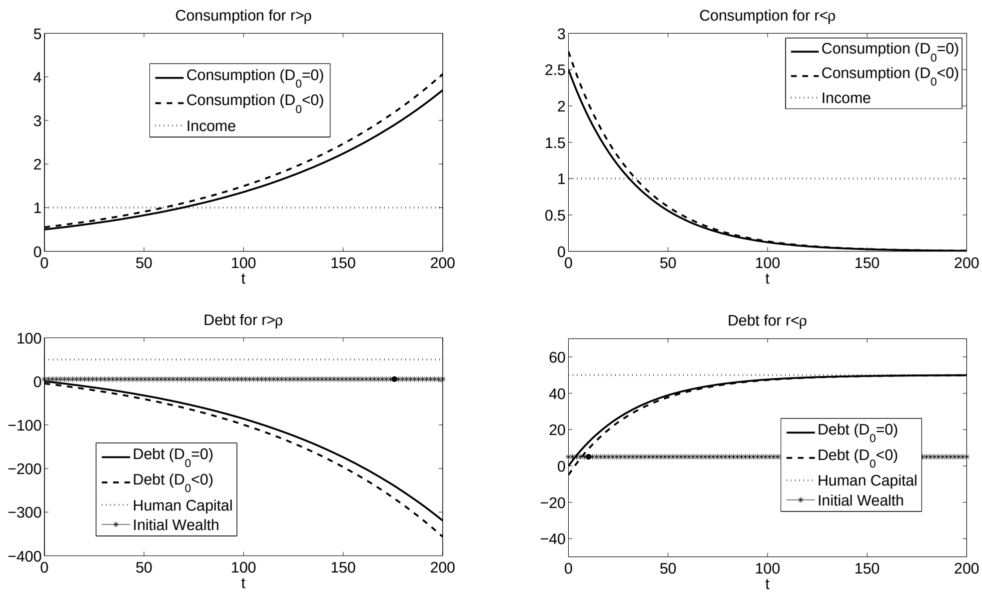


Figure 4.4.: Consumption $C(t)$ and debt $D(t)$ dynamics in time t for different relationships between r and ρ

Following the logic presented in figure 4.2 we can distinguish three cases. The first and most simple case would be $\rho = r$ (identical to the case of $\beta = R$ presented in figure 4.2). In fact, this is the standard equilibrium result in a representative agent case.[48] In this case consumption is constant in time:

$$C(t) = -\rho D_0 + \rho \frac{Y}{r} = rK_0 + Y. \tag{4.57}$$

In fact, agents behave like *hand to mouth consumers* consuming all current labor income and interest on capital. Capital / debt is constant in time ($D(t) = D_0$). Due to the triviality, we do not display this result graphically.

The second case emerges for condition $\rho = \rho_{impatient} > r$ identical with $\beta < R$ in the initial model leading to decreasing consumption in time as depicted in figure 4.2 and is presented for the derived closed-form solution in the right panel of figure 4.4.[49] In the long-run, consumption converges to a level of zero. In fact, in the long-run all income is used to pay interest on debt incurred earlier ($\lim_{t\to\infty} rD = r \cdot \frac{Y}{r} = Y$). Initial consumption increases with initial capital ($C(0) = \rho\left(\frac{Y}{r} - D_0\right)$). Debt converges to $\frac{Y}{r}$ being the time-discounted value of labor income defining the human capital of the agents. As a result, in the long-run the maximum debt level only depends on the labor income independent of initial endowment D_0. Note that realistic results can only

[48]If one interprets this in an open-economy setting it implies that there is neither export or import of capital from foreigners implying a balanced current account.

[49]For the numerical simulations we assume $Y = 1$, $0 < \rho_{patient} = 1\% < r = 2\% < \rho_{imaptient} = 5\%$, $-D_0 = K_0 = 5 > 0$.

be obtained for positive net worth agents ($D_0 < \frac{Y}{r}$). In the other case, agents would exert negative consumption in order to reduce their debt to the value of their human capital making them zero net worth in the long-run. For the special case of no initial debt or capital ($D_0 = 0$), the switching point between net lending and net savings can be easily computed. In particular, for $t^* > \frac{\ln(\rho/r)}{\rho - r} > 0$ agents consume less than their labor income Y. Assessing this result, one can say that the temporary violation of the budget constraint in the initial periods is traded-off for a long-run convergence to zero consumption. Given the assumption that there exists a subsistence level of consumption, this is not an attainable goal.

The third case corresponding to the case of a current account surplus emerges for $\rho = \rho_{patient} < r$ (equivalent to $\beta > R$ for the case depicted in figure 4.2). In this case, consumption diverges and therefore capital also diverges in the long-run (see the left panel of figure 4.4).[50] This seems to be at odds with the transversality condition. Note, however, that as consumption grows to infinity the marginal utility of consumption converges to zero in line with the transversality condition as depicted in equation 4.45. Keep in mind that capital grows with $r - \rho$ and therefore at a slower pace than the interest rate r. In the theoretical model, the accumulated capital is not subject to any risk - in particular default risk. In fact, the increasing consumption path of patient agents ($\rho_{patient} < r$) is financed via interest payment on accumulated capital.

We can also compare this complicated case with a simple Keynesian formulation. Assume an ad-hoc formulation of consumption with a subsistence level of consumption $\bar{c}$ and propensity to consume out of (disposable) flow income c_y and stock c_w:

$$C = \bar{c} + c_y(Y - rD) + c_w(-D). \tag{4.58}$$

We can insert the closed-form derivation of debt into this equation and compare it with the closed-form derivation of consumption:

$$\begin{aligned} C(t) = \rho\left(\frac{Y}{r} - D_0\right)\exp([r-\rho]t) &\stackrel{!}{=} \\ \bar{c} + c_yY + (c_yr + c_w)\left(\frac{Y}{r} - D_0\right)\exp([r-\rho]t) - (c_w + c_yr)\frac{Y}{r}&. \end{aligned} \tag{4.59}$$

By equating coefficients one can show that $\bar{c} \equiv c_w = 0$ and $c_y = \frac{\rho}{r}$. Consumption therefore only depends on current income. For the case of $\rho > r$ we have $c_y > 1$, implying negative savings ratios. Due to the disposable income effect, this nevertheless leads to a stable consumption path. This is the case since the accumulation of debt increases the credit costs, thus decreasing disposable income as well as consumption. This case, however, is accompanied by a long-run convergence of consumption to a level of zero. Therefore agents should be recommended to consume with a ratio of $\rho < r$, implying $0 < s < 1$, in line with the ad-hoc assumption of the simple models of Domar (1944) or Solow (1956).

[50] To rule out this case and thereby guarantee an ergodic distribution of wealth in the *Bewley*-type models the condition $\rho > r$ has to be satisfied (Sunel, 2013).

Note that up to this point - and in the contrast to the simple Domar case - we assumed that income is constant and does not vary in time. Following the rationale at the beginning of this section we now assume that income grows following an exponential process $Y_t = Y_0 \exp(gt)$. The first-order condition leading to the Ramsey equation remains unchanged. However, the flow constraint (formerly equation 4.33) now reads as follows:

$$\dot{D} - rD = C_0 \exp([r-\rho]t) - Y_0 \exp(gt). \tag{4.60}$$

Solving the equation leads to the following result:

$$D(t) = C_1 \exp(rt) + \frac{Y_0}{r-g} \exp(gt) - \frac{C_0}{\rho} \exp([r-\rho]t). \tag{4.61}$$

Using the transversality condition we know that:

$$\lim_{T\to\infty} [U'(C(0)) \exp(-rT) D(T)] = \lim_{T\to\infty} C_1 + \frac{Y_0}{r-g} \exp([g-r]T) \stackrel{!}{=} 0 \tag{4.62}$$

This condition not only requires $C_1 = 0$ (as before) but also $g < r$. Using the initial condition ($D(0) = D_0$) we can solve for C_0 resulting in:

$$C(t) = \rho \left(\frac{Y_0}{r-g} - D_0 \right) \exp([r-\rho]t). \tag{4.63}$$

This implies the following evolution of debt:

$$D(t) = -\left(\frac{Y_0}{r-g} - D_0 \right) \exp([r-\rho]t) + \frac{Y_0}{r-g} \exp(gt). \tag{4.64}$$

Let us assume a *Keynesian* consumption function of the type:[51]

$$c = \bar{c}_y Y + \bar{c}_w(-D). \tag{4.65}$$

Using the aforementioned method of comparing coefficients leads to:

$$\begin{aligned} C(t) = \rho \left(\frac{Y_0}{r-g} - D_0 \right) \exp([r-\rho]t) &\stackrel{!}{=} \bar{c} + \bar{c}_y Y \exp(gt) + \\ \bar{c}_w \left(\frac{Y}{r-g} - D_0 \right) \exp([r-\rho]t) &- \bar{c}_w \frac{Y}{r-g} \exp(gt), \end{aligned} \tag{4.66}$$

implying $\bar{c} = 0$, $\bar{c}_w = \rho$ and $\bar{c}_y = \frac{\rho}{r-g}$.

The most simple case would be to assume that $\rho = r - g > 0$, implying that the discount rate equals the wedge between interest rate and r and the growth rate g. This case is proposed for instance in Bertola (2000). In fact, this is also always required by the standard asset pricing theory. As formally shown in appendix A.3, this is the only

[51] The case without growth is nested in this result. For the case without growth ($g = 0$) we have $\bar{c}_y = \frac{\rho}{r}$ and $\bar{c}_w = \rho$.

feasible case for asset pricing. Any deviation ($r - g - \rho \neq 0$) at some point of time leads to an asset bubble with an ever-exploding or ever-contracting price path. It is eventually hard to measure whether the condition $\rho = r - g$ holds or if we have $r - g > \rho$ or $r - g < \rho$. While the interest rate r and the growth rate g are observable, the rate of time preference is not directly measurable. In fact, the equality condition is frequently assumed by defining $\rho \equiv r - g$.[52]

Sometimes it is even argued that the equality condition is also the only sensible calibration for models without growth ($g = 0$). In this case, the condition would read $r = \rho$. As already presented in figure 4.4 both the cases of growing or contracting consumption are thereby excluded. While the first case is accompanied by an ever increasing accumulation of capital ($\rho < r$), the latter leads to the unrealistic result of a long-run consumption of zero ($r < \rho$). In the case of $\rho = r$ holds exactly, agents consume all their current income, whereas their wealth is only determined by their initial endowment of inherited capital or debt D_0.

We can also discuss the condition $r - \rho = g$ in the light of our *Keynesian* consumption function. The latter implies $\bar{c}_y = \frac{\rho}{r-g} = 1$ once again going along with hand to mouth households that consume all their current income. Moreover, the time preference $\rho = \bar{c}_w$ can be interpreted as consumption out of stock. As a result, agents neither accumulate assets nor debt. In fact, the evolution of debt is given by:

$$D(t) = \exp([r - \rho]t) \left(\frac{Y_0}{r - g} - \frac{Y_0}{r - g} + D_0 \right) = \exp([r - \rho]t) D_0 = \exp(gt) D_0, \quad (4.67)$$

only depending on their initial endowment D_0. If agents start without initial endowment ($D_0 = 0$) they neither accumulate debt nor capital. The initial endowment grows as the same pace as the total economy does - namely with the rate g. For the realistic case that agents are initially endowed with assets ($-D_0 > 0$) rather than debt, this stock grows indefinitely. Or to use the terms of the Lucas-type fruit economy (Lucas, 1978): agents are provided with a seed from their parent generation which grows to be a large tree bearing new fruits. The growth rate itself is determined by the overall growth rate of the economy. More practically, agents invest in a portfolio of shares and risk-free assets yielding a return of r, but consume with a ratio of ρ resulting in the fact that their total capital grows at a pace of $r - \rho \equiv g$.

The rate of time preference is always positive ($\rho > 0$) capturing the tendency of agents to prefer current as opposed to future consumption. Thereby, the condition $r - g = \rho > 0$ also implies $r > g$. This condition is well-known as the condition for dynamic efficiency (Abel et al., 1989). For the nested case without growth ($g = 0$) this requires a positive interest rate $r > 0$. An economy that is characterized by dynamic efficiency aggregates a level of capital below the optimal level $k < k^*$. Or put differently, an economy that is characterized by dynamic inefficiency accumulated capital above the optimal level $k > k^*$.

[52] This variable is taken for e.g. tuning macro economic models. In fact, the factor β in the consumption decision problem is given by $\beta = \frac{1}{1+\rho}$.

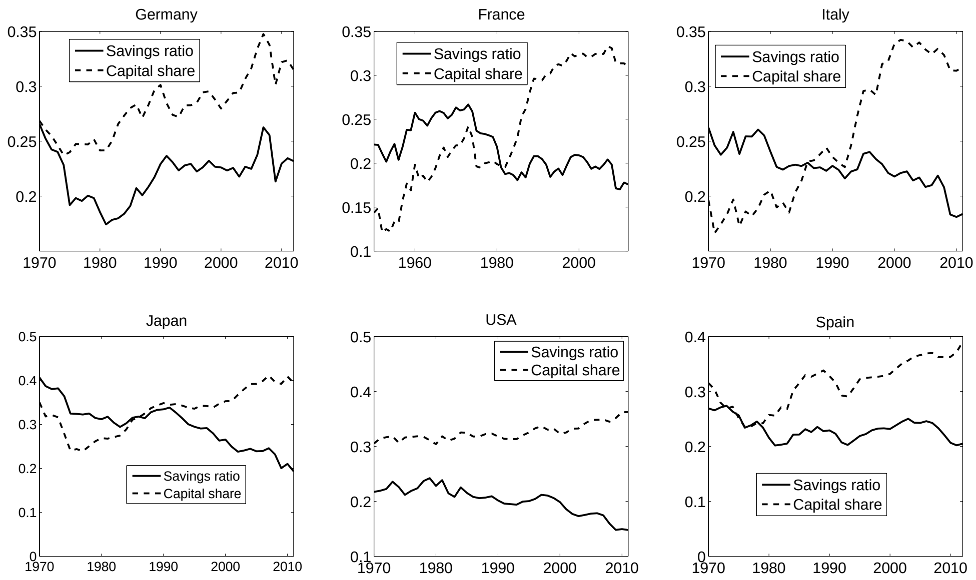


Figure 4.5.: Capital share α and savings ratio s in several OECD countries (Data source: OECD (2012))

In the standard growth literature in the golden-rule level of capital accumulation we have $f_k = r = g$.[53] [54] The optimal savings ratio is given by $s^* = \frac{gk}{f(k)} = \frac{rk}{y}$.[55] For the standard Cobb-Douglas type production function ($f(k) = k^\alpha$) we have $s^* = \alpha$.[56] Accordingly, in a dynamic efficient economy we have $s < \alpha$, implying that the aggregate savings ratio is below the capital share.

As put forward in Phelps (1961) this condition is identical to the condition of profits being larger than investment.[57] Abel et al. (1989) test this condition for OECD countries from 1929 (after the Great Depression) until 1989 (publication date of their paper) and show that dynamic efficiency prevails.

[53] Formally, this result is the first-order condition resulting from maximizing consumption $c = (1-s)y = (1-s)f(k)$ subject to the steady state condition $sf(k) = gk$.

[54] Note that in this case we abstract from depreciations δ. If included, we would have $r - \delta = g$.

[55] It is important to acknowledge that any savings ratio s can constitute a steady state.

[56] For a more general Constant Elasticity of Substitution (CES) type production technology the capital share α depends on the level of capital k. For the case of a capital-biased production function, the capital share increases with the level of capital. This implies that a highly developed economy with a high level of k accompanied by a large capital share, also requires a strong savings ratio to sustain the condition of dynamic efficiency. Or - if the savings ratio remains constant - an increase in capital also accompanied by an increase in the capital share is accompanied by a convergence to a situation of dynamic efficiency. This technology is empirically confirmed in the recent work of Karabarbounis and Neiman (2013). The inverse case emerges for a labor-biased technology. A more thorough discussion of this topic is provided in section 6.2.1.

[57] It is easy to verify that these condition once again equals the conditions $s < \alpha$. Recall that investment are $I = S = sy$ and profits are given by rk. With $I = sy < rk$ we have $s < \alpha$.

In figure 4.5 we show more recent data of several OECD countries. It is interesting to observe that at the moment all countries can be considered dynamically efficient by means of the condition $\alpha > s$. Eventually, before the mid 1980s for many countries the opposite was the case. This eventually is in line with the rationale of Piketty et al. (2014) who argue that the 1980s were a time of a paradigm change to lower redistribution and more deregulation also manifesting itself in a stronger capital share. It is also interesting to point out that the USA always exhibited a significantly positive gap between the capital share and the savings ratio ($\alpha - s > 0$) indicating that dynamic efficiency was always prevailing. Moreover, the gap eventually even grew. Similar behavior can also be observed for Japan and Spain.[58] This increasing gap is also interesting to point out as it implies increasing wealth inequality.[59]

The inverse case would be dynamic inefficiency. In this case we have $g > r$ or for the nested case without growth a negative interest rate $r < 0$. In this case, profits are lower than investments, the capital share is lower than the savings ratio ($s > \alpha$), and capital is accumulated above the golden-rule level ($k > k^*$).

It is interesting to point out that the condition of dynamic inefficiency $g > r$ is the condition required in the Domar (1944) type model for a stable level of debt. To compute a closed-form solution of the evolution of capital in our model we required $r > g$. This is made to have a convergence in the value of human capital resulting from the Gordon-growth model.[60] If the condition is not satisfied, human capital is infinite - by always growing in excess of the interest rate - allowing for an infinite absolute level of debt in line with the Domar type logic. Or put differently, debt can be sustainable even though the Domar condition $r < g$ is not satisfied.

In the long-run, it is assumed that the growth rate g is exogenously given. The interest rate decreases with the level of capital due to decreasing marginal productivity of capital.[61] Thereby in the long-run the interest rate could decrease below the level of growth $r < g$ leading to the condition of dynamic inefficiency. Piketty and Saez (2013a) argue that this case holds for a small open economy. One could, however, also argue that the low fertility prevailing reduces the growth rate g, whereas the the return to capital is exogenously given, implying a convergence to dynamic efficiency. This is eventually in line with the empirical evidence reported in figure 4.5. A policy maker that would like to achieve dynamic inefficiency could do so by means of imposing a tax on capital thereby lowering the effective rate of return.[62]

In an economy that is dynamically inefficient many market interventions can eventually be welfare improving. The issue of dynamic (in)efficiency is frequently discussed in Overlapping Generation (OLG) Models. In fact, in these models the presence of dynamic inefficiency can make a pay-as-you-go (PAYG) system of pensions superior to a capital-market-based systems. The rationale is obvious since the return of the PAYG is the total

[58] Note that these observations rely on simple descriptive statistics and eyeball evidence. Future research should consider this dataset more rigorously using econometric methods.

[59] This subject will be treated more rigorously in section 5.5.2.

[60] For more details the reader is referred to appendix A.3.

[61] In the (neo)classical setup $r = f_k$ and $f_{kk} < 0$.

[62] We will dwell more thoroughly on this topic in section 7.2.1.

growth rate of the economy g, whereas the return of the capital market based system is r with $g > r$ for dynamic inefficiency.[63] Boadway and Keen (2000) argue that public insurance may eventually be welfare increasing in the presence of dynamic inefficiency providing redistribution amongst generations. Thereby, *generational* risks such as being born in a period with wars or natural catastrophes can be hedged. This also provides a rationale for government debt as the costs of these disasters are financed by debt and thereby paid by future generations. An overaccumulation of capital also emerges in the *Bewley*-type model of Aiyagari (1995) due to uninsurable idiosyncratic risk. Following the logic of this model the problem can be solved by government debt. As emphasized by Piketty (2014) the OLG-models argue that there is an *age war* between current retirees and the current working population or even between the current living generation and the future generations (being the offspring of the current generation). In contrast to that, for dynastic models for which wealth is transferred within families a *class war* in the Marxist sense emerges in which wealthy individuals or dynasties compete with poor dynasties.

It is interesting to point out that dynamic inefficiency implies $r - g < 0 < \rho$ and thereby is accompanied by the accumulation of debt implied by $r - \rho < g$.[64] Yet, it is important to be precise about the relation. While the situations of the knife-edge case $r - \rho = g$ and the capital accumulation case of $r - \rho > g$ always go along with dynamic efficiency, the inverse case of debt accumulation $r - \rho < g$ *can* go along with dynamic inefficiency.[65] The condition $r - \rho > g$ also is accompanied by increasing wealth inequality. We will dwell more thoroughly on this issue in section 5.5.2. As an outlook one can, however, already state that dynamic efficient economies have a tendency to go along with increased wealth inequality thereby confirming the old *equity-efficiency* trade-off (Okun, 1975).

4.3. Behavioral and Heterodox Approaches to Consumption

As emphasized by Duesenberry (1949), consumption also has a social dimension.[66] Households imitate their social reference group when making consumption decisions. Since agents adjust their decisions by copying the decisions of others, this is more a behavioral than a rational expectations approach on consumption. Consumption not only reflects membership in a social class but in fact is used to express class identity. The

[63] If one takes a deeper look, other factors, however, are also of importance. One key determinant is the ratio of working-age population to the number of retirees.

[64] Formally, the debt presented in equation 4.64 diverges to a positive level $\lim_{t\to\infty} D(t) = \infty > 0$. In the inverse case, capital would be accumulated: $\lim_{t\to\infty} D(t) = -\infty < 0$. Note that the case with zero growth ($g = 0$) is nested within this case leading to long-run capital holding for $r > \rho$ and debt holding for $\rho > r$ (also compare with figure 4.4).

[65] Moreover, in this case, as discussed in the appendix A.3 we have a case of a negative asset bubble. A positive asset bubble emerges for $r - \rho > g$.

[66] The key idea even dates back to Veblen (1899) coining the term *conspicuous consumption*.

latter effect is nicely illustrated in the Agent-Based Model of Bruun (2001) implementing this idea leading to consumption clustering. The consumption theory of Duesenberry (1949) is often referred to as the relative income hypothesis of consumption. van Treeck (2012) emphasizes that the current situation requires for a revival of this thesis since in times of high inequality households increase consumption in positional and highly visible goods (Frank, 1997) such as houses, cars or also education while decreasing savings that are non-positional. Frank (2005) compares this consumption behavior to a welfare decreasing military arms race. Furthermore, he states that this might be an explanation to the *Easterlin Paradox* (Easterlin, 1995) measuring a rather constant level of happiness despite a strong increase in GDP. Even though the absolute level of consumption increased, most agents could not improve their relative position, leading to a stagnation in the level of happiness. Alpizar et al. (2005) are able to confirm the relative consumption effect in a lab experiment also classifying goods in the categories of non-positional (vacation and insurance) and positional goods (cars and housing). Moreover, they assert that economic majors and females are more prone to comparing consumption levels. As a policy conclusion, they suggest imposing taxes on positional goods to stop the over-consumption of these goods compared above a social optimum to non-positional goods (especially vacation).

Al-Hussami and Álvaro Martín Remesal (2012) provide a theoretical discussion of the relative income hypothesis in a two-period two-consumer model, in which low-income households imitate the consumption level of high-income households and discuss its effects on the current account. Furthermore, they also discuss the effect of increasing inequality as assumed by redistribution from low to high-income households. The result is that increasing inequality increases the current account deficit for borrowing economies in which $\beta < R$ (such as the USA) as well as increasing the surplus of savings economies with $\beta > R$ (e.g. China). The current account imbalances are not only amplified by increased inequality but also by higher financial development.

One can also account for relative consumption effects in a utility maximizing framework. To do so, we have to assume a modified CRRA utility function with a minimum consumption level $\bar{c}$:[67] [68]

$$U(C) = \frac{(C - \bar{c})^{1-\gamma}}{1-\gamma}. \tag{4.68}$$

This utility function was e.g. proposed in Uhlig and Ljungqvist (2000) to model *Keeping Up With the Joneses*.[69] In fact, for this version of the consumption function, consump-

[67] The Keynesian models are frequently blamed for their ad-hoc nature. The micro-founded models, however, do little else but transform this ad-hoc level to an earlier stage starting from an ad-hoc assumption about the functional form of the utility function.

[68] This is eventually the utility function assumed in the DSGE framework of Kumhof et al. (2014) to account for relative consumption effects.

[69] An alternative earlier version with ratios rather than differences is presented in Abel (1990), in which he assumes $U(C) = \frac{1}{1-\gamma}\left(\frac{C}{\bar{c}^{\theta}}\right)^{1-\gamma}$ with θ being a taste parameter increasing with strong taste for relative consumption. Uhlig and Ljungqvist (2000) furthermore discuss *Catching Up* with the Joneses, in which current individual consumption depends on a lagged form of aggregate consumption. Binder and Pesaran (2001) discuss social interactions in a life-cycle model and emphasize the im-

tion levels lower than a well-defined subsistence level lead to negative utility ($U(C) < 0$ for $C < \bar{c}$). Using the well-known constraint (cf. equation 4.3), the following first-order condition can be derived:

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{(C_t - \bar{c})^{-\gamma}}{(C_{t+1} - \bar{c})^{-\gamma}} = \frac{\beta}{R}, \tag{4.69}$$

which can be solved leading to the following recursive consumption equation (see also Bertola (2000)):

$$C_{t+1} = \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}} C_t + \bar{c}\left(1 - \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}}\right). \tag{4.70}$$

The steady state is equal to the subsistence consumption:

$$C = \bar{c}, \tag{4.71}$$

and only stable if $\beta < R$.[70] The latter can be rationalized when considering the phase diagram (see figure 4.6) as described by the following equation:

$$\Delta C = C\left(\left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}} - 1\right) + \bar{c}\left(1 - \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}}\right). \tag{4.72}$$

Moreover, for the special case $\beta = R$ the effect of subsistence consumption $\bar{c} \neq 0$ disappears.

This has some interesting economic interpretations. The current account deficit economy converges to a steady level of consumption equal to the subsistence level. Meanwhile, the equilibrium consumption level ($C = \bar{c}$) is unstable for the surplus case. However, only the case of an ever increasing consumption - as already presented in the previous sections - is economically meaningful.[71]

We can also make a statement about the relation between the savings ratio and the level of income. Following an argument presented in section 4.1 in which agents live for

portance of these factors in a dynamic rather than a static scenario. In particular, the presence the presence of *jealousy* and *conformism* within a peer group is able to explain time-series properties of aggregate consumption (such as excessive sensitivity to *anticipated* labor income changes as well as a general excess smoothness) which cannot be accounted for in a standard permanent income framework.

[70] We can also take the inverse view. As already presented in figure 4.4, the steady state level in deficit economy without a subsistence level is zero.

[71] The ever decreasing level of consumption has no deeper economic meaning. Technically, it can be explained by the fact that, if we start with consumption at a level lower than $\bar{c}$, we have negative utility and marginal utility ($U(C) < 0$ and $U'(C) < 0$ for $C < \bar{c}$). The first-order condition requires that this utility becomes larger - i.e. more negative in time - implying a convergence to zero consumption and negative utility. While this makes perfect sense in the formal framework it seems hardly feasible in reality. This case is excluded in a standard CRRA utility function as $U(C) > 0$ and $U'(C) > 0$ for all $C > 0$.

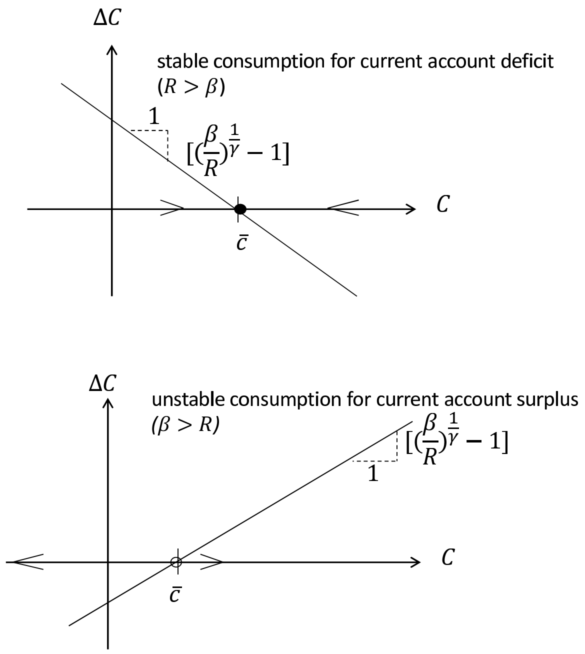


Figure 4.6.: Phase diagram for consumption with subsistence consumption effect $\bar{c} \neq 0$

two periods and are subject to simple life time budget constraint $(C_1 + RC_2 = Y_1)$[72] the consumption level for the young agents equals:

$$C_1 = \frac{Y_1 - \bar{c}R\left(1 - \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}}\right)}{1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}}. \tag{4.73}$$

For the case of a surplus economy $(\beta > R)$[73] this implies a higher level of consumption than without the relative consumption effect ($\bar{c} = 0$, cp. with the result of equation 4.12). Furthermore, this has the important implication that - in this case - the savings ratio increases with income:

$$s = \frac{Y_1 - C_1}{Y_1} = 1 - \frac{1}{1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}} + \frac{\bar{c}R\left(1 - \left(\frac{\beta}{R}\right)^{\frac{1}{\gamma}}\right)}{Y_1\left(1 + R^{1-\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\right)}, \tag{4.74}$$

as the partial derivative of the third term with respect to Y_1 is positive for the given case $(\beta > R)$. The basic idea underlying this result is that the average propensity to consume

[72] Without loss of generality and for sake of illustration we assume that there is no labor income in the second period and agents are born without heritage.

[73] As pointed out in Dynan et al. (2004) this is only the case for a surplus economy. In the converse case, the counterfactual result of low-income agents saving a higher ratio is derived.

exceeds the marginal propensity to consume.[74] A further impact of the rich saving more is that the distribution of wealth diverges. We will refer to this more precisely in section 5.5.2. It is also important to note that the inverse result holds for a deficit economy. In this case, savings decrease for high-income agents, leading to a convergence of wealth and - as already presented - in the long-run consumption also converges to the identical level $\bar{c}$.

Moreover, the model showed the proximity of relative consumption and subsistence consumption. Standard models assume that there is no minimum level of consumption resulting in the fact that a zero level consumption can be perfectly rational. This disregards the fact that individuals need a certain level of consumption (e.g. in the form of nutrition) to survive. The latter observation also justifies the existence of government intervention guaranteeing a minimum income. This, however, quickly leads into a debate about the required level of $\bar{c}$. A basic income level in a developing African country possibly takes a lower level than in a developed Western society resulting from externalities in consumption of others. For instance, the access to internet is considered as a standard in Western society. The associated problems are nicely summarized in the question "How much is enough?" (Skidelsky and Skidelsky, 2012).

The idea of relative consumption effects is elegantly captured in the OLG-type model of Alvarez-Cuadrado and Van Long (2011). Using a similar argument as above they show that in the presence of relative consumption effects the savings ratio increases for wealthy individuals.[75] They furthermore show that due to the linearity of the effect the aggregate savings ratio still remains unaffected by the distribution of income.[76] As Alvarez-Cuadrado and Van Long (2011) also include an explicit labor supply decision they are able to show that presence of relative consumption effects not only contributes to over-consumption and too little saving but also to an over-supply of labor compared to central planners results. To tackle this welfare-reducing results, a tax on the over-supplied or over-demanded goods (labor income and consumption) and a subsidy on the under-supplied good (savings) is proposed.

In contrast to neo-classical theory, heterodox economists were always concerned with the distribution of income. Rather than regarding a personal distribution of income the functional distribution of income was put in the center.[77] Post-Keynesian theory focuses on the distribution of total income Y between wage income w and profit/financial income Π. The idea dates back to the model of Kaldor (1955) arguing, however, from the

[74] The difference between the average and the marginal propensity to consume is given by: $APC - MPC = \frac{C}{Y} - C' > 0$. For all positive income this is equal to the condition in the upper equation: $\frac{ds}{dY} = \frac{d}{dY}\left(1 - \frac{C}{Y}\right) = \frac{C}{Y^2} - \frac{C'}{Y} > 0$. For a linear consumption function with offset this yields: $APC - MPC = \frac{\bar{c}}{Y} + c_y - c_y = \frac{\bar{c}}{Y} > 0$ being positive for positive levels of subsistence consumption $\bar{c} > 0$.

[75] In particular, they are able to make a very general statement not relying on restricting assumption about the relation between β and R.

[76] We will refer to this effect later more formally in this work. In particular, this behavior is captured in section A.2 showing that linearity in the underlying rules is necessary to avoid the fallacy of composition.

[77] In section 5.5.1 we discuss the relation of personal and functional distribution of income formally.

mirror perspective of savings.[78] Menz (2010) presents a simple model of consumption that accounts for changes in the distribution of income on total consumption. One crucial assumption in this model is that borrowing is allowed and thereby influences the disposable income Y^d. Disposable income Y^d in this case not only depends on current income Y, but also on interest payments (rD depending on the interest rate r and the stock level of debt D) and new borrowing B (flow of debt):

$$Y^d = Y - rD + B. \tag{4.75}$$

In this model, different MPC ($0 < c_i < 1$) are assigned to different sources of income including consumption of net borrowings:

$$C = c_w w + c_\Pi \Pi + c_B B. \tag{4.76}$$

If the share of profit ($\Pi' = \frac{\Pi}{Y} = 1 - \frac{w}{Y}$) is considered, this equation can be rewritten as follows:

$$C = (1 - \Pi')c_w Y + \Pi' c_\Pi Y + c_B B. \tag{4.77}$$

Furthermore, lending restriction is imposed defining a maximum level of new indebtedness B as a function of income less interest payments (this means *cash-on-hand*):

$$B = l(Y - rD). \tag{4.78}$$

The value l can be interpreted as maximum leverage of households. These assumptions yield an overall positive MPC:

$$\frac{\partial C}{\partial Y} = (1 - \Pi')c_W + \Pi' c_\Pi + l c_B > 0. \tag{4.79}$$

If we assume that the MPC out of wages c_W is higher than the MPC out of capital income c_Π ($c_W > c_\Pi$), a redistribution towards profit share decreases overall consumption:

$$\frac{\partial C}{\partial \Pi'} = (c_\Pi - c_W)Y < 0. \tag{4.80}$$

Furthermore, an increase in lending activities through better collateralization of assets via financial innovation (i.e. higher values for $l > 0$), also leads to higher consumption:

$$\frac{\partial C}{\partial l} = c_B(Y - rD) > 0. \tag{4.81}$$

This latter result only holds true in the no-Ponzi scheme ($Y > rD \leftrightarrow \frac{Y}{r} > D$) framework.

A similar argument is put forward in Gu et al. (2014) trying to rationalize the effect of inequality on aggregate savings in different geographical regions. As already shown,

[78] One key argument of Kaldor (1955) is that, if the share of capitalists increases, investment - as a premise for growth - increases. For a thorough discussion of the evolution of the argument in the Post-Keynesian theory the reader is referred to Kurz and Salvadori (2010).

the standard Post-Keynesian model predicts that an increase in inequality is also accompanied by an increased share of capitalists is accompanied by an increase in aggregate savings due to the lower marginal propensity to consume by capitalists. This behavior is e.g. witnessed in Asian countries and Germany. In a similar framework they, however, also argue that in the presence of consumer credit the link between inequality and aggregate savings can become negative implying that higher inequality contributes to lower aggregate savings. The latter effect is furthermore strengthened by the presence foreign financing rationalizing in particular the behavior witnessed in the USA.

Up to this point we focused on (neo)classical consumption theory based on rational expectations and also presented results of the less well-known consumption theory in heterodox economics. A very promising approach of consumption theory is grounded in behavioral theory abandoning the utility maximizing perfect foresight assumption in favor for a more realistic approach grounded on empirical results and psychological theory.

The perfect rational agent in this case is replaced by a bounded rational one. Due to limited resources of money and time (especially since the consumption process itself is time-consuming) as well as cognitive scarcity[79] agents rely on heuristics respectively rules of thumb. The consumption process is mostly guided by habit behavior and therefore shows little variance over time. This effect is rationalized by the the *Status Quos Bias* (Kahneman et al., 1991). Moreover, agents show *learning behavior* in consumption by copying and adapting patterns of other individuals (Nelson and Consoli, 2010). Thus, rules of thumb do not have to lead to suboptimal outcomes. To paraphrase Friedman (1953): an experienced pool player does not have to know Newtonian mechanics to be successful in his game. Deaton (1992) suggests a rule of thumb for consumption behavior which implies consuming all *cash-on-hand* - i.e. all current income and wealth - until median income. For levels above median income he suggests consuming 30% of cash-on-hand. This rule comes very close to the optimal consumption rule derived from a computational model. This result is reflected in a consumption function measured in empirical data (Carroll, 2001). This function exhibits a concave curvature resulting out of the risk aversion of the agents.[80] This furthermore reflects the effect that the elasticity of consumption is lower than one.[81] The presence of a credit market shifts the curve to the left and above the 45-degree line, implying that low-income households borrow whilst high-income households consume less than cash-on-hand.

D'Orlando and Sanfilippo (2010) emphasize that Keynes' original foundation of the consumption function is based on a behavioral approach rather than being utility maximizing. In 12th chapter Keynes (1936) already conjectured that uncertainty about the future plays an important role in the consumption process. Firstly, agents describe

[79] Allen and Carroll (2001) show that solving the consumption problem is only possible due to advanced computational solution methods. However, rule-of-thumb behavior can provide good approximations to the optimal solution.

[80] This implies that MPC decreases with level of income: $\frac{\partial MPC}{\partial Y^d} = \frac{\partial^2 C}{\partial Y^{D2}} < 0$.

[81] If we assume that the consumption function is given by $C = Y^\theta$ the elasticity of consumption is $\frac{\frac{dC}{C}}{\frac{dY}{Y}} = \frac{\log(C)}{\log(Y)} = \theta < 1$ also satisfying the concavity condition $\frac{C''}{C'} = \frac{\theta - 1}{Y} < 0$ for all $Y > 0$.

subjective probabilities to events not in line with their actual probabilities. The latter result is one of the key findings of prospect theory of Kahneman and Tversky (1979). Secondly, there is the problem of uncertainty in the sense of Knight (1933) implying that probabilities about events are not available and we therefore simply *do not know*.

When using rules of thumb economic agents express myopic behavior which can be considered by hyperbolic discounting in a theoretic framework (D'Orlando and Sanfilippo, 2010). This leads to the effect that households have a strong preference for current consumption and that interest rate changes barely affect the consumption decision. In contrast to the life-cycle hypothesis, empirical evidence also suggests that households do not accumulate savings for time of retirement but count on the interaction of the welfare state as *Good Samaritan*, thereby exhibiting moral hazard behavior. In particular this implies that in countries with well-developed *public* social security systems there are little *private* savings by low-income individuals. This myopic behavior suggests that consumption is closely tracked by current income.

Moreover, households are subject to *Mental Accounting*, implying that changes in different forms of income have different results on the change of consumption (D'Orlando and Sanfilippo, 2010). This goes along with the effect of debt aversion. Households refrain from taking debt since the awareness that debt has to be repaid with interest diminishes the pleasure out of the consumption process (D'Orlando and Sanfilippo, 2010).

In their paper D'Orlando and Sanfilippo (2010) built a consumption function associated with several behavioral effects. Besides a subsistence level of consumption they assume a variety of incomes associated with different MPCs to model the mental accounting effect. They categorize income in the categories of current income, future income and the stock level of wealth. This will also be the starting point for modeling the consumers in a positive rather than a normative approach in our model.

4.4. Estimating a Consumption Function for Germany

The microfounded consumption theory as well as the *buffer stock savings* theory provide a normative approach to savings, deriving an optimal consumption path. Empirical evidence, however, disconfirms theses theories. The overconsumption of the low-income households rationalized by behavioral theory can also be thought as an undersaving. Fulford (2012) documents that the poor save too little compared to the their target as indicated by the *buffer stock theory of savings*. This effect is further aggravated by the fact that they also require a higher amount precautionary savings as fraction of their income due to their higher income risk as well lower overall insurance against different hazards such as health risk.

On the other hand, high-income and old households exhibit higher savings than suggested by theory. The latter, in particular, is hard to capture in HAMs of the *Bewley*-type tradition, providing little explanation for the fat right tail of the wealth distribution (Cagetti and De Nardi, 2008). Therefore, the important research question to be answered by the theoretical literature might not be *Why do the poor save so little?*, but *Why do the rich save so much?* (Carroll, 1998). One key argument is the role of bequests typically modeled in an OLG-type framework. It is still important to point out that most bequests are inter-vivo in the form of human capital transfers (Becker and Tomes, 1986). Moreover, very large bequests, can be the results of a *sudden death* and are thereby accidental. As a result and as argued in Carroll (1998) dynastic models including a bequest motive cannot fully account for the measured high savings.[82]

An opposing and more promising approach assumes *Love of Wealth* (e.g. Rehme (2011)) or *Capitalist Spirit* (e.g. Zou (1994)) in the sense of Max Weber, in which agents derive utility directly from holding wealth. As put forward in Carroll (1998) this cannot only be explained by the motive of accumulating positional goods but also by a more altruistic *Joy of Giving* motive. It is also important to point out that accumulating capital is not the final purpose but money is rather thought of as a metric in measuring personal success (Carroll, 1998).[83]

The concept can be formalized in a simple model following the rationale of Carroll (1998). He assumes an additive utility function in which utility is derived from both current consumption and future wealth, in which both individual functions are of the CRRA type:

$$U = u(C_t) + v(W_{t+1}) = \frac{C_t^{1-\gamma_c}}{1-\gamma_c} + \frac{W_{t+1}^{1-\gamma_w}}{1-\gamma_w}. \tag{4.82}$$

The problem is subject to a standard flow equation:

$$W_{t+1} = (1+r)(W_t + Y_t - C_t), \tag{4.83}$$

resulting in the following first-order condition:

$$C_t^{-\gamma_c} = (1+r)W_{t+1}^{-\gamma_w}. \tag{4.84}$$

To derive a closed-form solution Carroll (1998) now assumes concrete values for risk aversion[84] being $\gamma_c = 2 > \gamma_w = 1$. Inserting the first-order condition in the given constraint yields a single feasible solution:

$$\begin{aligned} C_t^{\gamma_c} = \frac{(Y_t + W_t - C_t)^{\gamma_w}}{(1+r)^{\gamma_w - 1}} &\rightarrow C_t^2 + C_t - (Y_t + W_t) \stackrel{!}{=} 0 \\ &\rightarrow C_t = \frac{1 + \sqrt{1 + 4[Y_t + W_t]}}{2} \end{aligned} \tag{4.85}$$

[82] Intuitively, older households can accumulate wealth in their life time and are therefore richer in terms of stock. Models without bequest motives would, however, suggest a strong consumption preference due to low remaining life time. On the other side, older households are faced with higher health risk entailing monetary costs that motivate higher savings.

[83] Another rationale put forward for the high savings of wealth individuals is their high risk exposure due to their strong entrepreneurial activity (Cagetti and De Nardi, 2008). Moreover - in line with the Banerjee and Newman (1993) argument - workers that having entrepreneurial ideas and face financial market constraints accumulate more wealth in order to reach minimum capital requirements for running a business.

[84] To derive the result that savings increase with wealth, he has to assume that the relative risk aversion for consumption is lower than for wealth, in the line with the savings motive.

Consumption increases with both current income and current wealth. In fact consumption depends on cash-on-hand - being the sum of current income and current wealth. Due to the risk aversion, the relation between this input and the output (consumption) is not linear but concave. Furthermore there exists a specific thresholds value[85] below which consumption exceeds current cash-on-hand and vice versa for values above this threshold. The relation between cash-on-hand and consumption is also depicted in a later section in figure 5.22 where we discuss the impact of the curvature of the savings function on the distribution of wealth.[86]

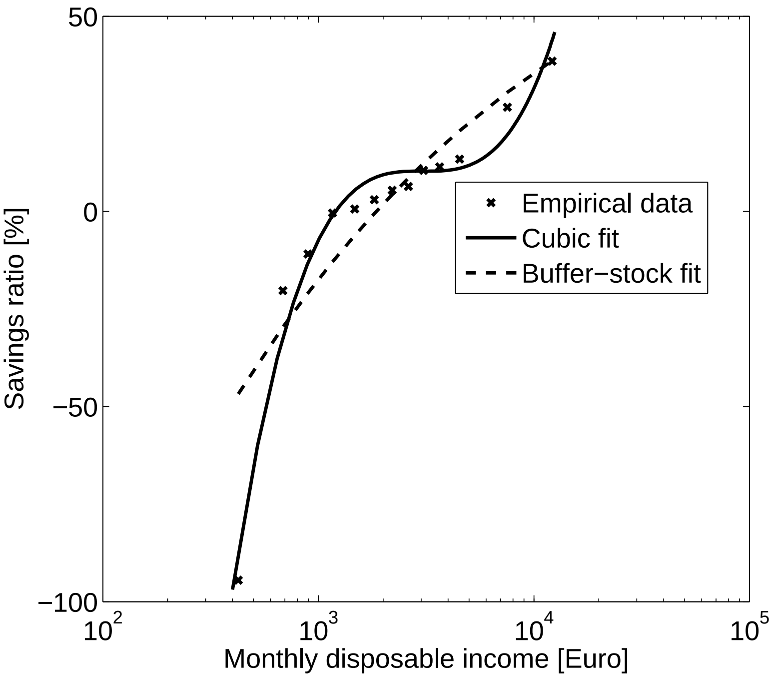


Figure 4.7.: Savings ratio [%] in Germany 2003 as a function of net income (log-scale, in Euro) (Data Source: Bach (2005))

Figure 4.7 presents the savings ratio of disposable income as a function of the level of income for Germany.[87] First of all, it is important to point out that the savings ratio increases with disposable income.[88] The curve exhibits a concave form at the lower end,

[85] In the concrete case, this would be: $Y_t + W_t = 2$.

[86] In this section, we also show that the curvature of the consumption function resulting from the presented optimization problem does not have to be concave, but rather depends on the relation between the degrees of risk aversion γ_c and γ_w.

[87] I am grateful to Christina Anselmann and Hagen Krämer for pointing to the data set.

[88] As intensively discussed in Dynan et al. (2004), this can be perfectly in line with the permanent income hypothesis of Friedman (1953), since an unexpected positive income shock does not lead to higher consumption but higher savings. They, however, are also able to show that the savings ratio increases with the level of permanent income (amongst others proxied by education) arguing that the wealthy save permanently more.

Dependent variable	$s = \frac{S}{Y}$
a_0	-6,594.286***
	(923.915)
a_1	2,469.373***
	(364.884)
a_2	-307.753***
	(47.578)
a_3	12.784 ***
	(2.049)
R^2	0.9696
adjusted R^2	0.9595

Table 4.1.: Polynomial regression (Standard errors in parenthesis, *** Significance at a 0.1% level)

but a convex shape at the higher income end. The empirical curve is well described by a polynomial function of third order. The regression is of the form:

$$\frac{S}{Y} \equiv \sum_{i=0}^{3} a_i \log(Y)^i, \tag{4.86}$$

for which Y is the monthly disposable income. The regression result is reported in table 4.1. The same exercise is executed in Fichtner et al. (2012) using higher frequency and updated data presenting a very good fit using a third order polynomial. Yet, the regression coefficients are not reported in this paper.

This result can be compared to the standard models. Following from the simple Keynesian consumption (equation 4.1) the following savings ratio can be computed:

$$\frac{S_t}{Y_t^d} = \frac{Y_t^d - C_t}{Y_t^d} = -\frac{\bar{c}}{Y_t^d} + 1 - c_y = -\frac{\bar{c}}{Y_t^d} + s, \tag{4.87}$$

for which we define the savings ratio $s = 1 - c_y$. This theory emphasizes the effect of the subsistence consumption leading to negative savings ratios for low-income households.[89] On the other hand the savings ratio curve is concave for which the richest households have a savings ratio amounting to s.[90] This function therefore cannot explain the behavior of very high-income households. As already put forward in the previous section this *Keynesian* approach with a subsistence level of consumption is close to the relative income hypothesis. The *relatively* high savings of the rich here in fact stem from too low savings of the poor. The contrasting approach in the spirit of Carroll (1998) with

[89] Eventually, for the limiting case the following result emerges: $\lim_{Y_t^d \to 0}\left(\frac{S_t}{Y_t^d}\right) = -\infty$.

[90] This is because the following relation holds: $\lim_{Y_t^d \to \infty}\left(\frac{S_t}{Y_t^d}\right) = s$. The concavity is ensured due to the relation $\frac{\partial^2 (S_t/Y_t^d)}{\partial (Y_t^d)^2} = -2\frac{c_0}{Y_t^{d3}} < 0$

wealth in the utility function as well as the *buffer stock theory of savings* (Deaton (1991), Carroll (1997)) attributes the high savings of the rich to an insurance motive.

We can generalize the approach of Carroll (1998) as presented in equation 4.85 in the following equation with $\theta < 1$:

$$C = (Y\bar{Y})^{\theta}. \tag{4.88}$$

Once again, there exists a certain threshold Y^* below which consumption exceeds income given by:[91]

$$Y^* = \bar{Y}^{\frac{\theta}{1-\theta}}. \tag{4.89}$$

Starting from this consumption function, the savings ratio is given as follows:

$$\frac{S}{Y} = s = \frac{Y-C}{Y} = 1 - \bar{Y}^{\theta}Y^{\theta-1}. \tag{4.90}$$

The function once again has a concave structure.[92] Eventually, the very rich save their total income.[93]

Taking the logs of consumption ratio ($c = 1 - s$ which in contrast to the savings ratio is strictly positive in the given dataset) leads to the following result:

$$\log(c) = \theta \log(\bar{Y}) + (\theta - 1)\log(Y). \tag{4.91}$$

We estimate this function using our data. In contrast to the ad-hoc polynomial regression this estimation has a theoretical underpinning in the model of Carroll (1998). The critical level of income going along with zero savings amounts to approximately 1,932 Euro per household. [94] The concavity coefficient yields $\theta = 0.74 < 1$. The regression results are summarized in the table 4.2.

Dependent variable	$\log(c)$
$\theta \log\left(\bar{Y}\right)$	1.97914*** (0.25063)
$(\theta - 1)\log(Y)$	-0.26157*** (0.03242)
R^2	0.8554
adjusted R^2	0.8423

Table 4.2.: Buffer stock OLS-regression (Standard errors in parenthesis, *** Significance at a 0.1% level)

The results of both fits and the empirical data are contrasted in figure 4.7 We can take the results of the regression for the savings ratio and compute the level of consumption as a function of disposable income Y.

[91]For the special case of $\theta = 0.5 \rightarrow Y^* = \bar{Y}$.

[92]Technically, this is because $\frac{\partial^2 S/Y}{\partial Y^2} = (1-\theta)(\theta-2)Y^{\theta-3}\bar{Y}^{\theta} < 0$ with $Y > 0$ and $\theta < 1$.

[93]Formally, this is because $\lim_{Y\rightarrow\infty}(S/Y) = 1$ for $\theta < 1$.

[94]This value can be computed from the regression as $\exp\left(\frac{1.9791}{0.2615}\right)$.

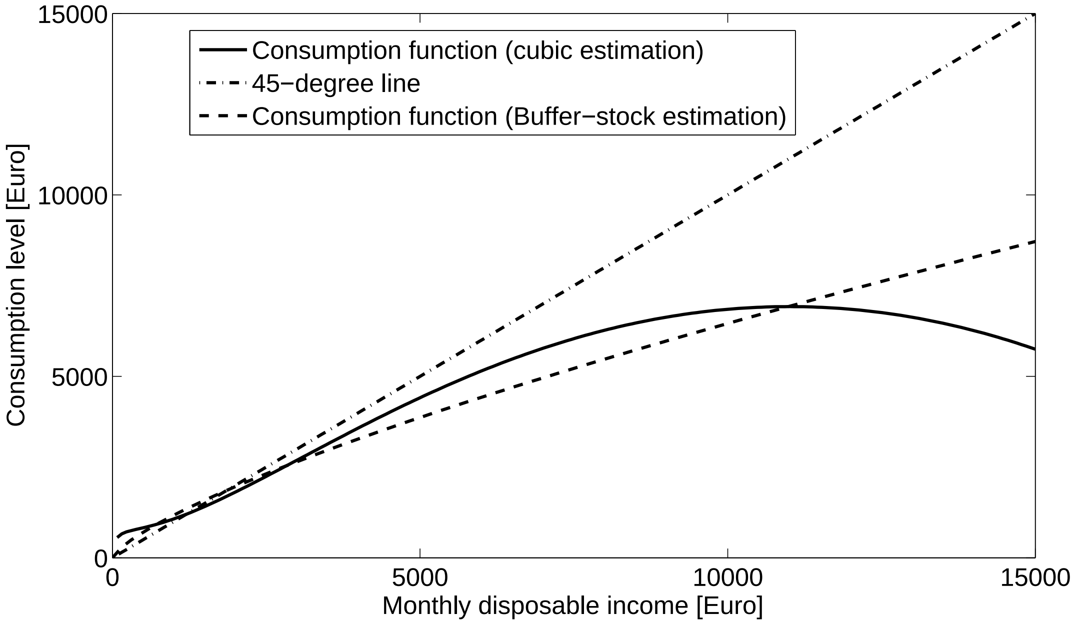


Figure 4.8.: Consumption function in Germany 2003 as a function of net income

Let us firstly discuss the polynomial specification. The result presented in figure 4.8 firstly shows that low-income households consume more than the current income resulting from their negative savings ratio. Even more interesting is the effect of a saturation level of consumption which can be tracked to a value of roughly 11,000 Euro per month amounting to an annual disposable income of 132,000 Euro.[95] This means that beyond a certain threshold - indicating a saturation level in consumption - individuals eventually consume *less* than other lower income individuals.[96] Or put differently, no saturation in the savings ratio corresponds to saturation in consumption for high-income levels.[97]

The latter result would be of far-reaching policy meaning. In fact, it would imply that taking away income from high-income persons (without redistributing it to low-income households) would increase aggregate demand. The result, however, should be treated with a pinch (or even a large amount) of salt. Firstly, the underlying data set only consists only of 13 observations is highly limited. Meanwhile, the convexity in the savings ratio driving this result was also reported in Fichtner et al. (2012) using a far richer dataset. On the other hand, the level of income going along with maximum

[95] The saturation behavior in consumption stems from the fact that the savings ratio exceed 100% respectively the average propensity to consume becomes negative.

[96] This level emerges for a savings ratio of approximately $\frac{S}{Y} \approx 40\% < 1$ since at this point the following condition holds: $C'(Y^*) = 0 \leftrightarrow 1 - f(Y^*) = Y^* \cdot f'(Y^*)$ with f being the estimated savings function.

[97] As discussed in Davis (1954) (Smith, 1776, p. 164f.) stated that "the desire of food is limited in every man by the narrow capacity of the human stomach...". However, he asserted that the desire for other goods (he names clothes and housing) are limitless. The empirical result seems to disconfirm the latter notion.

consumption roughly corresponds to the average income level of the top 1% (amounting to 12,143 Euro), which is our last available data point. On the other side, the data originating from the *Einkommens- und Verbraucherstichprobe* (EVS) are subject to right censorship bias by not including the top 0.1% due to privacy protection making it necessary to rely on regression to capture the *tip of the iceberg*. It might also be interesting to execute the presented exercise for different countries. As emphasized in Dynan et al. (2004), the fact that the savings ratio increases with income also implies that all consumption taxes (except on luxury goods) are regressive having a stronger impact on low-income individuals.

We can contrast this result with the consumption function resulting from the Buffer-stock regression. In contrast to the previous exercise, there is no absolute saturation in consumption. Note that by the construction of our regression - setting the log of the average propensity to consume as a dependent variable - this can never be the case. Nevertheless, the regression confirmed a value of $\theta < 1$ indicating a concave consumption function. In fact, the existence of a concave consumption function implies that a redistribution from high to low-income households increases aggregate consumption. For this case, aggregate consumption is maximized for an egalitarian income distribution. We will dwell more thoroughly on this issue in the later part of this work where we consider the role of redistribution.

4.5. A Short Summary on the Theory of Consumption and its Empirical Evidence

Before we begin with the modeling, we summarize the key findings of this section. There are two major attempts at modeling consumption behavior: the classic Keynesian, the heterodox, and the behavioral approach assume that consumption is mainly driven by current disposable income. While the standard Keynesian approach assumed linear relation between current income and consumption, recent empirical findings suggest a concave curve. On the other hand, (neo)classical theory in the form of the permanent income hypothesis and life-cycle hypothesis conjectures that consumption is roughly constant over time and driven by life-time wealth. Neither model perfectly explains reality, but the overview already revealed key determinants of consumption. Besides the level of current as well as future income and the level of accumulated wealth the main driver for consumption is the rate of time preference β. Behavioral models suggest that individuals show myopic behavior and thereby tend to overconsume in the present. On the other hand, since agents are risk averse and future income is uncertain, agents try to smooth consumption, whereas future growth prospects increase current consumption activity. As confirmed by empirical evidence the *substitution effect* outweighs the *income effect*, resulting in the the fact that a rise in interest rate has negative effect on current consumption. But since behavioral studies suggest hyperbolic discounting this effect is rather small. Consumers are scarcely characterized by rational behavior but use rules of thumb and especially adapt consumption behavior of social peer groups. A higher level of financial market development and lower collateralization values allow for higher consumption than current income and therefore for consumption smoothing. Households react differently to different sources of income. Heterodox theory, in particular, believes that the MPC out of wages is higher than out of profits. Theory confirmed by empirical data suggest that very high-income agents have a saturation level of consumption and have a tendency to *oversave*.

5. The Model

Neither a borrower
nor a lender be.

William Shakespeare (1603) - Hamlet
Act 1, scene 3, 75–77

In this section, we will present our model. Our aim is to represent and explain behavior in an economy with inequality - in particular for the situation of the recent financial crisis as detailed in section 2. The general model is presented in section 5.1. Besides making assumptions about individual behavior, we also have to model the distribution of quantities - in particular the distribution of labor income, which we assume to be exogenous (cf. section 5.2). Moreover, we will discuss the model based upon closed-form solutions. In particular, we shed light on the issue of financial (in)stability and its relation to inequality, which represents the core of this work (cf. section 5.4). Furthermore, we will also discuss the issue of wealth inequality and the functional distribution of income 5.5. This issue regained popularity and public attention beyond the scientific community after the publication of the work of Piketty (2014). We will discuss these issues based upon our model and compare it to the existing literature as already detailed in section 4. The analytical discussion already presents some very interesting facts and especially points to the underlying mechanisms without relying on seemingly arbitrary numerical calibrations. The true non-linear and complex nature can, however, not be captured by the analytical part. The complex dynamics are therefore considered in section 6.

5.1. Model Formulation

This section presents the general formulation of the model. The key actors in the model are the households. This means that all form of income is basically distributed amongst this group. The other (passive) agents are the foreign countries.[1] Households can engage in consumption of non-durables and in consumption of durables. Furthermore, they have the ability to accumulate debt or savings. *Non-durable* consumption is consumption in the more immediate sense, for instance clothing or food. *Durable* consumption can be thought of as cars, but in particular self-used real estate. These goods are not destroyed

[1] If, as will be presented in simulation results, savings are higher (lower) than investments (or more precisely dissavings), a current surplus (or deficit) emerges, implying a transfer from domestic households to foreigners (from foreigners to domestic households). Foreigners are not modeled explicitly but only fill the savings gap.

in the process of consumption and therefore can also be taken as collateral for borrowing activities. Besides the market for durables there is also a market for credit/savings.

The general timeline of the model is documented in figure 5.1. The simulation runs for $t = 1, \cdots, T$ periods and $i = 1, \cdots, N$ agents. In contrast to standard homogeneous agent models, the labor income of households therefore is not a vector Y_t but a matrix $Y_{i,t}$.[2] The wage income as well as the initial distribution of assets $q_{i,0}$ [3] is taken as exogenous. The details about the distribution functions are described in section 5.2. Based on their current wage income $Y_{i,t}$, their current asset holdings at current prices $q_{i,t}P_t$, and their indebtedness $D_{i,t}$ households make their decisions about consumption (normal and durables) as well as (dis)savings.[4] As soon as all N costumers have made their consumption plans, new prices in the market for durables and savings are established in order for those markets to clear. At the end of every simulation period aggregate quantities such as Gross Domestic Product (GDP) as well as distributional indices such as the Gini coefficient of net worth are calculated.

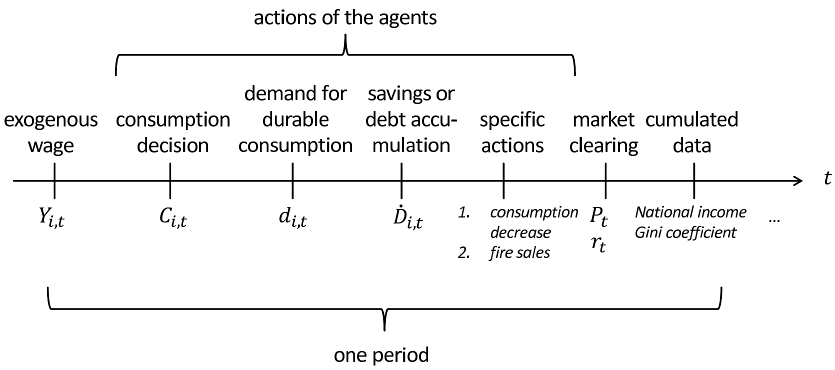


Figure 5.1.: Timeline of the events in the model

Households have a rank order in their decisions. First and foremost, they decide on how much to consume. The consumption function is therefore related to the result of section 4. Since we assume bounded rational agents rather than utility maximizing agents, this consumption function is based upon behavioral theory instead of a micro-foundation.[5] In its results it also resembles more the Keynesian approach rather than

[2]It is important to acknowledge that in our case $Y_{i,t}$ does not represent income out of capital, which is endogenously determined in the model by means of debt-credit interlinkages between agents.

[3]This is in the unit number of assets. To get wealth in the unit of prices, the current asset holding $q_{i,t}$ has to be multiplied with current real prices P_t.

[4]In a future model extension it would interesting to discuss the case for which households can also decide to default on their debt thereby making default a model endogenous variable rather than being exogenous like in many standard models dealing with the effect of default.

[5]However, as extensively discussed in section 4.3, for a specific underlying utility function a similar result can also be derived using a microfoundation.

the permanent income or life-cycle approach. Closely related to D'Orlando and Sanfilippo (2010) we assume a consumption function with the different input factors current income $Y_{i,t}$ and net worth $W_{i,t}$.[6] Both factors contribute to the size of consumption reflecting the effect of Mental Accounting (Kahneman and Tversky, 1979). As shown in section 4.2, they can also be derived from an intertemporal optimization problem. Furthermore, there is also a subsistence level of consumption $\bar{c}_t$ independent of individual characteristics. To achieve a concave consumption function - suggested by empirical evidence shown in section 4.4 - we introduce a variable $\varepsilon > 1$:[7]

$$C_{i,t} = \bar{c}_t + c_y Y_{i,t}^{\frac{1}{\varepsilon}} + c_w \cdot max\{0; W_{i,t}^{\frac{1}{\varepsilon}}\}. \tag{5.1}$$

The MPC is higher for flow income, but always lower than one ($1 > c_y > c_w > 0$). To introduce the effect that consumption only depends on the disposable income we can rewrite the equation as follows:

$$C_{i,t} = \bar{c}_t + c_y \cdot max\{0; (Y_{i,t} - r_t D_{i,t})^{\frac{1}{\varepsilon}}\} + c_w \cdot max\{0; W_{i,t}^{\frac{1}{\varepsilon}}\}. \tag{5.2}$$

The introduction of the maximum operator ensures that households with strong indebtedness, reducing both net worth and disposable income through the effect of interest payments, do not end up with the unrealistic result of negative consumption. To introduce a social effect of consumption in the sense of Duesenberry (1949) we assume that the subsistence level of consumption $\bar{c}_t$ representing a minimum level of consumption is a function of the general income distribution at time t:

$$\bar{c}_t = quantile_j(Y_{i,t}). \tag{5.3}$$

The quantile j describes the minimum level households consume. This level therefore crucially depends on the income distribution function. In industrial countries with higher mean income the subsistence level of consumption is therefore higher than in developing countries.[8] Apart from that, the subsistence level crucially depends on the distribution of income.[9]

As already discussed in section 1 houses present the most important durable consumption good. For sake of convenience we disregard the important factor of lumpiness in housing consumption - allowing only consumption of large bundles of houses - in our model. Instead we assume that households can buy any size of durables. If the effect of lumpiness would be included this would only furthermore strengthen our points. In particular, the employment of debt to finance durable consumption would be underscored.

[6] In their work D'Orlando and Sanfilippo (2010) also emphasize the role of future price expectations $P^e_{i,t+1}$ as a factor of influence. For the sake of clarity, we do not want, however, to include it.

[7] As presented in footnote 81 of section 4, in the case of single input consumption function the parameter can be interpreted as the inverse consumption elasticity.

[8] Industrial countries for example would consider a TV set or PC as a basic need.

[9] As will be presented in the following simulations, increased inequality, ceteris paribus, leads to a lower level of subsistence consumption for values of $j < 0.5$ (i.e. the subsistence level of consumption is below the median income). If $j > 0.5$ increased inequality increases the level of subsistence consumption.

The formation of the durable demand in our model is closely related to the demand in financial markets with heterogeneous agents as presented in recent surveys by Hommes and Wagener (2009) and Chiarella et al. (2009). This approach seems convenient for our model since it can produce boom/bubble behavior endogenously and thereby introduce financial fragility into the model.

We assume mean-variance portfolio optimization in a world with two assets: the durables represent the risky asset with expected return $E_t^i(p_{t+1} - p_t)$ [10] and savings as the risk-free asset with safe return of r_t.

The risky asset is the durable good as the the (expected) return is uncertain. In contrast to that, providing credit to another agent promises a certain return. Note that this return r_t is also subject to time variation, however, exhibits a lower volatility[11] than the return of the durable and - as will be confirmed in the numerical simulations in section 6 - is constant in a long-run equilibrium justifying the assumption of zero variance for the credits. The presence of margin requirements ($0 < m < 1$) furthermore ensures that the amount of credit is always lower than the pledgeable collateral making it *safer* than directly engaging in the durable market. To put this in the context of the recent financial crisis, the *safe* credits to low income households can be considered as Mortgage Backed Securities that in parts received top ratings and were bundled in liquid securities such as money market funds.

The demand for durables ($d_{i,t}$) is derived with mean-variance portfolio optimization (Hommes and Wagener, 2009) with constant relative risk aversion γ (CRRA).

The results can be derived following Chiarella and He (2001) and Chiarella et al. (2006). Wealth W_t evolves as follows:

$$W_{t+1} = W_t \cdot \omega \cdot (1 + R_t) + W_t(1 - \omega)(1 + r_t) = W_t(1 + r_t + \omega[R_t - r_t]), \tag{5.4}$$

where ω indicates the relative proportion invested in risky assets yielding a return R_t as opposed to risk free asset r_t. Using a second order Taylor approach assuming that risky assets variance is given by $Var(R) = \sigma_R^2$ [12] the following result can be derived:

$$\begin{aligned} E[U(W_{t+1})] &= E[U(W_t + W_t(\omega(R_t - r_t) + r_t))] \\ &= U(W_t) + (\omega W_t(E(R_t) - r_t) + r_t)U'(W_t) + \frac{1}{2}(W_t^2\omega^2\sigma_R^2)U''(W_t). \end{aligned} \tag{5.5}$$

This yields the following (approximated) first order condition:[13]

$$\omega = \frac{E(R_t) - r_t}{\sigma_R^2}\left(-\frac{U'(W_t)}{U''(W_t) \cdot W_t}\right). \tag{5.6}$$

[10] We use the nomenclature of small case letters for log-values and upper case letters for real values ($p_t = \log(P_t)$). In a first-order approximation the difference in log-values can be interpreted as the returns: $R_t \approx \ln(1 + R_t) = \log\left(\frac{P_{t+1}}{P_t}\right) = p_{t+1} - p_t$.

[11] The latter is formally ensured by the realistic assumption of a more liquid market for credits than for durables ($\mu_r < \mu$). Note also that we exclude the possibility of default and thereby no default premium is charged.

[12] The risky asset is assumed to have zero variance as well as zero covariance with the risk-free asset.

[13] This result is derived by setting $\frac{\partial(E[U(W_{t+1})])}{\partial\omega} \overset{!}{=} 0$.

In the case of CRRA utility function[14] this yields a constant relative risk aversion γ:

$$\gamma = -\frac{U''(W_t) \cdot W_t}{U'(W_t)}. \tag{5.7}$$

For heterogeneous households i the demand in unit of assets is given by the product of the ratio and the current net worth:[15]

$$d_{i,t} = \omega \cdot W_{i,t} = \frac{E(R_t) - r_t}{\gamma \cdot \sigma_R^2} \cdot W_{i,t}. \tag{5.8}$$

There are k different expectation formation strategies with an individual time-varying market weight w_t^k:

$$d_{i,t} = W_{i,t} \sum_k w_t^k \frac{E_t^k(p_{t+1} - p_t) - r_t}{\gamma \cdot \sigma_R^2} = W_{i,t} \cdot MPCD_t. \tag{5.9}$$

This implies that agents engage in durable consumption if, based upon their strategy, durables will yield higher returns than savings. Therefore, durable consumption is timed on market and agents are not in a hurry to buy a house but can also decide to stay in their rented flat. In time of low risk aversion γ [16] and low interest rate r_t demand for durable consumption increases. The latter not only captures the effect that for wealthy households risk-free credit provision is unattractive, but also that for lower income households debt-financing of housing is cheap. The CRRA approach, moreover, links the demand of durables to the net worth of the individual agent leading to a higher demand for durables by wealthy households.[17] Comparable to the MPC we can define a marginal propensity to consume durables $MPCD_t$.

Note that in contrast to the normal MPC the values of $MPCD_t$ are not constrained, vary in time, and can be even greater than one (levered purchases). We assume that there is a total MPCD in the market at each time t, which derives from the different trading strategies and their weights w_t^k. Consistent with the well-established literature on heterogeneous agent financial market models we assume three trading strategies, namely (1) fundamental trading, (2) chartism, and (3) noise trading (cp. e.g., Westerhoff (2008), De Grauwe and Grimaldi (2006)). The weights of the different strategies vary

[14] The function is given as follows: $U(W_t) = \frac{W_t^{1-\gamma}}{1-\gamma}$.

[15] In Fischer and Riedler (2014), we take a slightly different approach to target a specific ratio in the balance sheet $A_{t+1} = \frac{E(P_t)(q_{i,t}+d_{i,t})}{E(W_{i,t+1})}$ leading to $d_{i,t} = A_{t+1}\frac{W_{i,t}}{E(P_t)} - q_{i,t}$. Qualitatively, the result presented in this work, however, is similar, with demand being proportional to the product of current net worth and targeted ratio of risky assets. By disregarding the effect of already accumulated assets $q_{i,t}$, in the long-run we only have $\omega_t > A_t$ since in the long-run the price equals its fundamental value $E(P_t) = F_t = 1$.

[16] In the simulations, we take the simplifying assumption that the product of risk aversion and market volatility is constant and exogenous to avoid further non-linear feedback effects between markets and demand for assets. Furthermore, we normalize the variance $\sigma_R^2 = 1$.

[17] For a CARA utility function the demand for assets would be independent of the current wealth position. We explore this topic in more depth in section 5.5.2.

in time, depend on the attractiveness of a certain strategy A_t^k, and are derived using a Multinominal Logit Model as presented in Manski and McFadden (1981):

$$w_t^k = \frac{e^{\Gamma A_t^k}}{\sum_{i=1}^n e^{\Gamma A_t^k}}. \tag{5.10}$$

The application of the Multinominal Logit Model as a strategy switching model was introduced in Brock and Hommes (1997), whilst its application in the financial market context dates back to Brock and Hommes (1998). Due to the construction of the equation, the individual weights sum up to one. The parameter Γ presents a degree of rationality in choosing a strategy. In the case for which Γ equals zero, the weights of the groups are constant and amount to $1/n$. The other extreme case with Γ converging to infinity represents the case in which all individuals choose the optimal forecast. De Grauwe and Grimaldi (2006) therefore interpret this parameter as a model of the behavioral effect of Status Quos Bias as presented in Kahneman et al. (1991). This effect implies that individuals find it difficult to change a decision rule they used in the past. In a more general way, this parameter can also be considered as a value for bounded rationality in the sense of Simon (1955). Due to the limited resources of time and money, individuals use suboptimal rules.

The weight of a strategy w_t^k in the market is evaluated by its attractiveness A_t^k in a period t. This parameter is modeled in the following way:

$$A_t^k = \frac{E_t^k(p_t - p_{t-1}) - r_{t-1}}{\gamma} \cdot (p_t - p_{t-1} - r_{t-1}) + \Lambda A_{t-1}^k. \tag{5.11}$$

This parameter measures the precision of predictions made by a given strategy. Note that a strategy becomes more attractive in the case for which durables are bought when returns are higher than risk-free returns, or risky assets are sold when their return is lower than the return of the risk-free asset. One might also consider the attractiveness as a measurement of correct market timing. The parameter $0 < \Lambda < 1$ represents the memory of the agent. If it is set to zero, myopic traders who only value the very last success of the strategy are considered. The case in which $\Lambda = 1$ represents the case of perfect memory. This modeling approach enables us to investigate the effect of short-term focusing. The parameters Γ and Λ are therefore the key to measuring the degree of irrationality in markets.

Fundamental traders know the true fundamental log-value of an asset f_t and expect the prices to converge to it. Their expectations can therefore be modeled in the following way:

$$E_t^F(p_{t+1} - p_t) = \beta_F(f_t - p_t). \tag{5.12}$$

The parameter $\beta_F > 0$ measures the speed at which fundamentalist traders expect prices of durables to converge to their true underlying value. Their action contributes to higher market efficiency. We assume that the fundamental value is constant in time and normalized to one ($F_t = 1 \leftrightarrow \log(F_t) = f_t = 0$).

Chartists on the other hand do not consider fundamental prices, but derive order signals from past prices. Chartism is especially important for short-term forecast horizons and is often also referred to as Technical Trading, since it derives its trading signals from clear rules that can be automated. For this reason it is also very easy to implement these rules in a ABM. We use a simple trend-following strategy:

$$E_t^C(p_{t+1} - p_t) = \beta_C(p_t - p_{t-1}). \tag{5.13}$$

The important feature of this rule is that it shows Momentum behavior by generating buying signals in case of increasing prices and selling signals in case of decreasing prices. The parameter $\beta_C > 0$ measures the aggressiveness with which the chartist traders take positions in the market.

The last trading strategy is *noise trading*. According to Black (1986), noise traders trade on noise as if it were information. Noise trading d_t^{noise} is modeled as an independent and identically distributed (i.i.d.) process with mean zero and variance σ_i^2. This is consistent with the consideration of Shleifer (2000) that noise should, on mean, cancel itself out. The noise trading in our context can also account for durable consumption that is not related to market timing but to exogenous effects such as buying a house when a child is about to be born. In contrast to the chartists and fundamentalist strategy, we assume that the market weight of noise trading does not change over time. Therefore, we can summarize the marginal propensity to consume durables as follows:

$$\begin{aligned} MPCD_t &= \sum_k w_t^k \frac{E_t^k(p_{t+1} - p_t) - r_t}{\gamma} \\ &= \frac{1}{\gamma}\left[w_t^C E_t^C(p_{t+1} - p_t) + w_t^F E_t^F(p_{t+1} - p_t) + d_t^{noise} - r_t\right]. \end{aligned} \tag{5.14}$$

Basically this assumes that, even though there are N agents, agents switch between only three investment strategies, and agents all follow the same strategy mixture in the market. This is surely a strong simplification mixing up real heterogeneity (concerning income and wealth) and weak heterogeneity (concerning durable consumption strategies) is chosen to keep the already complicated model analytically tractable.

If comparing the modeling of the durable and non-durable consumption, we follow two different paradigms. While the non-durable consumption is modeled as a rule-of-thumb-strategy[18], non-durable consumption follows a micro-founded approach with heterogeneous expectations. This is arguable since households, when deciding to buy durables such as houses, cars, or even stock shares, deliberate more deeply. This is especially the case since these goods entail high monetary costs requiring to lever the purchase. The debt aversion of households therefore also contributes to a more rational and sober decision. Non-durable goods (especially food and clothing) are goods of every day consumption and the process of consuming them is more a process of habit without deeper cognitive involvement. One potential shortcoming of our model is surely that we introduce relative consumption only for durables. As already discussed, non-durable goods (especially houses and cars) are highly positional and therefore subject to relative consumption. This effect is not accounted for in the model.

[18] As already put forward in section 4.2, this *Keynesian* approach can, however, also be microfounded.

Each households' activity can be summarized in a cash-flow equation, for which negative terms on the right hand side of the equation represent cash-outflow and positive cash-inflow. In this case D_t represents the stock level of debt at time t:

$$D_{i,t+1} - D_{i,t} = -Y_{i,t} + C_{i,t} + P_t d_{i,t} + r_t D_{i,t}. \tag{5.15}$$

This is similar to the budget constraint, even though this budget does not really *constrain* since agents have the opportunity to increase debt ($D_{i,t+1} - D_{i,t} \equiv \dot{D}_t > 0$). In fact, this is the *instantaneous* budget constraint, rather than the *intertemporal* budget constraint as emphasized in section 4.1 (cf. equation 4.3). A negative term for dissavings ($\dot{D}_t < 0$) represents savings and thereby leads to the classic relation that disposable income ($Y_{i,t} - r_t D_{i,t}$) can be split into consumption and savings. Since we assume that households are the only actors in our model, the savings have to be redistributed amongst them. This is certainly a strong assumption since a lot of actual savings go to firms reinvesting it. Note that we impose a negative definition of debt. Negative debt, however, can be interpreted as capital $K_{i,t} = -D_{i,t} > 0$. If we consider $Y_{i,t}$ as labor income, capital income $r_t K_{i,t}$ can only be generated by supplying credit to other private households due to the lack of firms in the model. It is, moreover, important to point out that there are no direct links between agents in a network sense, but all debts and assets are treated as a common pool. This is eventually a good representation of the structured products seen in the recent financial crisis for which income streams resulting from mortgages were pooled in common marketable securities which once again were tranched into pieces and sold to other individuals holding the mirror position. Thus - and in contrast to individual contracts e.g. persisting in relationship banking - idiosyncratic risk is hedged away. Nevertheless, all agents are exposed to the same common risk. The presence of this abstract relationship also allows us to disregard financial institutions as autonomous actors in our model as they *originate and redistribute* rather than *originate and hold*.[19]

Starting from the flow equation, we can also derive the stock equation of wealth. Since by definition $D_{i,t} > 0$ represents debt, $D_{i,t} < 0$ can be thought of as accumulated savings, also dividing the households into net debtors and net lenders (see figure 5.2).

The net worth $W_{i,t}$ is defined as follows:

$$W_{i,t} = P_t q_{i,t} - D_{i,t}. \tag{5.16}$$

By requiring $q_{i,t} \geq -d_{i,t}$, we can model a short-sale constraint linking flows to stocks.

On the other side, we impose a collateral constraint in the sense of Geanakoplos (2003) and Kiyotaki and Moore (1997) that constrains the ability of households to increase debt. The maximum debt level depends on the amount of durable consumption good $q_{i,t}$ evaluated at its current price P_t and a required equity ratio m:

$$(1 - m) P_t q_{i,t} \geq D_{i,t+1}. \tag{5.17}$$

The parameter m can also be thought of as a haircut on securities or a margin and gives a maximum ratio of net worth to securities. This parameter depends on the risk-sharing

[19]By not having *skin in the game*, this, however, imposes a severe incentive problem when screening credits.

Net lender	
$P_t\, q_{i,t}$	$W_{i,t}$
$-D_{i,t}$	

Net debtor	
$P_t\, q_{i,t}$	$W_{i,t}$
	$D_{i,t}$

Figure 5.2.: Balance sheets for net lenders and debtors

ability of financial markets. Sophisticated and evolved financial markets are expected to exhibit low values for m. In the model the parameter is given exogenously and not the result of the optimization of a financier. In section 5.4.1, we, however, discuss factors that should be accounted for by an optimizing entity when setting the optimal level of equity requirement. Stock and flow can now be linked as follows:

$$(1-m)P_t q_{i,t} \geq -Y_{i,t} + C_{i,t} + P_t d_{i,t} + (1+r_t)D_{i,t}. \tag{5.18}$$

This condition is more binding in depressed markets for durables (low prices P_t) and for households with low level of wealth $q_{i,t}$.

Another important insight from the previous equation is that lenders only decide on lending based upon the presented collateral. The use of debt therefore does not matter. Lenders in our model simply do not care whether the debt is used for buying durables, net worth decreasing consumption of non-durables or even to pay interest on past debt. A microfounded view of this modeling approach could be that there is non-observability of the use of debt on the side of lenders. This is especially the case - as already emphasized - if there is no direct relation between lenders and debtors (e.g. in the form of house bank relation), but debt securities are traded in an anonymous market (a market-based system) as it was the case with Mortgage-Backed-Securities in the current crisis.

After households have formed their plans for consumption and consumption of durables, they try to execute them. If necessary, they therefore employ debt. Problem arise when these households are now collateral constrained and would like to borrow up more than they are allowed to do. As presented in figure 5.1, the simulation will present different options for the collateral constrained households and investigate their results on financial stability and the whole macroeconomy.

Firstly, they can reduce their consumption. We label this the *Austerity*-case. These households are forced to save by credit market conditions. In the case in which prices for collateral P_t drop significantly or households have already piled up high amounts of debts, this might force some household to consume less than subsistence level of consumption. If we further do not impose a non-negativity condition on consumption this might eventually lead to unrealistic result that households engage in negative consumption.

A second option for these households would be selling their durables and using the proceeds for consumption. This is not as painful as the first possibility, but only a short-term solution. By selling their assets they reduce their stock of collateral and eventually aggravate their problem. Furthermore the *fire sales* might eventually reduce the prices of assets and thereby worsen not only their very own situation, but also impose a negative externality on other agents in reducing their ability to borrow up.[20]

This *rule of thumb* based behavior can be nicely implemented in a software-code since it follows the *if-then-else* logic of a computer algorithm. The details will be presented in the simulation section 6.

At the very ending of each period prices change according to market conditions. We follow the rationale of standard financial market ABMs and model the price reaction with a market maker mechanism (e.g., Chiarella et al. (2006), Westerhoff (2008)). Even though this approach is still very simplified, it comes closer to price determination in actual markets. The key idea here is that an institution named market maker takes an offsetting long or short position to assure that excess demand in period t equals zero. In the next period, the Market Maker announces a new log-price p_{t+1} to reduce excess demand:

$$p_{t+1} = p_t + \frac{\mu}{N}\sum_{i=1}^{N} d_{i,t} = p_t + \frac{\mu}{N}\sum_{i=1}^{N} MPCD_t W_{i,t}. \tag{5.19}$$

The parameter $\frac{\mu}{N}$ can be thought of as the illiquidity of the market.[21] In illiquid times prices show high volatility. The formulation using log-prices p_t is made in order to avoid negative prices.[22]

The clearing of the debt/savings market is made in a similar way:

$$r_{t+1} = r_t \cdot \exp\left(\frac{\mu_r}{N}\sum_{i=1}^{N} \dot{D}_{i,t}\right). \tag{5.20}$$

High demand for credit leads to the fact that interest rates increase and vice versa. We assume that these markets are more liquid and therefore impose $\mu_r < \mu$.

At the end of each simulation period aggregate results comparable to results of standard homogeneous macroeconomic models can be calculated. The national income Y_t can be thought of the as the income of all N agents within a certain period t:[23]

$$Y_t = \sum_{i=1}^{N} Y_{i,t} = C_t + P_t d_t - \dot{D}_t + r_t D_t. \tag{5.21}$$

[20] Another important option which, however, will not be considered in this model is the option of default - being the most drastic option. The default on debt by low-income households poses a negative externality on the high-income agents holding the mirror position.

[21] Note that we normalize the illiquidity by means of the number of agents N. This is done in order to make the price reaction independent of the number of market participants N and thereby the model results independent of the number of assumed agents.

[22] The equation can also be reformulated as follows: $P_{t+1} = P_t \cdot \exp\left(\frac{\mu}{N}\sum_{i=1}^{N} MPCD_t W_{i,t}\right)$.

[23] This results from the flow equation 5.15.

The national income (which is exogenously determined by the growth of wages and which in turn depends on the labor productivity) can be decomposed as follows: the first two terms capture consumption for non-durables and durables. The third term represents change in excess savings ($-\dot{D}_t$) that can be thought of as a current account surplus.[24] In a society which has a balanced current accounts at any time (i.e. demand for savings always equals its supply and aggregate debt is zero $\sum_{i=1}^{N} D_{i,t} = 0$) the least term is zero. In a country that accumulated deficits in the past($D_t > 0$), the last term presents interest payments that are a transfer from domestic debtors to foreign lenders and vice versa for a country with a history of a surplus. If we consider Y_t as income out of labor, in the aggregate a positive share of capital income can only be sustained if we argue in a surplus economy $K_t = -D_t > 0$. Nevertheless, even in a balanced economy ($D_t = 0$), some agents receive capital income ($r_t K_{i,t} = -r_t D_{i,t} > 0$) while others pay interest on debt ($r_t D_{i,t} > 0$).

Usually in models with small open economies, countries pay a world rate of interest that is exogenously given. In the long-run, the current account is balanced by means of the alignment of the flexible exchange rate. Concretely, a current account surplus leads to appreciation of the exchange rate deteriorating the terms of trade, which rebalances the current account. Vice versa, current account deficit countries achieve balanced accounts by means of currency devaluation. In our model, we consider a large economy that eventually influences the world interest rate by its supply and demand of savings. The argument of the flexible exchange rate also does not hold for the cases we aim to model (the pair: USA and China, the European Monetary Union), in which the exchange rates are *de facto* respectively even *de jure* fixed. In a regime with fixed exchange rates the devaluation of the currency can be stopped by the intervention of the central bank. Concretely, the central bank can sell foreign reserves in order to counteract the devaluation pressure of the currency persistent in a current account deficit environment. This activity, however, is limited by means of their foreign reserves. In contrast to that, a currency appreciation can always be impeded by the central bank by issuing new domestic currency resulting from their monopoly of issuing legal tender. This monetary expansion, however, also comes at the cost of national inflation. The latter is especially problematic if the country is indebted in foreign currency. As will be discussed more thoroughly in section 5.4.1, the balancing of the current account is achieved by means of the interest rate mechanism and depends crucially on the existence of consumption out of net worth ($c_w > 0$).

In the case of a deficit country the latter terms reduce the amount of aggregate demand ($AD = C_t + P_t d_t$). As put forward in Biggs et al. (2009) for the case of a steady state with no technological progress ($\dot{Y} = 0$) growth in aggregate demand ($\dot{AD} > 0$) can only be achieved by an increase in the growth rate of debt ($\ddot{D} > 0$) which is accompanied by further capital account deterioration. Meanwhile for $\dot{Y} = 0$ even with growing debt ($\dot{D} > 0$), a decrease in the growth rate ($\ddot{D} < 0$) can already lead to a decrease in

[24] This can be attributed to the fact that in this simple model we only consider private households and no firms. The abstraction from investment ($I = 0$) thereby simplifies the flow equation to $S_t \equiv -\dot{D}_t = EX_t - IM_t$ with EX being exports and IM imports.

aggregate demand. This surprising result concerning the second-order derivative of debt is due to the fact that debt is a stock, but GDP a flow quantity.

In contrast to standard homogeneous agent models we can also calculate distributional measurements of flow (e.g. income, consumption) and stock (e.g. net worth, asset holdings) sizes in different forms (coefficient of variation, Gini coefficient). This will be presented more focused in the following section.

If one critically reviews this model, it is important to state that the model emphasizes particular activities in the economy and can even be considered a partial model of the economy. Since we focus on the role of private households, the model can be considered a *Keynesian* model.[25] In fact, we do not model private firms, which, however, are finally owned by (other) private households and therefore do not require explicit modeling in a heterogeneous agent economy. The government sector is also not considered explicitly. We assume that the government satisfies a balanced budget. We dwell more thoroughly on this issue in section 7.2, where we assume that the government income out of taxes (mostly imposed on high-income households) is equal to the transfers to low-income households and thereby zero-sum. If the budget of the government was not balanced the difference between aggregate supply and demand for debt $\dot{D}_t \neq 0$ conceived as current account imbalances could also considered as an unbalanced government budget.[26]

Moreover, we do not consider labor markets explicitly and assume that the labor - and in particular its distribution - is exogenous. Furthermore, agents supply labor inelastically. We also consider markets for durable goods only from a demand side perspective. Essentially, this implies a vertical Phillips-curve in the inflation-output domain representing a constant aggregate supply (AS) curve as emphasized in the RBC-literature. The latter also implies that - at least for consumption decisions - money is only a veil and has no real effects.

In the following, we dig deeper into the outcomes of the model presented so far. Closed-form solutions of models are desirable since they can deliver insights about the existence and stability of equilibria as well as the models dynamics. Rather than (seemingly arbitrary) tunings, global conditions for variables can be derived allowing for economically meaningful interpretation. However, due to the complexity of the non-linear interactions of this model closed-form solutions beyond a certain level are virtually impossible making it necessary to rely on simulations as provided in section 6.

Firstly, section 5.2 discusses the role of an exogenous given distribution function on measurements of inequality. Afterwards, we give a general overview of the workings of the model focusing on the different *classes* of households. We especially consider the important issue of financial stability and its drivers in section 5.4. We distinguish between two forms of stability - stability of debt (section 5.4.1) and durable prices 5.4.2 as well as

[25] Some other modelers (e.g. Dosi et al. (2010)) also emphasize the role of the entrepreneur and label the latter feature as *Schumpeterian*.

[26] More formally, if $EX_t - IM_t \equiv 0$, we have $\dot{D}_t = T_t - G_t$, for which G_t is government consumption (e.g. in the form of transfers) and T_t is aggregate tax income. Excess private savings $-\dot{D}_t < 0$ in a closed economy ($EX_t - IM_t = 0$) thereby have to go along with excess government consumption ($G_t > T_t$) buying the produced goods and vice versa. In fact, the part of equalizing the excess supply or demand of savings is shifted from foreign countries to the government.

their interaction (section 5.4.3). Moreover, we derive some important general macroeconomic properties. While we assume the distribution of labor income to be exogenously determined the functional distribution and the distribution of net worth are determined endogenously. The relation between in personal and the functional distribution of income are considered in section 5.5.1, whereas the section 5.5.2 analyses the factors driving the distribution of net worth and the necessary conditions for convergence.

5.2. Relation between Income Distribution and Measurements of Inequality

Income inequality in our model is treated as an exogenous process. The measuring of inequality is far from trivial since it requires to making a statement about a distribution in the form of an easy to interpret scalar. Therefore, in this section we present our modeling approach for the initial conditions and relate it to some common indices of inequality and also discuss the implications of this approach.

We assume that prices in the beginning ($t = 0$) equal fundamentals ($P_0 = F \equiv 1$). Furthermore, we presume that all agents are born without debt ($D_{i=1;\cdots;N,0} = 0$). Also note that our model implicitly assumes infinite-living agents or at least clans that transfer intergenerationally.[27] We assume that agents are not born bare-handed but are endowed with an initial stock of assets as a heritage of their parent generation. This amount $q_{i,0}$ is unequally distributed, implying wealth inequality.

We assume that both the initial wealth $q_{i,0}$ as well as the initial wage $Y_{i,0}$ are distributed log-normally.[28] This assumption has two major advantages. First, it accounts for the non-negativity condition.[29] Furthermore, it represents one of the most simple forms of positive skewed-distributions reflecting the effect of inequality. Moreover, the log-normal distribution is also easy to justify from a theoretical point of view. In a world in which labor is paid according to its marginal product, to make a statement about the distribution of incomes we have to make a statement about the distribution of skills or abilities. If we assume that general ability is a function of several complementary[30] input factors $e_{j,i}$ such as language, mathematical, and social skills, $A_i = \prod_{j=1}^{k} e_{j,i}$ in which all $e_{j,i}$ are i.i.d. Using the central limit theorem, it is easy to verify that $log(A)$ is normally

[27] Yet, our model does not take into account the age structure of agents. In fact, we implicitly assume that all agents are of the same age. As emphasized in Lane and Milesi-Ferretti (2012) low savings emerge for a high amount of young agents (due to low-incomes at the beginning of a career) as well as high amount of retirees drawing on their savings. In this case it is also important to note on the different age structure in growing economies and developed economies. These important factors are left to further research. Hence, the age distribution also has an important impact on the distribution of wealth. This aspect is emphasized in the OLG-models.

[28] This approach was proposed e.g. in Cowell (2000).

[29] This means $Y_{i,0} \geq 0$ for $Y_{i,0} \sim L(\mu, \sigma_y^2)$. The same result holds for $q_{i,0}$.

[30] The notion of complementarity requires that no factor can be completely substituted. If, for example, a particular agent posses a large amount of programming skills but does not have any social skills, he ends up with zero (aggregate) ability.

distributed and A and thereby wages Y are log-normally distributed (Aitchison and Brown, 1957). We assume the following distribution for the initial labor income $Y_{i,0}$:[31]

$$Y_{i,0} \sim L(\mu, \sigma_y^2) = L(\log(\bar{Y}), \sigma_y^2). \tag{5.22}$$

The median of this distribution equals $\exp \mu = \bar{Y}$. The first moment (mean), however, is larger than the median and a function of the parameter σ_y indicating a right-skewed distribution (Evans et al., 2000):

$$E(Y) = \bar{Y} \exp\left(\frac{1}{2}\sigma_y^2\right). \tag{5.23}$$

The ratio of the two indices is larger than one and completely controlled by the parameter σ_y:

$$\frac{E(Y)}{\bar{Y}} = \exp\left(\frac{1}{2}\sigma_y^2\right) > 1. \tag{5.24}$$

The second moment (variance) often taken as measurement of inequality can be described by the following equation:

$$Var(Y) = \bar{Y}^2(\exp(\sigma_y^2))(\exp(\sigma_y^2) - 1). \tag{5.25}$$

The variance as an index for inequality therefore increases with higher values of σ_y^2. As presented in Atkinson (1970) the coefficient of variation (CoV) which is the ratio of standard deviation and mean of a distribution is another frequently used measurement of inequality also being scale-invariant - i.e. independent of the median $\bar{Y}$. In the log-normal case this is also only a function of σ_y:

$$CoV = \frac{\sqrt{Var(Y)}}{E(Y)} = \sqrt{\exp(\sigma_y^2) - 1}. \tag{5.26}$$

The same holds true for the third moment (skewness) which is independent of the median income:[32]

$$Skew(q) = (\exp(\sigma_y^2) + 2)\sqrt{(\exp(\sigma_y^2) - 1)}. \tag{5.27}$$

We can also conduct a study of the quantiles that will be important since we model the subsistence level of consumption to be a function of the underlying income distribution. Figure 5.3 shows the evolution of the quantiles as a function of the distribution parameter σ_y for the case of $\bar{Y} = 1$. Consistent with the calculation the median equals $\bar{Y} = 1$, in which the non-monotonous behavior can be attributed to the finite number of 10,000 draws from the function. While quantiles n_j with $j > 0.5$ increase with increasing σ_y the opposite is the case for $j < 0.5$.[33] The important effect on the income distribution will

[31] The same assumptions also apply for the initial distribution of assets $q_{i,0}$.

[32] Similar behavior is visible for the excess kurtosis indicating fat tail behavior for positive values for σ_y^2: $Exc\,Kurt(Y) = \exp(\sigma_y^2)^4 + 2\exp(\sigma_y^2)^3 + 3\exp(\sigma_y^2) - 6$.

[33] Formally the quantiles of the log-normal distribution l_j can be related to the quantiles of the standard normal distribution n_j as follows: $l_j = \bar{Y}\exp(n_j \cdot \sigma_y)$. Therefore, as presented in figure 5.3 they are an exponential function of the standard deviation in which for $j < 0.5$ $n_j < 0$.

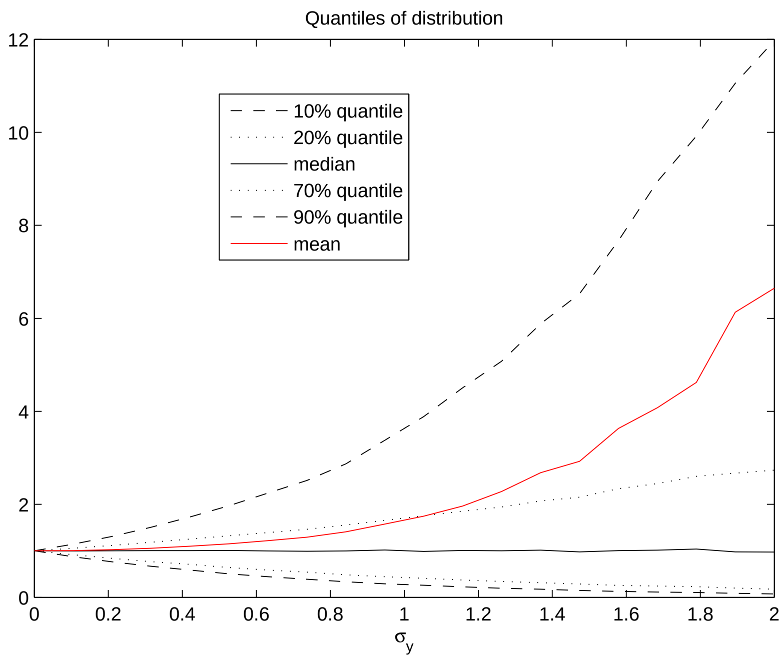


Figure 5.3.: Quantiles of the log distribution for different values of σ_y

be discussed in the following sections. Apart from that it is important to point out that the mean value grows following an exponential process (as shown in the theory) and due to the positive skewness of the distribution function not only exceeds the median but even higher quantiles.

The ratio of quantiles are also frequently used measurements of inequality Cowell (2000). They can be directly related to the single variable σ_y. Consider e.g. the frequent case of the 90% to 10% quantile:

$$\frac{Y_{90}}{Y_{10}} = \exp([n_{0.9} - n_{0.1}]\sigma_y). \tag{5.28}$$

As presented in figure 2.5 another widely used index for inequality is the variance of log-wages. Since we assume that wages $Y_{i,t}$ follow a log-normal distribution, log-wages follow a normal distribution. The standard deviation of this distribution is directly given by the variable σ_y creating a direct link to this measurement of inequality.

There is also a strong relation between the variable σ_y^2 and the Gini coefficient, which is probably the most important and commonly used index of inequality. It equals twice the area between the equal distribution and the Lorenz curve. It ranges between 0

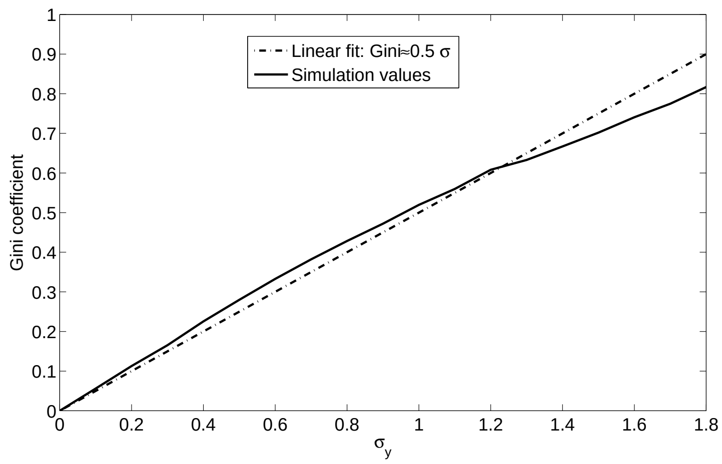


Figure 5.4.: Relation between standard deviation of log distribution σ_y and Gini coefficient for 5,000 agents

(complete equality) and 1 (maximum inequality) (Dagum, 2008).[34] [35] Figure 5.4 shows simulation results for the Gini coefficient for different values of σ_y.[36] It is clearly visible that the Gini coefficient increases with σ_y and for low values of σ_y can furthermore be well approximated by the linear fit $Gini = 0.5 \cdot \sigma_y$. This result, however, does not hold for high levels of σ_y since the Gini with an upper bound of 1 enters a saturation area. If we take a first-order Taylor approximation for small values of σ_y the relation can even be approximated with $Gini = \frac{1}{\sqrt{\pi}}\sigma_y$.[37]

The presented computations showed that inequality - as measured by different proxies - is completely determined by the single value σ_y. If all income distribution follows a log-normal distribution this allows us to unambiguously rank distributions as the Lorenz-curves do not cross (Cowell, 2000). This, however, does not have to be the case in reality. If two countries e.g. exhibit the identical Gini coefficient, their concrete distributions can be completely different if the log-normal assumption does not hold. In particular, the share of different quantiles might be completely different, yet - by chance - lead to the same aggregate quantity (here: the Gini coefficient). The reader therefore should

[34]The Gini coefficient is given by the following integrals (Atkinson, 1970): $Gini = \frac{1}{2E(Y)} \int_0^{Y_{max}} [YF(Y) - E(Y)\Phi(Y)] f(Y) dY$. In this case, Y_{max} is the highest value of the distribution and $E(Y)$ equals the mean. $f(Y)$ represents the Probability Density Function (PDF) and $F(Y)$ the Cumulated Probability Distribution Function (CPDF): $F(x) = \int_0^x f(Y)dY$. The Lorenz curve is given as follows: $\Phi(x) = \frac{1}{E(Y)} \int_0^x Yf(Y)dY$.

[35]Note that the case for the income (or wealth) is uniform (implying that there is an equal number of people in any income category), the Gini coefficient is $\frac{1}{3}$.

[36]In this simulation, we assumed 5,000 agents. With a high number of agents the law of large numbers holds and the presented relation emerges. The result is independent of the median $\bar{q}$. As already presented, higher median values $\bar{q}$ only increase the variance of the distribution requiring for a higher number of draws to make the law of large numbers apply.

[37]The proof for this result is presented in the appendix A.4.

keep in mind this caveat when interpreting empirical data, but also acknowledge that this shortcoming does not apply to our abstract model world.

All in all, the log-normal distribution has some very intriguing properties. On the other hand, it fails to mimic the behavior of the very top income distribution. As e.g. put forward by Diamond and Saez (2011), top incomes are better described by a Pareto-distribution, for which the probability density function is given by:

$$f(Y) \sim Y^{-(a+1)}, \tag{5.29}$$

with $a > 1$ being the Power-law exponent, for which smaller values indicate fatter tails and thereby more unequal distributions. Empirical literature confirms a value of approx. $a = 2$ for the USA (Jones and Kim, 2014).[38]

There are also some formal models that are able to capture the Pareto-distribution. Formally a Pareto-distribution emerges for two conditions holding (Jones and Kim, 2014): (1) the underlying variable governing the distribution of income must be exponentially distributed and (2) the process linking the underlying variable and the level of income is an exponential growth process. A natural candidate for the underlying variable is age whose distribution is well-described by an exponential distribution (Blanchard, 1985). As widely documented in the empirical literature (Mincer, 1958) income grows with age, resulting in the fact that a Pareto-distribution emerges. In order that this process does not explode in the long-run (implying a Gini of 1) Jones and Kim (2014), furthermore, introduce *creative destruction*. Due to the latter mechanism successful employees are displaced once a new variety emerges (e.g. a worker is replaced by a robot). The literature frequently also emphasizes the *span of control* as a mechanism contributing to a Pareto-distribution of incomes especially allowing top-managers to scale their activities. Gabaix and Landier (2008) shows that the distribution of firm-sizes can also explain the cross-country heterogeneity in income distribution (e.g. CEO-compensation in Germany and the USA).

The problem with this type of distribution is that it only captures the right tail of the total distribution.[39] The reader should therefore keep in mind that by the assumption of the log-normal distribution we essentially underestimate the role of very top incomes on the overall dynamics. Future research could take the model at hand and replace the pure log-normal distribution with log-normal distribution with a fat tail to study the aggregate impacts.

We assume that wage income $Y_{i,t}$ evolves over time. The future income is a function of current income, implying that there is a strong autocorrelation in income mimicking that those with a better education get better jobs, get better career opportunities, leading to even better jobs in the future. If we assume a uniform wage growth-rate for the whole population the Gini coefficient of wage remains unchanged. We therefore present

[38]The Pareto- distribution also has the nice property that the Gini is given as $\frac{1}{2a-1}$. The share of quantile x [in percent] as emphasized e.g. in the work of Atkinson et al. (2011) can be computed by $\left(\frac{x}{100}\right)^{1-\frac{1}{a}}$ (Jones and Kim, 2014).

[39]For the wealth distribution it is sometimes argued that it follows a double Pareto-distribution in which both tails are given by Power-law behavior (Benhabib et al., 2014).

a formulation in which the growth rate of income depends on the size of income itself resulting in higher inequality:

$$Y_{i,t+1} = (1 + g_{ineq} \log(Y_{i,t})) \cdot Y_{i,t}. \tag{5.30}$$

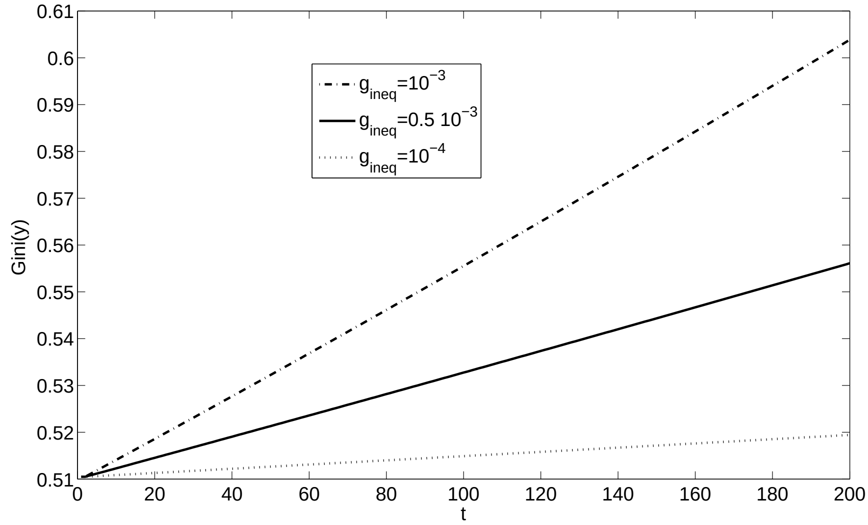


Figure 5.5.: Level of Gini coefficient over time for different values of inequality growth g_{ineq}

Growing inequality is related to the fact that high-incomes have high income-growth rates, while lower-income households might have low or even negative income growth rates as documented in empirical results (Eckstein and Nagypál, 2004).[40] The newly introduced parameter g_{ineq} scales the complete model. In the presented case, the individual growth rate is given as follows:

$$g_{i,t} = \frac{Y_{i,t+1} - Y_{i,t}}{Y_{i,t}} = g_{ineq} \cdot \log(Y_{i,t}). \tag{5.31}$$

As shown in Evans et al. (2000) applying the log-operator to values of a log-normal distribution results in a normal distributed function. Following *Gibrat's Law* a vector whose growth rate obeys a normal distribution results in a log-normally distributed variable (Sutton, 1997).[41] Thereby, the assumption of log-normality is conserved in the growing inequality scenario. Moreover, the Gini-ratio can be approximated as follows:[42]

$$Gini_T = (1 + g_{ineq})^T Gini_0. \tag{5.32}$$

[40]The latter is formally the case for $Y_{i,t} < 1$.

[41]This result is easy to verify. If we assume $x_t - x_{t-1} = g_t x_{t-1}$ for small normally distributed values of g_t in a first-order approximation the following result holds: $\log(x_t) = \log(x_0) + g_1 + g_2 + \cdots + g_t$, implying a log-normal distribution for x_t (Sutton, 1997).

[42]The proof for this is presented in appendix A.5.

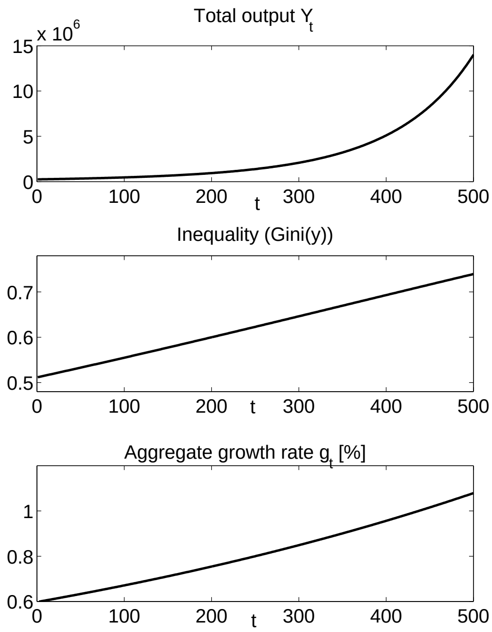


Figure 5.6.: Aggregate output Y_t, inequality $Gini(Y_t)$, and aggregate growth g_t

For low values of $g_{ineq} \cdot T$ results can be further simplified as follows:

$$Gini_T = T \cdot g_{ineq} \cdot Gini_0. \tag{5.33}$$

This quasi-linear behavior is documented in simulation results as presented in figure 5.5. These analytically appealing features resulting from the characteristics of the log-normal function help to tune the model.

We can also compute the aggregate growth rate:

$$g_t = \frac{\sum_{i=1}^{N} Y_{i,t+1} - \sum_{i=1}^{N} Y_{i,t}}{\sum_{i=1}^{N} Y_{i,t}} = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{g_{ineq}}{Y_t} \sum_{i=1}^{N} \log(Y_{i,t}) \cdot Y_{i,t}. \tag{5.34}$$

It is important to point out that the aggregate growth rate depends amongst others[43] on the distribution of income as captured by the factor σ_y. In particular, aggregate growth increases with inequality. Thereby, this result only captures the left part of the Kuznets curve, suggesting a positive correlation between growth and inequality. As the modeling leads to the fact that inequality increases in time, the growth rate also increases in time. The latter behavior is displayed in figure 5.6 showing a quasi-linearly increasing growth rate and an exponential growth process.[44] Note that modeling does not follow from an underlying formal microfounded model but is of an *ad-hoc* nature to capture a

[43]It also unambiguously increases with the growth rate of inequality g_{ineq} and the median income $\bar{Y}$.

[44]For the numerical simulation we chose the values $\sigma_y = 1$ and $\bar{Y}$ used in the benchmark simulations in section 6 as well as $g_{ineq} = 10^{-3}$.

rising inequality. It is, however, in line with empirical evidence. Campos-Vazquez et al. (2013) using data from the World Top Income Database show that income of wealthy individuals exhibits a higher growth rate than the income of lower income agents.[45] They summarize their findings in the way that *growth is (really) good for the (really) rich*.

It is important to point out that in our model we do not impose noise shocks on the income process. Implicitly we assume that an insurance or a social security system exists that is able to insure against any form of idiosyncratic income risk. In contrast to that, the presence of uninsurable idiosyncratic shocks is the key building block of *Bewley*-type models. Introducing uninsurable income risk would further increase the level of income inequality not only amongst agents but also for every particular agents during his life-span.

5.3. General Mechanisms of the Model

To illustrate basic simulation results, we assume a simplified version of the model. To get rid of non-linearities we also presume a curvature of the consumption function of $\varepsilon = 1$ resulting in the standard linear Keynesian consumption function as presented in equation 4.1. Furthermore, we assume that there is perfect correlation between wealth and income. In the general instance, it was superimposed that wealth and income were independently distributed. To connect stock and flow we assume a linear relation between the two described by a variable $H > 1$ for heritage, which can be interpreted as the number of annual incomes that is passed on from the parent generation:

$$q_{i,0} = H \cdot Y_{i,0}. \tag{5.35}$$

This is certainly a strong - yet not unrealistic - assumption since it entails the idea that individuals born rich can afford a better education, leading to higher future incomes, which in particular can be rationalized by the presence of credit constraints (Galor and Zeira, 1993). Besides economic heritage, there can also be biological heritage for which high ability (positively correlated with earnings) is transmitted by means of genes (Piketty, 2000). In the first instance, we also disregard interest on debt ($r = 0$) and thereby exclude the Ponzi case. We will dwell intensively on this issue in section 5.4.1.

The main workings of the model can now be presented in figure 5.7 in the static case. The left panel shows the total consumption function ($C_i + d_i P$) and the distribution of income. The intersection of the total consumption function ($C_i + Dd_i$) with the 45-degree line divides the households into borrowers (left side) and savers (right side).[46] Starting from this, the right panel plots the dissavings ($\dot{D}_i = C_i + d_i P - Y_i$) as well as the credit

[45] As we will elaborate more thoroughly in section 5.5.2 considering wealth inequality, the existence of a growth rate that positively depends on current level leads to the fact that inequality diverges to maximum inequality ($Gini = 1$).

[46] Debtors can be classified as households with an income $Y < Y_{II/III}$, while lenders have an income $Y > Y_{II/III}$ with $Y_{II/III} = \frac{\bar{c}}{1-c_y-H(c_w+MPCD_t)}$. For special long-run case of $E(MPCD) = 0$ we have $\frac{\bar{c}}{1-c_y-Hc_w}$.

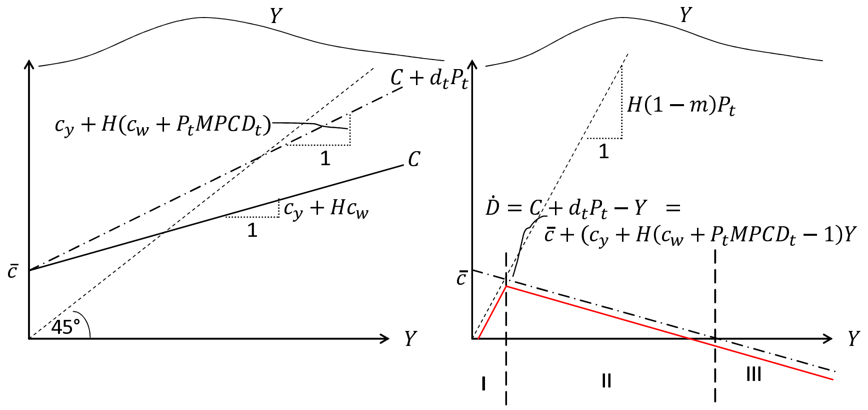


Figure 5.7.: Consumption and durable consumption ($C+Pd$) as function of labor income Y (left panel); dissavings $\dot{D}$ as function of labor income Y (right panel)

constraint ($\dot{D}_{max} = H \cdot P \cdot (1-m)Y_i$ [47]). The left panel in figure 5.7 therefore represents a stock quantity while the right one considers flow.

This allows us to split the population into three social groups. Group III (the high-class) engages in savings that are redistributed to lower income households. Individuals who have an income within the group II interval (the medium-class) increase their debt but are not subject to a collateral constraint since they have enough assets to borrow against. The most important group driving the systems dynamics are group I members (the low-class). They also would like to borrow, but face a restricting collateral constraint. For a given income distribution $Y_{i,t}$, group III (the high-income group) increases in size for low subsistence levels of consumption $\bar{c}$, low marginal propensity to consume c_y as well as c_w and to consume durables $MPCD_t$. Note that the ranking of individuals according and to classes is identical. While the assignment to a specific income group is determined by the level of income (measured in a monetary unit), the assignment to *class* is defined functionally by means of conducting specific activities.

Moreover, there is also a movement from group I to II for high prices of collateral P and low equity requirements m. The latter reflects the *American story*: Due to a

[47]Since we assume that initially the debt is zero ($D_{i=1;\cdots;N,0} = 0$), the maximum flow level in this case equals the maximum stock level of debt ($\dot{D}_{max} = D_{max}$). With this assumption the income which splits collateral constrained and non-collateral constrained households is given as follows: $y_{I/II} = \frac{\bar{c}}{H(P(1-m)-c_w-MPCD_t)+1-c_y}$. This ratio, however, only holds in the first simulation period in which no agent has accumulated debt so far.

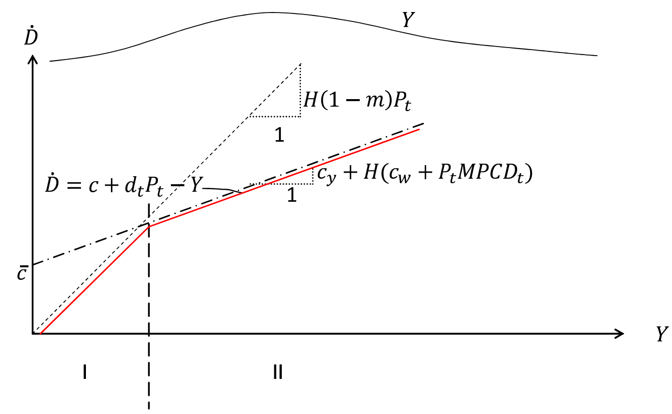


Figure 5.8.: Dissavings as a function of income in the case of a strong boom

boom in real estate prices and financial innovation (requiring for lower levels of m) the individuals who formally were part of the *lower-class* could transform into *middle-class*.

If we assume that $\varepsilon > 1$ and thereby suppose a low elasticity of consumption, the savings of high-income households will be more accentuated. This might lead to the fact that credit markets do not clear ($\sum_{i=1}^{N} \dot{D}_{i,t} \neq 0$) and a current surplus emerges. As presented in section 4.4, the concave consumption is emphasized if we assume a strong *love of wealth* motive modeled by higher risk aversion for consumption than wealth ($\gamma_c > \gamma_w$) in which agents accumulate wealth in order to protect against possible adverse future income shocks. Hence, a candidate explanation for the strong current account surpluses witnessed in countries such as China and Germany could be higher risk aversion.

A special case can be presented if we assume that there is a high $MPCD_t$. As presented in equation 5.14 this boom situation can emerge because of low interest rates, good fundamentals, or even a self-sustaining trend-following strategy of traders. Low market volatility and low risk aversion as experienced in the Great Moderation also contribute to this very behavior. If we assume that $H \cdot c_w + MPCD > 1 - c_y$ the right panel of figure 5.7 can be redone as presented in figure 5.8. Note that group III disappears in the boom situation because the dissavings function has a positive slope resulting in a positive relation between income and debt accumulation. The positive slope, however, can also emerge due to strong heritage H combined with strong MPC out of wealth c_w and a strong MPC out of income c_y. The positive slope of the dissavings function eventually represents a negative savings ratio. Thus, in a strong boom situation no domestic savings are provided and it will be financed via a current account deficit.

If we assume $\varepsilon = 1$, we end up with a simple linear consumption function of the Keynesian type. As, however, presented in the right panel of figure 5.7 the dissavings function has a kink leading to the fact that the aggregate behavior loses its linear properties. The latter can be attributed to the existence of the collateral constraint. If we disregard heterogeneity in an economy with a binding collateral constraint we end up being subject to the *fallacy of composition* (e.g. Caballero (1992), Kirman (1992)).[48] While the representative agent concept is feasible to provide a general idea about the workings of the model, macro behavior should not be confused as the behavior of a representative agent multiplied by the number of agents. The non-linear behavior as well as the complex interaction with distributional features makes it necessary to rely on simulation results since elegant analytical solutions are not feasible. The second reason why heterogeneity matters is the concavity of the consumption function as emphasized in Carroll (2001). Or put inversely, if consumption functions were linear - implying an underlying homothetic utility function - and financial market would not constrain, the actions of the representative agent (with mean income) multiplied by the number of agents perfectly describe the evolution of the total economy. In fact, the behavior of the complete economy can be described by the first statistical moment making distributional issues (described by higher moments such as the variance or skewness) irrelevant. We discuss this statement more thoroughly in appendix A.2 and provide formal conditions.

In the following, we give some intuition on the effects of the collateral constraint in a dynamic environment. This does not supersede the simulation results presented in section 6, but aims at providing a clearer insight into the underlying mechanisms.

If we consult our consumption/dissavings-diagram, positive flow rates of dissavings $\dot{D}_t$, increase the stock level of debt and thereby reduce the collateral constraint curve (see figure 5.9). However, by definition, this collateral constraint cannot fall below zero, since this would imply that agents would hold a higher ratio of debt (respectively a lower ratio of equity) than allowed by the market requirements m. In fact, the case for a collateral curve equal to zero represents the case for agents who have stretched their debt holding to the maximum. Not surprisingly, low-income individuals are the first to reach this level due to their lower holding of collateral. We also have to distinguish between debt for durables and non-durables.

If an agent attains credit in order to buy a durable - such as a house or a car - and times this purchase to the market, this increases her net worth, eventually leading to a higher financial scope. Consider an agent with a stock q_t of assets deciding to buy or sell the amount of $d_t \equiv \Delta q_t$ assets. The purchase price is given by the current market price P_t while the asset is activated in the balance sheet at the market price in the subsequent period P_{t+1} already accounting for the effect of the trading. To finance the purchase of this asset employs a certain debt ratio $0 \leq l \leq 1$ (leverage). The net worth evolves as follows:[49]

$$W_{t+1} - W_t \equiv \Delta W_t = (P_{t+1} - P_t)q_t + P_{t+1}d_t - P_t \cdot l \cdot d_t, \tag{5.36}$$

[48] An overview of the subject dating back to the seminal paper of Sonnenschein (1972) is given in Stoker (1993).

[49] Note that this simple computation assumes that the equity part of the purchase is completely financed by means of current cash-flow not considered in the stock. If a purchase touches another active

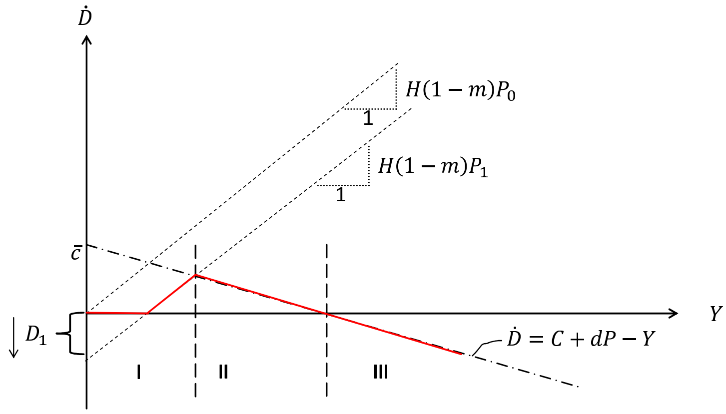


Figure 5.9.: Dissavings in the dynamic case

for which the first term captures the capital gains (or losses) of the given balance sheet, the second term activates the asset on the balance sheet, and the third term captures the decrease in net worth due to the employment of debt. The equation can be summarized as follows:

$$\Delta W_t = \Delta P_t(q_t + d_t) + P_t d_t(1 - l) = \Delta P_t(q_t + d_t l) + P_{t+1}(1 - l)d_t. \tag{5.37}$$

First of all, if agents do not trade ($d_t = 0$), a bull market ($\Delta P > 0$) always increases net worth as the previously accumulated assets have a higher value ($\Delta W_t = \Delta P_t q_t$). Another extreme case emerges if the purchase is completely financed by external capital ($l = 1$):

$$\Delta W_t = \Delta P_t(q_t + d_t), \tag{5.38}$$

for which the trade only increases net worth if the trader taking the long position correctly anticipated a price increase ($d_t \Delta P_t > 0$). As a result, the lower the leverage, the higher the gain when buying ($d_t > 0$) an asset ($\frac{\partial \Delta W_t}{\partial l} = -P_t d_t < 0$) as the effect of the net worth decreasing debt also has to be accounted for.

We can also consider the case of no price changes ($\Delta P = 0 \leftrightarrow P_{t+1} = P_t \equiv P$):

$$\Delta W_t = Pd(1 - l), \tag{5.39}$$

in which any long position ($d > 0$) increases the net worth. The only exception emerges for the case for which the purchase is completely financed by external capital ($l = 1$)

balance sheet position (e.g. cash), the trade itself would only alter the structure of the balance sheet but not its size (balance sheet sum).

and the higher active position is completely canceled out by debt of the same amount ($\Delta W_t = 0$).

Going short ($d_t < 0$) can be written as follows:

$$\Delta W_t = W_{t+1} - W_t = P_{t+1}(q_t + d_t) - P_t d_t - P_t q_t = \Delta P_t(q_t + d_t), \tag{5.40}$$

making it identical to the case with a completely leveraged transaction for the *long* case. The latter is, however, the case since the complete proceeding of the sale increases the net worth. In fact, in the second period $t + 1$ a new active position (i.e. cash) is created by the sale of the existing asset ($-d_t P_{t+1} > 0$ for $d_t < 0$). For no price changes ($\Delta P_t = 0$) the trade would have no effect on net worth and only alter the composition of the active side of the balance sheet ($\Delta W_t = 0$).

To implement a complete *stop-loss strategy* - also absorbing the losses of the existing stock of assets q_t - for the case in which agents always correctly predict the market results ($\Delta P \Delta q > 0$), would require selling *all* assets in a bear market ($\Delta q_t = -q_t \Rightarrow \Delta W_t = 0$). This extreme strategy, however, is hardly feasible due to the lumpiness of the investment.

This result, however, does not hold for the case of leveraged non-durables purchases. While increasing the amount of debt, non-durables do not contribute to net worth, since they are consumption goods in the *more immediate sense* being destroyed in the process of consumption.[50] Therefore, credit for non-durables always decreases net worth narrowing the financial scope of agents. This phenomenon is most important for low-income agents especially in a context of the social effect of consumption.

In our model context, levering up always satisfies the condition that the factual equity ratio is higher or equal to the required equity ratio m. However, we can consider shocks that destroy this relation. In the context of the model we can identify two shocks: a policy shock increasing the required ratio of equity capital $m_{t+1} > m_t$ would lead to forced savings or deleveraging. In contrast to this exogenous shock we can also consider a shock that is endogenous to our model, in the form of a bust in the market for durables ($P_{t+1} < P_t$). Since assets are valued at current market prices, whereas debt is given in nominal terms and thereby not inflation indexed, a price shock decreases net worth. Once again, this especially hits the low-income group. Rather than having only a class I we create a class 0 that are forced to save ($\dot{D} < 0$) (see figure 5.10). This class consists of those agents already leveraged up to the (former) maximum as well as some agents that still had some financial scope in the former case. Moreover, some middle-class agents (class II) cannot realize their consumption desires and therefore also fall into class I. The composition of the upper-class, however, is unaffected.

The reaction of the group 0 to their financial dire strait is important. Finally, we therefore compare fire sales and the Austerity-case. In the static case, allowing for fire sales can increase the size of the middle-class, since - in the short-run - households can take the proceeds from the sale of the housing asset for non-durable consumption. Technically, for one single period the collateral constraint is given by a curve with the slope HP rather than $HP(1 - m)$. Since $m < 1$ the size of the middle-class increases as presented in figure 5.11. This effect is even more pronounced if households already

[50] The most tangible example for this case is food.

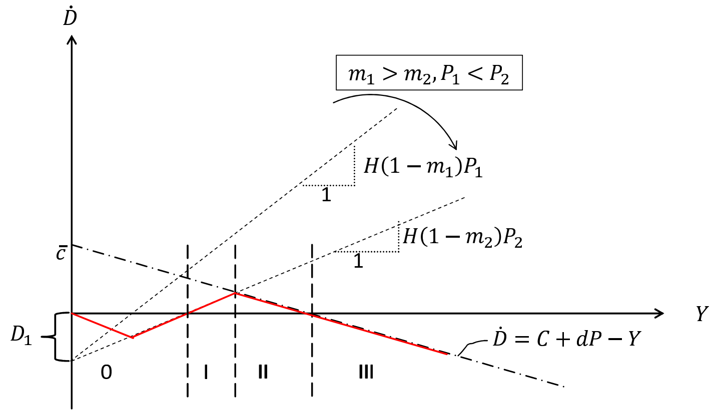


Figure 5.10.: Dissavings in the dynamic case with a positive shock to equity requirements and negative shock to prices

accumulated high amounts of debt, shifting the collateral constraint curve downward (as seen in figure 5.9) further decreasing the size of the middle-class. This fire sale can contribute to a delay of consumption decrease for a certain group of households for one single period. If, however, this group already piled up debt, in a dynamic case this leads to overindebtedness since the debt is not backed by any collateral, resulting in forced savings through a consumption decrease. In summary, fire sales are only a desperate measure with some short-run gains that in the long-run, even aggravate the problem.

This stylized analysis emphasized the reaction function of the very low-income agents, however, not accounting for aggregate effects. The negative externalities of fire sales or consumption decreases for the aggregate economy are more precisely considered in the section presenting the numerical simulations (section 6).

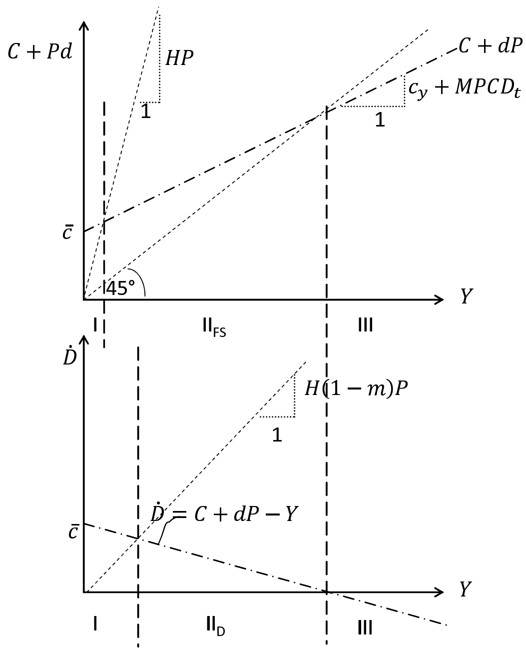


Figure 5.11.: Comparison of the cases with debt accumulation (bottom) and fire sales (top)

5.4. Financial Stability

The major topic of this dissertation is to rationalize a theoretical link between inequality and financial stability. The latter can be separated into stability of debt (ruling out *Ponzi schemes*, see section 5.4.1) and stability of the durable market (ruling out ever growing positive or negative bubbles, see section 5.4.2). It is also important to point out that while debt is a nominal asset (we abstract from inflation-indexed debt contracts), the durable is a real asset. The two asset classes, however, are interconnected by means of the collateral constraint. As we show in section 5.4.3, there are common factors that stabilize one market at the expense of destabilizing the other and vice versa.

5.4.1. Dynamics of Debt

In this section, we analytically shed some light on the stability of the debt level (D) in a slightly simplified version of the model. This also gives insights into the dynamics of interest rates and current accounts ($\dot{D}$) as well as the necessary assumptions. In fact we combine the key equations presented in section 5.1 to derive a *master equation* governing the aggregate dynamics. In particular, we consider whether financial instability can prevail in the form of *Ponzi schemes*. Thus, this section is also closely related to section 4.2 where we discuss general conditions for the (non)-existence of *Ponzi schemes*. This section, however, provides a more concrete picture for the model as developed in our work. The major difference of our model as compared to the case presented in section 4.2 is the variability of the rate of interest. In particular - and in line with empirical evidence - the rate of interest increases if a high amount of debt is already accumulated. A high amount of debt decreases the distance to default[51] and thereby leads to the fact that the mirror party holding the claim requires a higher rate of interest for risk compensation purposes. This, however, induces an unstable equilibrium for which the level of debt increases the rate of interest, requiring more debt, increasing the rate of interest even more and so forth.

Using the equation for dissavings eq. 5.15 and consumption based upon disposable income eq. 5.2 we can derive the following result:[52]

$$\dot{D} = -Y + C + dP + rD = \bar{c} + c_y(Y - rD) + c_w W - Y + dP + rD. \tag{5.41}$$

The MPCD can be decomposed into two factors depending on the return expectation of the risky asset and the interest rate in the market for debt/savings leading to the following result:

$$\dot{D} = -Y + C + dP + rD = \bar{c} + c_y(Y - rD) + c_w W - Y + P \cdot \frac{E(\Delta p)}{\gamma} W - P \cdot \frac{r}{\gamma} W + rD. \tag{5.42}$$

For sake of simplicity we assume that the stock of durables is constant ($\frac{\partial(Pq)}{\partial t} = \dot{P}q + P\dot{q} = \dot{P}q + Pd \equiv 0$). As presented in appendix A.6, the long-run distribution of wealth is constant. This leads to the fact that net worth only changes if indebtedness changes:

$$\dot{W} = -\dot{D}. \tag{5.43}$$

By integrating with the initial conditions that all households are born without debt holdings ($D_0 = 0$) but with a heritage of durables ($P_0 q_0 = HY$ [53]) we find the following relation between debt and net worth:

$$W = -D + H \cdot Y. \tag{5.44}$$

This very simple equation provides some deep economic insight. In the model, we have two sources of wealth: real assets HY and monetary assets $-D = K$. While real assets are exogenously given, monetary assets can be created by agents. We might consider the real assets as a form of land whose size is determined and not subject variation in time. Despite some temporary valuation gains or losses, which, however, are nil in the long-run ($E(\dot{P}) = 0$), the total amount does not change. Agents can only change their individual endowment with the real asset by means of trading. In contrast to that the agents, however, can create monetary assets $\dot{D}$ representing claims to one agent $D < 0$ and debt to another agent $D > 0$. In a closed economy, these assets should amount

[51] The latter term was coined by Merton (1974) and nowadays is a common index for computing the financial soundness of a banking institution (e.g. in the form of a stress test).

[52] Note that we remove the time index t since we now argue in a continuous time region rather than a discrete time thereby replacing difference equations with differential equations. This approach already provides an interesting insight into the workings of the model. This, however, does not incorporate the non-linear behavior of the *collateral effect*.

[53] This result is derived by assuming $P_0 = F_0 = 1$ and $q_0 = HY_0$.

to zero. In an open economy the supply of credit to foreigners $\dot{D} < 0$ is accompanied by a current account surplus which in turn creates claims against them and increases national wealth ($\dot{W} = -\dot{D}$). The creation of credits is made by agents who reap a larger amount of fruit than they are capable of digesting. The mirror position is the case for agents who *eat the credit*.[54]

Formally, this can be captured by the following differential equation in which we insert the last equation into eq. 5.42:

$$\dot{D} = \bar{c} + c_y(Y - rD) + c_w(HY - D) - Y + P \cdot \frac{E(\Delta p)}{\gamma}(HY - D) - P \cdot \frac{r}{\gamma}(HY - D) + rD. \quad (5.45)$$

Using the market clearing condition for the savings market (eq. 5.20) this results in the following non-linear differential equation:[55]

$$\begin{aligned} \dot{D} &= Y(-1 + c_y) + YHc_w + YP\frac{E(\Delta p)}{\gamma}H + \bar{c} \\ &+ r_0 exp(\mu_r D)\left[-\frac{HYP}{\gamma} + D\left(1 - c_y + \frac{P}{\gamma}\right)\right] - D\left(c_w + \frac{E(\Delta p)}{\gamma}P\right). \end{aligned} \quad (5.46)$$

The first term is always negative since we assume a positive savings ratio $s = 1 - c_y > 0$ due to $c_y < 1$. The second term - representing consumption out of wealth - is always positive. The third term is time-varying in which the sign depends on the expectation about future prices. The fourth term, accounting for subsistence consumption, is always positive and thereby increases debt. We will subsume these terms as the primary deficit K.

If we take the simplifying assumption that long-run demand for durable is zero ($E(\Delta P) = 0$), we can also compute a simple long-run condition for a primary surplus. A surplus is reached if the following conditions hold:[56]

$$K = -Ys + YHc_w + \bar{c} < 0 \leftrightarrow 0 < \frac{\bar{c}}{Y} < s - Hc_w. \quad (5.47)$$

This implies that the primary surplus increases for low conspicuous consumption $\frac{\bar{c}}{Y}$, high savings ratios s, low consumption out of wealth c_w and low heritage H. The latter seems somewhat surprising. However, one should keep in mind that a high level of heritage increases the level of wealth available for consumption purposes. If we consider no consumption out of wealth ($c_w = 0$), heritage H does not matter. Without conspicuous consumption effects ($\bar{c} = 0$), the condition simplifies even more to:

$$Hc_w < s. \quad (5.48)$$

Note that this condition is less restrictive than the condition presented in the penultimate equation, implying that an economy with conspicuous consumption does not have to be stable only if the last equation is satisfied.

---

[54] This catchy phrase was coined by Rajan (2010).

[55] This result can be computed as follows: $r_{t+1} = r_t \exp(\mu_r \dot{D}_t) = r_{t-1} \exp(\mu_r \dot{D}_{t-1}) \exp(\mu_r \dot{D}_t) = r_{t-1} \exp(\mu_r [\dot{D}_{t-1} + \dot{D}_t]) = r_0 \exp(\mu_r [\dot{D}_{t-1} + \cdots + \dot{D}_t]) = r_0 \exp(\mu_r D_t)$.

[56] This is also the condition separating middle from high-class households for the case without durable consumption ($MPCD = 0$) as shown in footnote 46.

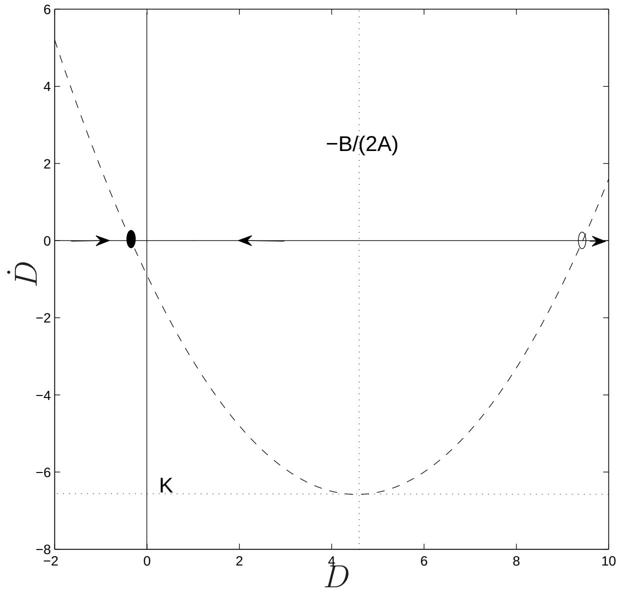


Figure 5.12.: Phase diagram for the case without zero lower bound and wealth effect

To analyze this result we firstly assume no consumption out of stock ($c_w = 0$). By using the exponential function in determining the market interest rate, we implemented a zero lower bound. A more simple approach also allowing for negative interest rates would be to assume a linear relation $\dot{r} = \mu_r \dot{D}$ which for the given initial conditions yields $r = \mu_r D$. This would simplify the function to the following form:[57]

$$\begin{aligned} \dot{D} &= Y(-1+c_y) + YP\frac{E(\Delta p)}{\gamma}H + \bar{c} \\ +\mu_r D &\left[-\frac{HYP}{\gamma} + D\left(1 - c_y + \frac{P}{\gamma}\right)\right] - D\left(\frac{E(\Delta p)}{\gamma}P\right) \\ &= \tilde{K} + D^2\mu_r\left(1 - c_y + \frac{P}{\gamma}\right) + D\left(-\mu_r\frac{HY}{\gamma} - \frac{E(\Delta p)}{\gamma}P\right) \\ &\equiv \tilde{K} + D^2 A + DB. \end{aligned} \tag{5.49}$$

This is the classic Saddle-Node-Bifurcation (Strogatz, 2001). For the case with a primary deficit ($\tilde{K} > 0$), there is no equilibrium and a debt bubble emerges. For the case with a primary surplus ($\tilde{K} < 0$), this yields two equilibria as presented in figure 5.12. The lower equilibrium at a negative level of debt corresponding to a situation where an economy holds claims against foreigners is stable. Once the equilibrium has surpassed

[57]In this case we assume for the modified primary deficit $\tilde{K} = K - YHc_w$.

a threshold level D^* [58] - graphically depicted as the intersection of the phase curve and the D-axis ($\dot{D} = 0$) - the destabilizing *Ponzi mechanism* sets in. Paying the interest on debt requires new debt, thereby increasing the interest rate further, and aggravating the debt problem. This results in infinite debt growth with steadily deteriorating current accounts. The stability of the lower equilibrium, however, crucially depends on the possibility of negative interest rates. Increased savings resulting in claim positions ($D < 0$), decrease the interest rate. Once the interest rate has fallen to a negative ratio, the flow of interest payments from debtors to lenders is reversed. The fact that lenders now have to pay a fee to invest their savings, increases their demand for debt and decreases their savings. This mechanism is stabilizing.

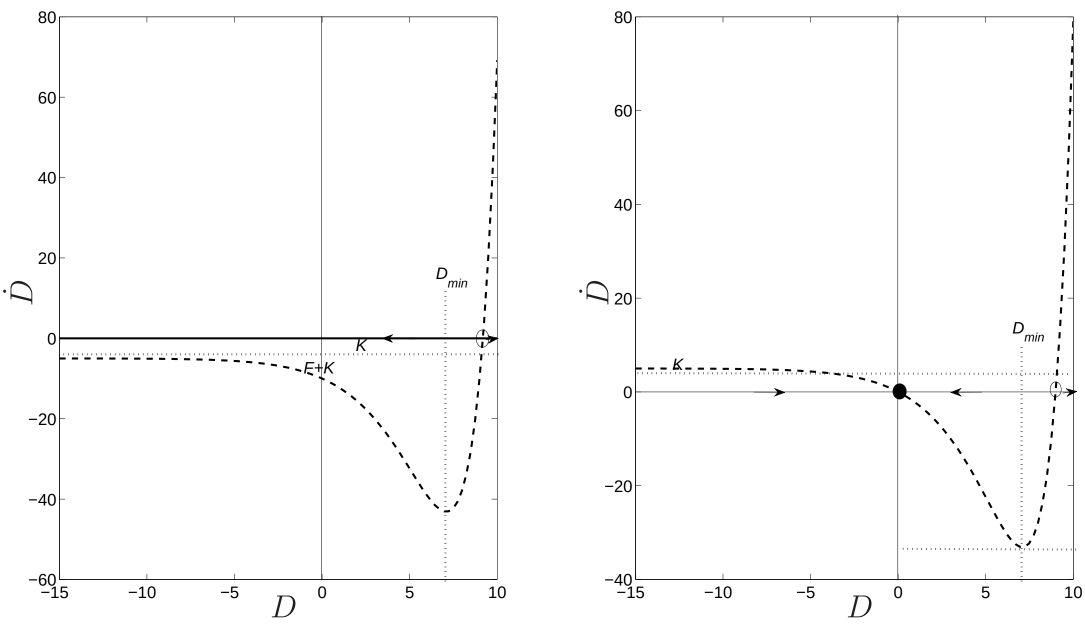


Figure 5.13.: Phase diagram for the case with zero lower bound and no wealth effect for primary surplus ($K < 0$, left panel) and deficit ($K = -F > 0$, right panel)

If we, however, assume the zero lower bound, the differential equation without consumption out of stock is given as follows:

$$\dot{D} = \tilde{K} + r_0 exp(\mu_r D) \left[D \left(1 - c_y + \frac{P}{\gamma} \right) - \frac{HYP}{\gamma} \right] - D \frac{E(\Delta p)}{\gamma} P. \tag{5.50}$$

If we further assume that there is no expected price change for durable goods ($E(\Delta p) = 0$), the lower stable equilibrium disappears (see figure 5.13 [59]). Besides the already

[58] This amounts to $D^* = -\frac{B}{2A} + \sqrt{\frac{B^2}{4A^2} - \frac{K}{A}}$. For the case with $B = 0$ there is no *flight effect* and no net worth effect for durable consumption. These two effects that increase the level of stable debt will be covered later.

[59] For the figure 5.13 we define $F \equiv -\frac{HYP}{\gamma}$ and $\tilde{K} \equiv K$.

discussed *Ponzi effect* for the case with a primary surplus ($\tilde{K} < 0$) there is an converse case with ever increasing lending. This can be explained by the fact that agents do not consume (neither durable nor non-durables) out of wealth and keep increasing savings, leading to a deterioration of the interest rate to zero. The annual current account surplus $\dot{D}_\infty$ converges to the annual savings surplus $\tilde{K}$.

On the other hand, in this economy without consumption out of wealth a primary deficit ($\tilde{K} > 0$) can be sustainable. This effect can be attributed to finite market liquidity ($\mu_r < \infty$) and the *flight effect*. The latter is captured by the value $(-exp(\mu_r D)\frac{HYP}{\gamma})$ in the previous equation leading to a global minimum of the function in the phase diagram at $D_{min} = \frac{HYP}{\gamma(1-c_y)+P} - \frac{1}{\mu_r}$.[60]

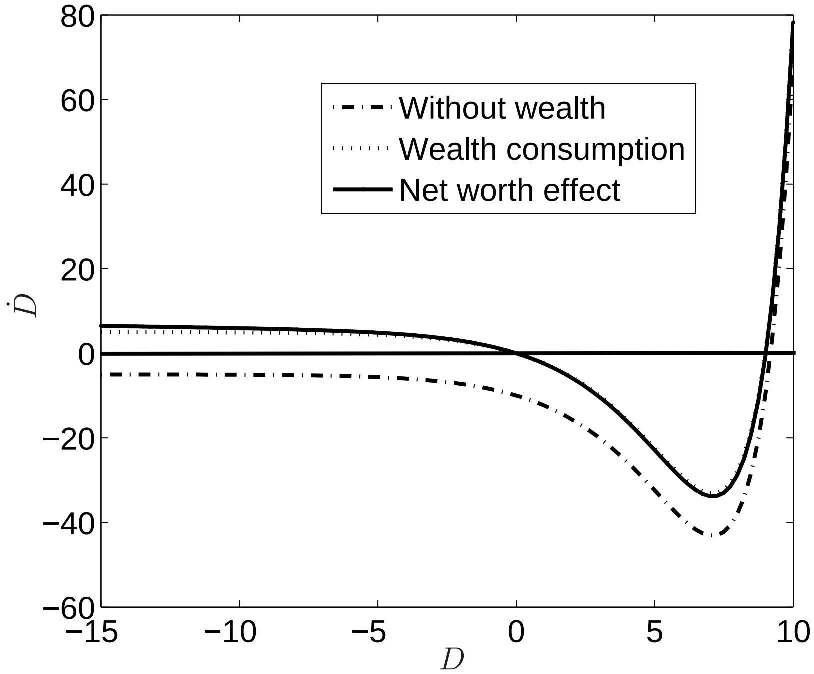


Figure 5.14.: Phase diagram for the case with zero lower bound and wealth effect

For the case without the flight effect ($\gamma \to \infty$), the local minimum as presented in figure 5.13 is at $D_{min} = -\frac{1}{\mu_r}$ amounting to $\dot{D}_{min} = \tilde{K} - \frac{r_0 s}{e\mu_r}$ lowering the amount of debt in the equilibrium. Therefore, the existence of the flight effect can contribute to a stable equilibrium with a high amount of debt. The missing flight effect also restricts the primary deficit to a level of $0 < \tilde{K}^* < \frac{r_0 s}{e\mu_r}$. If this restriction is not met, there would be no equilibrium since $\dot{D} > 0$ for all D. Translated into economically meaningful terms, this implies that only the *Ponzi case* could persist, for which any primary deficit beyond the computed one K^* would result in a vicious debt cycle.

[60] This result is derived by setting $\frac{\partial \dot{D}}{\partial D} \stackrel{!}{=} 0$.

The case is even stronger if we furthermore assume an infinite interest rate adjustment speed ($\mu_r \to \infty$) making any primary deficit unsustainable since $\frac{\partial \dot{D}}{\partial D} > 0$ and $\dot{D} > 0$ for all D. Moreover, for this situation with a primary surplus $\tilde{K} < 0$ there would only be one unstable equilibrium leading to either a Ponzi growth of debt or an unstoppable growth of claims. The extreme case of $\mu_r \to \infty$ can be considered the Walrasian case, for which an excess demand/supply for savings would lead to a prompt convergence of the interest rate to zero/infinity. Or put positively, the existence of finite market adjustment speed allows maintaining a stable amount of debt in the case of a primary deficit.

The *flight effect* works, since an increase in consumption increases demand for debt, increases interest rates, thereby making savings - rather than consumption - attractive. This is a stabilizing mechanism. Besides this *flight to quality*, there is also a *flight to risk* for which increased savings, lower the interest rate, thereby increase the demand for risky assets (being the durable consumption good), resulting in lower savings. A high level of debt, therefore, can be sustainable for strong switching in the portfolio, as indicated by low risk aversion γ.[61] This can be considered the period of the Great Moderation. Lastly, it should be mentioned that the *flight effect* also has a destabilizing element which further promotes the *Ponzi effect*. Technically, this effect is captured by the factor $r_0 exp(\mu_r D)\frac{P}{\gamma}$. To put it verbally, this factor acts through net worth. High debt decreases net worth and thereby lowers the *flight to quality* reaction to increased interest rates. While slightly promoting the *Ponzi case*, this, however, cannot outdo the stabilizing *flight* effect.[62]

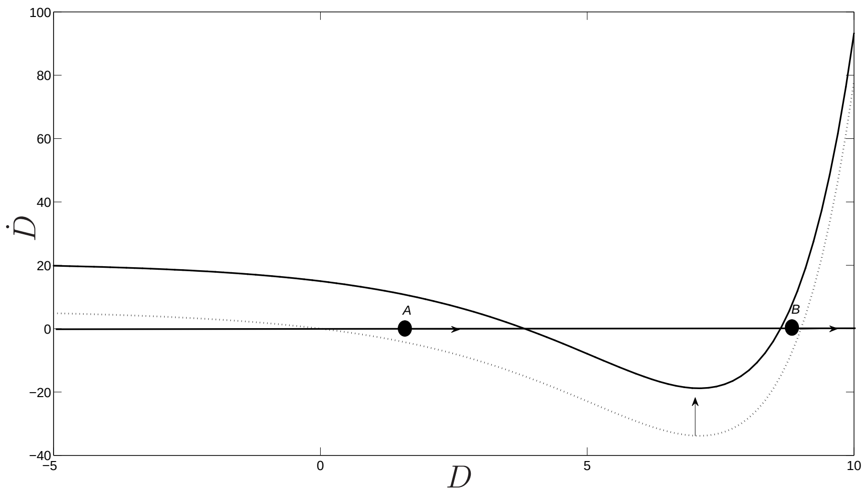


Figure 5.15.: Comparative statics for the case with consumption shock for which the debt level A still converges while the level B implies infinite growth of debt

[61] This also holds true for low market volatility, which in our model is normalized to 1.

[62] Technically, this is ensured due to the condition $HY > D_{max} = (1 - m)HY$ with $0 < m < 1$ forbidding purchases that are completely financed by debt.

Introducing consumption out of wealth ($c_w \neq 0$), leads to an increase in primary deficit. In figure 5.14, we label this effect as the *wealth consumption effect*. This increase in primary deficit contributes to the emergence of a second *stable* equilibrium. On the other side there is also a stabilizing effect represented by the very last term in the following equation:

$$\dot{D} = K + r_0 exp(\mu_r D)\left[D \cdot s - \frac{P}{\gamma}(HY - D)\right] - D\left(\frac{E(\Delta p)}{\gamma}P + c_w\right). \tag{5.51}$$

The latter effect once again passes through net worth. Increased wealth increases consumption, increases debt, which in turn decreases net worth lowering consumption. This is a stabilizing mechanism. In figure 5.14, the latter is introduced as the *net worth effect*.

In the resulting phase diagram, we can perform some comparative statics. Strong heritage H increases the primary deficit through the consumption effect while also increasing the level of the equilibrium debt through an increased *flight effect*.[63] Technically, this moves the curve upwards (primary deficit) and to the left (increased flight effect). The increase in subsistence level of consumption $\bar{c}$, marginal propensity to consume (out of wealth and income, c_y and c_w) as well as positive trend perception in the market for durables leads to an upward shift of the curve ($E(\Delta p) > 0$). This can lead to the fact that markets converge to a new higher stable equilibrium value of debt (starting from level A in figure 5.15). On the other side, a debt level B that formerly would have converged to a stable equilibrium now leads to a debt explosion. The latter is especially the case when trend-following trading in the market for durables leads to a series of self-enforcing positive shocks accompanied by an upward shift of the curve.

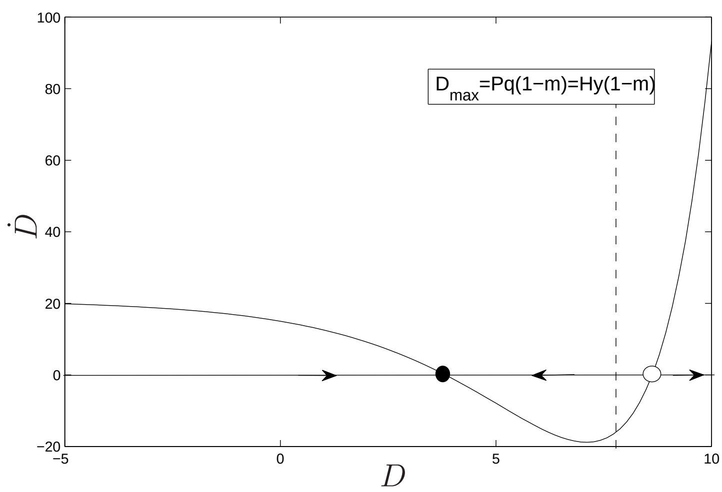


Figure 5.16.: Phase diagram for debt dynamics

[63] This analysis, however, does not take into account the important *collateral effect* of strong heritage for low-income households.

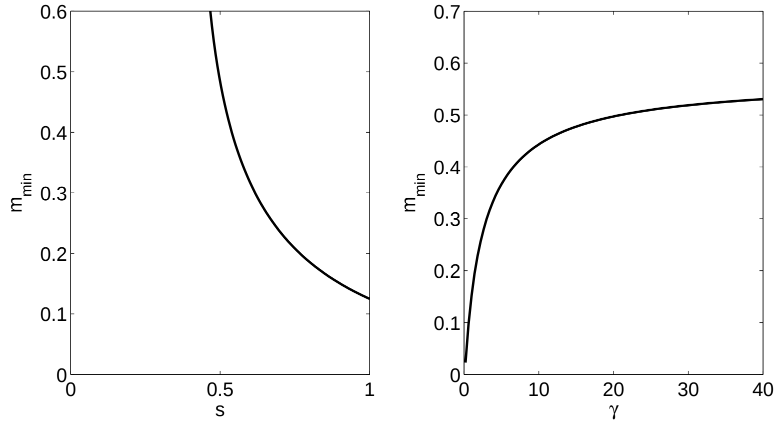


Figure 5.17.: Numerical solution for the minimum equity ratio for variation of savings rate s (left panel) and risk aversion γ (right panel)

To prevent the Ponzi effect lenders require collateral. By setting an equity ratio m, the maximum debt level is implicitly set to $D_{max} = (1-m)HY$. As presented in figure 5.16, the equity ratio m can be chosen in such a manner that D_{max} is always smaller than the value of the unstable equilibrium D^*. Using the complete non-linear differential equation presented in equation 5.51 and taking the numerical values of our simulation (refer to table 6.1) we can compute a numerical solution of D^*. For the threshold case $D^* = D_{max}$ we can use this case to derive a minimum equity requirement m_{min} following from:

$$m_{min} = 1 - \frac{D_{max}}{HYP}. \tag{5.52}$$

Note that for the benchmark tuning the ratio would be as high as $m_{min} = 50\%$. In figure 5.17, we provide comparative statics for a variation of risk aversion γ and the savings rate $s \equiv 1 - c_y$. The figure shows that an increase in risk aversion γ lowers the strength of the flight effect and thereby increases the possibility of Ponzi schemes prevailing. Despite high rate on interest lenders would not be willing to supply credit - as e.g. witnessed in the European debt crisis - strengthening the hazard of Ponzi schemes. As a result, lenders should set higher values of m_{min} to counteract these effects. Conversely, higher savings ratios s lower the amount of consumption and thereby the employment of debt and thereby decrease the hazard of Ponzi effects. The lender thereby can charge lower equity ratios m_{min}.[64] Note if savings fall below a specific threshold we end up with a primary deficit ($K > 0$) that would lead to complete instability.[65] To rule out this case,

[64]We also computed the reaction to variations of consumption out of net worth c_w, expected price change $E(\Delta P)$, and heritage H. Not surprisingly, in all three cases the minimum equity ratio increases with the parameters since all cases promote stronger consumption accompanied by an increased demand for debt. We spare the results for lack of space.

[65]Furthermore, savings increase capital, also increasing disposable income, leading to stronger consumption and thereby debt.

theoretically an equity ratio $m_{min} > 100\%$ would be required. This case is not depicted in figure 5.17 for the purpose of readability.

This simple analysis does not take income heterogeneity into account. In fact for the numerical results presented in figure 5.17 we assumed the mean agent with an income of $E(Y) = \bar{Y} \exp\left(0.5\sigma_y^2\right) \approx 8.24$. Albeit delivering some important insights into the mechanisms leading to (in)stability, the results cannot be imposed on the total model without being subject to the fallacy of composition (cf. section A.2). While the interest on debt r is a uniform market-wide variable determined by the aggregate level of debt, the demand and supply of debt are determined on an individual level. Lower income is accompanied by a lower primary surplus (or even a deficit) graphically shifting the curve in the phase diagram upwards (e.g. refer to figure 5.15). As will be presented in more detail in section 5.5.1, low-income agents have an equilibrium holding of debt, whereas high-income agents hold claims (or capital) which furthermore increases with the level of debt. The interaction between the groups determines the level of the interest rate. As a result, the *Ponzi effect* is crucial for indebted low-income households while the stabilizing *flight to quality* is important for high-income households. The overall result therefore depends on the distribution of income amongst agents, which will be discussed based upon simulation. Moreover, the presented analysis implicitly assumed that $E(\Delta p) = 0$ and $P = const$ for the market for durables letting us focus on the market for savings. In the following section we take the inverse position by considering the market for durables independently.

5.4.2. Dynamics of Prices of Durables

This section investigates the market for durables especially considering the stability issues. The dynamics of the log-prices p are given as follows:

$$p_{t+1} - p_t = \frac{\mu}{N} \sum_{i=1}^{N} MPCD_t \cdot W_{i,t} = \mu \cdot MPCD_t \cdot E(W_t). \tag{5.53}$$

In this case, $E(W_t)$ represents the mean net worth. Using the construction of the $MPCD$ we can arrive at the following result:

$$p_{t+1} - p_t = \frac{\mu}{\gamma} \left[w_t^C E_t^C(p_{t+1} - p_t) + w_t^F E_t^F(p_{t+1} - p_t) + d_t^{noise} - r_t\right] E(W_t). \tag{5.54}$$

Following Fischer (2012), this non linear differential equation can be linearized by assuming $\Gamma = 0$. This is the extreme case in which agents totally stick to one strategy. More technically, the weights of chartists and fundamentalists do not vary in time and are of equal size $w_t^C = w_t^F = 0.5$. Transforming the difference equation into a continuous time differential equation results in the following equation:[66]

$$\dot{p} = \frac{\mu}{\gamma} \left[\frac{\beta_F}{2}(f - p) + \frac{\beta_C}{2}(\dot{p} - \ddot{p}) + d_t^{noise} - r_t\right] E(W). \tag{5.55}$$

[66] For the details of this transformation the reader is referred to Fischer (2012). It is important to point out that the backward-looking behavior of chartists results in a second-order differential equation.

After transforming this equation into the frequency domain the following transfer function can be derived:[67]

$$\frac{p(s)}{d^{noise}(s) - r(s)} = \frac{\frac{2}{\beta_F}}{s^2 \frac{\beta_C}{\beta_F} + s\left(\frac{2\gamma}{\mu E(W)\beta_F} - \frac{\beta_C}{\beta_F}\right) + 1}. \tag{5.56}$$

This function gives the log-price effect on a positive shock in noise trading or a negative shock in interest rates. If we would assume that interest rates are solely determined by the central bank, this yields the strong result that an expansionary monetary shock can not be distinguished from a positive noise trading shock both leading to a boom in asset prices. However, in the basic representation of this model noise trading is assumed to follow a stochastic process while the interest rate is determined endogenously. As section 6 will show, an increase in inequality can also lead to a decrease in the interest rate (as suggested by the *global savings glut hypothesis*) resulting in a price boom for assets. Therefore in our model we can link inequality and asset bubbles.

The computed transfer function resembles the so-called PT_2 transfer function:

$$F(s) = \frac{K}{\frac{1}{\omega_0^2}s^2 + \frac{2D}{\omega_0}s + 1}. \tag{5.57}$$

The factor $K \equiv \frac{2}{\beta_F}$ determines the peak to an impulse shock. In our case, this effect is lower for high values of β_F indicating strong fundamental trading. The given function has the following eigenvalues[68]:

$$s_{1/2} = \omega_0(-D \pm \sqrt{D^2 - 1}). \tag{5.58}$$

The stability conditions requiring the real part of the eigenvalue to be negative ($Re\{s_i\} < 0$) yields the following condition:[69]

$$D = \frac{2\gamma}{\mu E(W)\sqrt{\beta_F \beta_C}} - \sqrt{\frac{\beta_C}{\beta_F}} > 0 \Rightarrow \gamma > 0.5\mu E(W)\beta_C. \tag{5.59}$$

This condition implies that if the risk aversion falls below a specific threshold the durable market becomes unstable. The rationale for this is that low risk aversion empowers strong trading that in the presence of chartist traders ($\beta_C \neq 0$) results in boom/bust-cycles. This result is also interesting in the light of the results of the previous section. Low risk aversion can help to maintain a stable amount of debt due to the *flight effect*.

[67] The transformation of a function into the frequency domain is given by the solution of the Fourier integral: $y(s) = \int_0^\infty y(t)e^{-st}dt$. For an introduction to the techniques of discussing differential equations in the frequency domain the reader is referred to Franklin et al. (1998).

[68] Note that the imaginary part of the eigenvalues also gives the length of the cycle in the financial market.

[69] This is only a lower bound. The exact threshold value is even higher since the non-linear destabilizing effect resulting from the switching to the chartist strategy (neglected in our analysis by setting $\gamma = 0$) kicks in.

Meanwhile, this is accompanied by high price volatility. This in turn acts through the collateral constraint and can promote (unsustainable) debt growth (in time of asset bubbles) and debt-deflation dynamics (in times of asset price busts). The stabilizing effect of low risk aversion for debt markets therefore can be outdone by the destabilizing effect on asset markets.

Another factor contributing to market instability is a high value of market illiquidity μ. Illiquid markets (for instance housing market) react severely on excess demand causing strong price fluctuations that are further amplified by chartist traders. This result also holda in the market for savings for which strong illiquidity contributed to instability.

Financial volatility is characterized by high absolute returns in the market for durables. Starting from equation 5.19 in a first-order approximation, absolute returns can be described as follows:

$$|p_{t+1} - p_t| = \frac{\mu}{N} \sum_{i=1}^{N} |MPCD_t| \cdot |W_{i,t}| = \mu \cdot |MPCD_t| \cdot |E(W_t)|. \tag{5.60}$$

This implies that high mean levels of absolute net worth $|E(W_t)|$ lead to high financial volatility.

Opposed to the results of Fischer (2012), (implicitly) assuming CARA traders rather than CRRA traders, the mean net worth $E(W_t)$ (which in a first approximation is assumed to be constant in time) also contributes to market instability. High net worth allows traders to massively engage in trading in the market and therefore has the analog effect of low risk aversion. In our simulations, we assume that agents start with zero debt. However, as simulation time progresses low-income agents will lever up to finance consumption while high-income agents, as a mirror image, hold the resulting claims. For low-income households the debt reduces net worth, while increasing debt for high-income households. The inequality of net worth thereby increases. Yet, in net this is a zero-sum game not changing the mean. However, in open economies with income inequality characterized by an excess supply of savings rich households hold claims against foreigners. This effect is further amplified by underdeveloped financial markets posing strong collateral constraints on low-income households, thus not enabling them to lever up. An unbalanced current account thereby increases mean net worth and thereby makes financial markets more unstable. This case emerges for a current account surplus economy.

5.4.3. Interaction between the Market for Durables and the Market for Debt

Summarizing the two preceding sections, we derive an argument for more equality on a domestic and a world-wide level. A primary deficit can lead to a *Ponzi* style debt bubble. On the other side, a primary surplus through the channel of net worth can contribute to high volatility and even instability in the market for durables. Our model framework traces back both factors to income inequality and financial markets.

In effect, *financial stability* in our model has two specifications. Stability of debt - i.e. the absence of *Ponzi* schemes - and stability of durable markets - implying the absence of unsustainable asset price booms. Both markets are interconnected by means of the collateral constraint. Thus, they have common factors. These common factors, however, act in inverse directions.

Low risk aversion γ promotes the *flight effect* leading to stable markets for debt. On the other side, low risk aversion can enhance speculation in markets for durables. Therefore, times of low risk aversion (e.g. the subprime crisis) caused by *new-era thinking* or by the introduction of new hedging instruments can go along with asset price bubbles. Opposed to that, times of high risk aversion (e.g. the current European sovereign debt crisis) can lead to prevailing Ponzi schemes.

Another inverse effect can be seen for the equity ratio m. While high levels of equity ratio help to stabilize the market for debt by restricting the amount of debt to a manageable level, they negatively impact the market for durables. Strong constraints in the presence of income inequality lead to a current account surplus accompanied by increased mean net worth feeding back into destabilizing asset market speculation. More generally, the equity ratio m can also be considered as a proxy for financial development, whereas high values can be interpreted as times of *financial repression*.[70] While the latter helps to maintain debt manageable (not only for the government, but also for private households), it promotes speculation in durable markets.

In the presented analysis of the market for debt, we did not consider the effect of booms in the market for durables ($E(\Delta p) > 0$). As already presented in figure 5.8, an asset price boom is mostly financed via a current account deficit (as seen in Ireland, USA, and Spain)[71]. A current account deficit lowers the mean net worth, eventually stabilizing the market for durables, although this effect may not be sufficient to stabilize markets in times of growing prices, allowing for higher indebtedness. Hence, an asset price boom is frequently followed by a debt crisis (as seen in the recent crisis) rather than being in the reverse chronological order.

5.5. The Functional Income Distribution and the Distribution of Wealth

The last sections discussed the issue of financial stability in the framework of our model. In the following, we consider the issue of wealth inequality. In the presented model, we cannot make a statement about inequality of labor income as we exogenize all decisions in the domain of the labor market. The distribution of wealth, however, endogenously evolves in our model. As emphasized in the previous sections rather than the distribution of income, the distribution of wealth matters for the issue of financial stability. Therefore, in this part we discuss the functional distribution of income (the split between labor and

[70] The return of this phenomenon in the recent years is documented in Reinhart and Rogoff (2011).
[71] For more rigorous empirical evidence the reader is referred to Aizenman and Jinjarak (2009). The authors show that real estate prices are a key driver of current accounts.

capital income) as well as the distribution of wealth. These two concepts are closely intervened as capital income is merely the product of accumulated capital multiplied by the current rate of interest. In particular, we investigate conditions of stability. Note that a divergence of wealth inequality also leads to increased financial instability.

5.5.1. Relation between Personal and Functional Distribution of Income

The existing empirical literature (e.g. Atkinson et al. (2011)) focuses on the personal distribution of income. In contrast the Post-Keynesian literature has always focused on the functional distribution of income differentiating between income out of labor and capital. As already put forward in section 3.2, there has been some recent evidence reported in Karabarbounis and Neiman (2013) showing that there has been a long-run decline of the labor share. As put forward by Milanovic (2014), the causality could run from an increase in the capital share to an increase in income inequality.

To verify this statement consider the following thought experiment. We have two forms of income - labor income Y and capital income rK - and mix them without changing the aggregate level of income. Total income X is given by the sum of labor Y and capital income rK:[72]

$$X = Y + rK \leftrightarrow 1 = \frac{Y}{X} + \frac{rK}{X} = (1-\alpha) + \alpha, \tag{5.61}$$

for which α describes the share of capital income.

Taking a scale invariant inequality measurement ($ineq(X)$)[73] has the effect that the (uniform) rate of interest does not have an impact on the functional distribution of capital income for a given distribution of capital. Furthermore, assume that the distribution of income and capital are not negatively correlated:[74] [75]

$$\begin{aligned} ineq(X) = ineq([1-\alpha]Y + \alpha rK) &\approx (1-\alpha) ineq(Y) + \alpha \cdot ineq(rK) \\ &= (1-\alpha) ineq(Y) + \alpha \cdot ineq(K), \end{aligned} \tag{5.62}$$

[72] Note that our annotation is in disaccord with the traditional literature in which Y does not signify labor income, but total income being the sum of labor and capital income. For sake of consistency, we, however, keep the notation from our work.

[73] For this case, a scaling of the underlying measurement - e.g. in the form of currency conversion - does not change the reported level of inequality. Amongst others, the popular indicators Gini coefficient and coefficient of variation satisfy this condition.

[74] For the unrealistic case that income and capital are negatively correlated - implying that agents with a high income have a low level of wealth and vice versa - an increase in the capital share would lead a decrease in the personal distribution of income. In this back of the envelope computation, there is no correlation between income and wealth. As we, however, show in the following, in line with empirical evidence labor income and wealth are positively correlated.

[75] For some readers it might seem surprising to compute the following equation from the previous equation. To grasp the result consider the following simple analogy. Consider mixing a cocktail out of two ingredients, a non-alcoholic beverage and a liquor. The total amount of cocktail X is given (e.g. 0.3 l). Properties of this cocktail - in particular the total alcohol content - can be derived from the share of the alcoholic ingredient. In fact, the total share of alcohol is the product of the share of the spirit $0 < \alpha < 1$ and its alcoholic content A: αA.

with the result:

$$\frac{\partial ineq(X)}{\partial \alpha} = -ineq(Y) + ineq(K) \stackrel{!}{>} 0 \leftrightarrow \frac{ineq(K)}{ineq(Y)} > 1, \tag{5.63}$$

which always holds, since - as widely documented in the empirical literature - the stock measurement (capital) is more unequally distributed than the flow measurement (income).[76]

However, the effect can also go in the opposite direction - implying that higher income inequality is followed by higher capital ratios. The rest of this section discusses this direction of causality - which we consider to be more relevant - in the framework of our model.[77]

Using the flow of debt equation, we can compute between stock and flow. Starting from equation 5.46, the stock level of debt ($D = -K$) in the steady-state ($\dot{D} = 0$) can be derived:

$$\begin{aligned}\dot{D} = Y(-1 + c_y) + YHc_w + YP\frac{E(\Delta p)}{\gamma}H + \bar{c} \\ + r_0 \exp(\mu_r D)\left[-\frac{HYP}{\gamma} + D\left(1 - c_y + \frac{P}{\gamma}\right)\right] - D\left(c_w + \frac{E(\Delta p)}{\gamma}P\right) \stackrel{!}{=} 0,\end{aligned} \tag{5.64}$$

with $s \equiv 1 - c_y$, no demand for durable assets ($\gamma \to \infty$)[78] and the interest rate being set exogenously $r_0 \exp(\mu_r D) \equiv r$ leading to:[79]

$$Y(Hc_w - s) + \bar{c} = D(c_w - rs), \tag{5.65}$$

and resulting in:

$$K = -D = Y\frac{c_w H - s}{rs - c_w} + \frac{\bar{c}}{rs - c_w}. \tag{5.66}$$

We can present this result in a function as depicted in figure 5.18. The function is of the general type:

$$K = k_y Y + k_0. \tag{5.67}$$

For the case without consumption out of wealth ($c_w = 0$), the case boils down to the Keynesian case (cf. figure 4.1) in which the level of income splitting net savers and indebted households is given by $Y^* = \frac{\bar{c}}{s}$.

[76] For the special case in which flow and stock level share the same degree of inequality ($ineq(Y) = ineq(K)$), a change in the functional distribution would not have an impact ($\frac{\partial ineq(X)}{\partial \alpha} = 0$).

[77] The numerical results presented in section 6.2.2, show that there is no clear cut relationship. The underlying mechanisms that trade-off are discussed in detail in the mentioned section.

[78] This crucial assumption shuts down the effect of demand for durables. Since we are interested in a long-run relation, this assumption is justifiable since the long-run demand for durables is zero and does not impact on wealth inequality. For a formal argument refer to section 5.5.2.

[79] Another way of deriving this result would be to solve the differential equation of 5.46 leading to $D(t) = D_0 \exp([rs - c_w]t) - \frac{Y(c_w H - s) + \bar{c}}{rsc_w}$. For the initial condition of no debt $D_0 = 0$ the same result is derived. Furthermore, for the case of $rs - c_w < 0$ the exponential term converges to zero if time goes to infinity - leading to the same steady state.

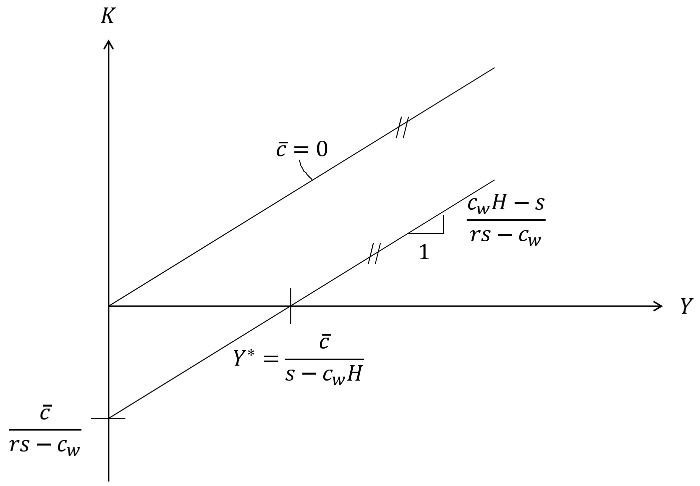


Figure 5.18.: Capital K as a function of labor income Y in the labor dominated economy ($Hr < 1$) and in the economy without conspicuous consumption ($\bar{c} = 0$)

Note that the ratio of capital (monetary assets) to labor income $\Psi \equiv \frac{K}{Y}$ depends on the level of income, if we assume the presence of a relative income motive ($\bar{c} \neq 0$):

$$\Psi = \frac{K}{Y} = \frac{c_w H - s}{rs - c_w} + \frac{\bar{c}}{Y(rs - c_w)}. \tag{5.68}$$

In fact, the ratio of capital to income increases with income:

$$\frac{\partial \Psi}{\partial Y} = -\frac{\bar{c}}{rs - c_w} \frac{1}{Y^2} > 0, \tag{5.69}$$

as long as:

$$rs < c_w, \tag{5.70}$$

is satisfied, since $Y > 0$ and $\bar{c} > 0$. This condition was already required for a convergence of capital for a non-zero initial debt ($D_0 \neq 0$) (cf. the discussion in footnote 79). To speak in economic terms, the condition requires that the return of saved income is lower than the consumption of wealth, implying that the level of capital does converge.[80] Meanwhile, for the case without subsistence consumption ($\bar{c} = 0$), the capital income ratio is identical for all agents:

$$\Psi(\bar{c} = 0) = \frac{s - c_w H}{c_w - rs}. \tag{5.71}$$

[80] As put forward more thoroughly in the following section, this also implies a negative real interest rate net of depreciation.

In this case, a change in the personal distribution of income would not have an effect on the aggregate functional distribution of income. Or, to argue in the negative direction, the presence of conspicuous consumption leads to the fact that changes in the personal distribution of income also lead to changes in the functional distribution of income. Besides the condition $c_w > rs$ previously introduced, we have to assume:

$$s > c_w H, \tag{5.72}$$

to generate a positive amount of capital for high-income agents ($\lim_{Y\to\infty}\Psi = k_y = \frac{s-c_wH}{c_w-rs} \overset{!}{>} 0$) rather than debt. In contrast to the first condition, impacting on the stock quantity debt respectively capital (see equation 5.70) the latter relates to the flow Y. The condition requires that agents save more than they consume out of the flow income. As will be discussed in the subsequent section, the same conditions are also required for a convergence of wealth.

The two conditions are also necessary to construct a realistic ratio between capital and income as presented in figure 5.18. The condition 5.70 - influencing the offset k_0 of equation 5.66 - ensures that low-income agents are indebted in the presence of relative consumption effects. Meanwhile, the combination of the two conditions - driving the slope k_y of the curve described in equation 5.66 - ensures both that capital increases with income and high-income agents hold capital (rather than debt).

We can also combine the two conditions yielding a third condition:

$$\frac{s}{H} > c_w > rs \to Hr < 1 \leftrightarrow H < \frac{1}{r}. \tag{5.73}$$

Keeping in mind that heritage H is defined as initial endowment with assets ($\frac{P_0 q_0}{Y} = H$), the condition implicitly requires that the initial endowment with assets is lower than the present value of human capital:[81]

$$P_0 q_0 < \frac{Y}{r}. \tag{5.74}$$

This is a very interesting result as it implies that the majority of wealth should come from human capital and thereby from one's own labor rather than inherited wealth. As summarized in the survey of Davies and Shorrocks (2000), there is major disagreement in the empirical literature about the level of self-made wealth as a share of total wealth. However, there is a slight consensus that in the US self-made wealth (i.e. non-inherited wealth) dominates and that its share of total wealth amounts to approximately 60% (Davies and Shorrocks, 2000, p. 656). In the term of our model, this would imply $Hr = \frac{2}{3} < 1$.[82]

The ratio of capital to labor income is a function f of the following inputs:

$$\Psi(\bar{c} = 0) = f(s[+], r[+], c_w[-], H[-]), \tag{5.75}$$

[81] The subject of human capital was already considered in section 4.2.

[82] This is the case since total wealth is given by $W = \frac{Y}{r} + HPY = \frac{Y}{r} + HY$ with $P = P_0 = 1$ making the share of self-made wealth $0.6 = \frac{1}{1+Hr} \to Hr = \frac{2}{3}$.

for which the symbols in brackets indicate the direction of the marginal effect if conditions 5.70 and 5.72 hold. While a higher savings ratio s and a higher rate of interest r increase the level of capital, higher consumption out of wealth c_w - acting like a depreciation on wealth - decrease the capital ratio. Somewhat surprisingly a high level of heritage H decreases the equilibrium value of the ratio of capital to labor income Ψ. This can be explained by the fact that this variable always works together with consumption out of wealth c_w adding to the depreciation of capital.

We can also compute the relation between net worth - i.e. capital and durable assets - and income. By assumption $P_0 q_0 = HY$ and - as shown in A.6 - this result is also stable in the long-run implying that inequality of assets is only driven by initial conditions.[83] Thus, we have:

$$W = Pq - D = HY + Y\frac{c_w H - s}{rs - c_w} + \frac{\bar{c}}{rs - c_w} = Y\frac{s(rH-1)}{rs - c_w} + \frac{\bar{c}}{rs - c_w}. \tag{5.76}$$

For the case of no subsistence consumption ($\bar{c} = 0$), the net worth to labor income ratio $\Omega \equiv \frac{W}{Y}$ is of a functional form h:

$$\Omega(\bar{c} = 0) \equiv \lim_{Y\to\infty} \Omega = \frac{s(1 - rH)}{c_w - rs} = h(s[+], r[+], c_w[-], H[-]), \tag{5.77}$$

for the conditions given in equation 5.70 and 5.72 holding. Identical to the case of capital to labor income the input factors have the same effect. Furthermore, the ratio that would prevail in a scenario without conspicuous consumption ($\bar{c} = 0$) for all agents is only reached for the richest agent in the relative consumption case ($\bar{c} \neq 0$). The case without the relative consumption effect ($\bar{c} = 0$) is also depicted in figure 5.18. It is important to point out that in this case inequality of wealth equals inequality of income ($ineq(K) = ineq(Y)$). This is the case since wealth and income in this case are just scaled by a uniform factor Ω. This result is clearly at odds with empirical evidence showing that wealth is considerably more unequally distributed than the flow quantity of income (e.g. Davies and Shorrocks (2000)). Thereby, our model requires a conspicuous consumption effect to generate realistic features of wealth inequality.

Moreover, the factor Ω is in line with the capital coefficient as computed in Piketty (2014) and also referred to in section 4.2 as κ, which includes both nominal capital (already captured in the factor Ψ) as well as real assets. The latter are in fixed supply in our model and are transferred to agents by means of inheritance. This ratio, however, is a bit more complicated than the simple ratios discussed in section 4.2. The underlying factors, however, have the same marginal impact regarding their sign.

Using aggregate data Piketty (2014) computes a total capital to total income of approx. 4.5 for Germany in 2010.[84] Using data from Bundesbank (2013) we can recapitulate the

[83] The following section elaborates more on this topic considering necessary assumption and practical implications.

[84] This value can also be interpreted in the way that it take 4.5 years to produce the amount of capital already accumulated.

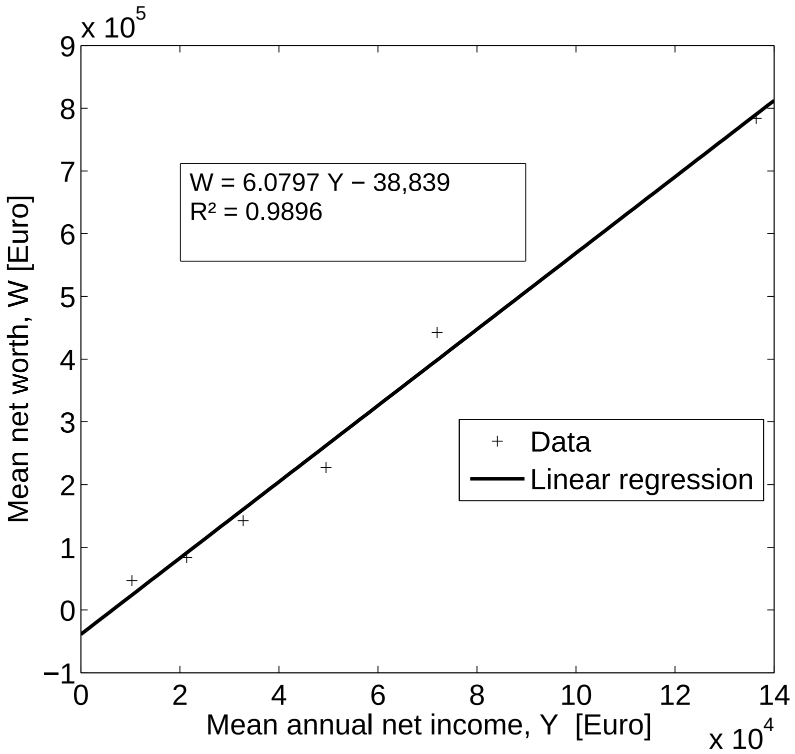


Figure 5.19.: Mean net worth as a function of mean net annual income [Euro] (Data source: Bundesbank (2013))

result from an individual perspective[85] (see figure 5.19). We confirm the negative offset - which in theory is rationalized by the presence of a minimum consumption level - and a slope of $\Omega_y \approx 6$. Note that the average or aggregate capital income ratio ($\frac{\Omega}{Y} = \Omega_y + \frac{k_0}{Y}$) is lower than the individual ratio $\Omega_y = \frac{s(1-rH)}{c_w - rs}$ due to the presence of the relative consumption term $\bar{c} \neq 0$ making $k_0 < 0$. Hence, these *micro* results are in line with the aggregate or macro results of Piketty (2014).

The aim of this section is to compute a closed-form version of individual profit share depending on individual income and resulting from the given consumption function. We can start by relating the capital income rK to labor income:

$$\frac{rK}{Y} = \frac{r(c_w H - s)}{rs - c_w} + \frac{r\bar{c}}{Y(rs - c_w)}. \tag{5.78}$$

[85] The underlying data come from the HFCS micro survey. In Bundesbank (2013), however, only 6 data points are presented representing the average net worth respectively income of 6 bins.

If this ratio exceeds one, capital income has a larger share. The profit share can be computed as follows:

$$
\begin{aligned}
\alpha \equiv \frac{rK}{rK+Y} = \left[1+\left(\frac{rK}{Y}\right)^{-1}\right]^{-1} &= \left(1+\frac{Y(rs-c_w)}{Yr(c_wH-s)+r\bar{c}}\right)^{-1} \\
&= \frac{Yr(c_wH-s)+r\bar{c}}{r\bar{c}+Yc_w(Hr-1)}
\end{aligned}
\tag{5.79}
$$

This implies that the individual capital share is a function of income. In particular, there exists a threshold level of income under which the ratio turns negative, implying that these agents are net debtors. This level is given by (also compare with figure 5.18):

$$
Y < \frac{\bar{c}}{s-c_wH} \tag{5.80}
$$

In effect, the aggregate capital share ($\alpha = \sum_{i=1}^{N} \alpha(Y_i)$) can be positive, negative or even zero depending on the distribution of income Y. Note that in the case considered in our model an aggregate capital share of zero is accompanied by a balanced current account, whereas a positive (negative) capital share is the signal of a series of current account surpluses (deficits) accompanied by claims (debt) to foreigners. Consider the following thought experiment: there is a given distribution of income, for which there is immigration into the economy of very high-income individuals with a positive capital share. This increases the level of inequality and the aggregate capital share. However, the converse can be the case. Assume that for a given distribution of income there is immigration of low-income agents who are net debtors. While - once again - the inequality increases the capital share decreases. As a result, there is no clear-cut relation between personal income distribution and the aggregate share of capital income.

It is important to point out that - given that conditions 5.70 and 5.72 hold - the individual capital share increases with total income, implying that rentiers are individuals with high (total) incomes and workers agents with low income. This is eventually in line with the empirical results as presented in (Piketty, 2014, figure 8.4 and 8.10 for France and the USA).

Once again, for high levels of income the measurement converges to the level we would perceive without a relative consumption effect:

$$
lim_{Y\to\infty}\alpha = \alpha(\bar{c}=0) = \frac{r(s-c_wH)}{c_w(1-Hr)} = h(s[+], r[+], c_w[-], H[-]) > 0, \tag{5.81}
$$

since the effect of relative consumption becomes negligible. To speak in economic terms, this implies that for a framework with relative consumption only the richest household reach the capital share any households would have in a scenario without relative consumption effects ($\bar{c} = 0$). The input factors have the well-known marginal effects. Furthermore, given the conditions 5.70 and 5.72, the ratio is always positive, implying a current account surplus economy. Moreover, this implies that the personal distribution of income does not impact on the capital share. This also repeats the argument - already presented - arguing that labor income Y and capital K in this case would follow the same distribution, which is at odds with empirical evidence. Once again, the presence of relative consumption effects is crucial for deriving realistic features.

If we assume conspicuous consumption ($\bar{c} \neq 0$) and, moreover, the specific case $rs = c_w$, high-income households extract their total income from capital rents ($\lim_{Y\to\infty} \alpha = 1$). Other than that we have, $\lim_{Y\to\infty} \alpha < 1$.[86]

Up to this point we discussed the case in which conditions 5.70 and 5.72 hold and showed that this generates realistic results. In particular, in this case low-income agents hold debt and high-income agents are lenders. As a result, the individual capital share of income α_i increases with non-capital income Y_i as well as total income ($X_i = Y_i + rK_i = rk_0 + (1 + rk_y)Y_i$) as the two are positively correlated.[87] We also showed that the conditions - which will also reoccur as necessary conditions for convergence of wealth (cf. section 5.5.2) - implied that the prime share of agents' wealth results from labor income and therefore from human capital rather than inherited wealth ($P_0 q_0 < \frac{Y}{r}$).

In his controversial work, Piketty (2014) argues that we are currently witnessing a turning point in history and he foresees a return of *Patrimonial capitalism*, for which the prime source of wealth and income is not labor but (inherited) capital. In the terms of our model this implies:

$$P_0 q_0 > \frac{Y}{r}. \tag{5.82}$$

It is easy to show that this implies a reverse of the conditions 5.70 and 5.72:

$$\frac{s}{H} < c_w < rs \to 1 < rH \to 1 < r\frac{P_0 q_0}{Y} \to \frac{Y}{r} < P_0 q_0. \tag{5.83}$$

This case is depicted in figure 5.20. Once again, individual capital K_i increases with individual labor income Y_i. However, in this case, agents with zero labor income eventually hold capital rather than debt:

$$K(Y = 0) = \frac{\bar{c}}{rs - c_w} > 0, \tag{5.84}$$

since $rs > c_w$. Thereby, all agents are net lenders and we have a current account surplus economy. The economic implication of this condition - concerning the stock - is that agents earn a higher return on capital than consuming out of stock. As a result, the stock of capital is ever increasing. Going along with the underlying assumption that the aggregate level of income is constant the capital ratio κ diverges.[88] The second condition - considering flows - necessary to sustain this result requires $s < c_w H$. This condition implies that the agents consume more than they save out of their flow income. Nevertheless, agents do accumulate wealth due to their high inherited wealth. The latter, however, also implies that the capital to income ratio decreases with income:

$$\frac{\partial \Psi}{\partial Y} = \frac{\bar{c}}{c_w - rs}\frac{1}{Y^2} = -\frac{k_0}{Y^2} < 0. \tag{5.85}$$

[86] This follows directly from the condition 5.70 requiring $c_w > rs$.
[87] This is the case since $k_y > 0$.
[88] As will be discussed in section 5.5.2, this also implies a divergence in wealth inequality.

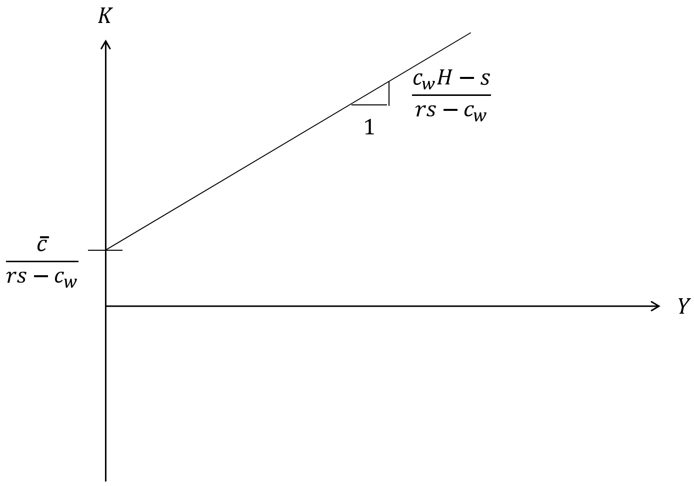


Figure 5.20.: Capital K as a function of labor income Y in a society dominated by inheritance

The same result holds true for the capital share α. This implies that those agents with a low level of current labor income have a high share of capital income. Or put more bluntly, we have poor rentiers and wealthy workers - in discordance with the empirical evidence presented in Piketty (2014). This is certainly not the case for the working population. However, this result can prevail for pensioners. In fact, the rentier society foreseen by Piketty (2014) might eventually just be a result of demographic problem. As the labor market participation rate decreases since - for a given legal age of retirement - in a world with decreasing fertility and increasing longevity, a large share of the population must rely on capital income rather than labor income to earn a living. If we redo our thought experiment, this implies that an economy with a given distribution of labor income Y experiencing migration of very high-income agents increasing the inequality of labor income eventually witnesses a decrease in the capital share. Vice versa, a migration of agents with low labor income leads to the fact that the capital share increases.

As a result, we consider the inheritance dominated society (with $Hr > 1$) an interesting thought experiment, but not a realistic description of reality. Moreover, we feel that the emphasis on the functional distribution of income is interesting and simple (mainly relying on a two agents case with workers and capitalists), although it fails to capture important underlying features. Therefore, in the following section we consider the case of wealth inequality with N agents.

5.5.2. Inequality of the Distribution of Wealth

This section discusses the necessary conditions for a convergence in wealth inequality, implying that there is no ever-expanding path for which the *rich always get richer* but in the long-run all agents are equal or at least that inequality converges to a finite level ($Gini < 1$). In a nutshell, we emphasize two conditions. Firstly, decreasing returns to capital lead to the fact that the wealth distribution converges to total equality.[89] The second condition requires a decreasing savings ratio for rich individuals. In the following, we will formalize these arguments, consider them within the normative economic literature and in the context of our model. We also relate the results to empirical evidence.

Our review follows the macroeconomics and finance literature to explain wealth inequality. We, however, do not consider the important aspect of inheritance and intergenerational transfers which is at the heart of the rationale of Piketty (2014) and discussed in the literature of family economics.

Key results of the literature of family economics (cf. e.g. Davies and Shorrocks (2000)) are that wealth inequality increases in time if wealth and fertility are negatively correlated - implying that low net worth agents split their wealth amongst a large number of descendants.[90] Assortative mating in the marriage *market* leads to increased wealth inequality. The concrete ways of inheriting also matter: As emphasized in Stiglitz (1969), primogeniture - the tradition of only bequeathing the oldest (male) offspring - increases wealth inequality. In contrast, Becker and Tomes (1976) emphasize that bequests eventually decrease wealth inequality as altruistic parents bequeath more to offspring with less human capital, providing a form of insurance. This argument, however, only applies within a particular family, yet not amongst different families for which abilities are far more unequally distributed and therefore can be considered slightly flawed.

Linear Savings Functions

We follow the argumentation of Stiglitz (1969) using a standard neo-classical growth model. The first condition requires decreasing returns to capital k or negative scale effects. This requires decreasing returns:

$$f''(k) < 0, \tag{5.86}$$

respectively convergence of returns to zero for high amounts of capital:

$$\lim_{k\to\infty} r(k) = \lim_{k\to\infty} f'(k) = 0. \tag{5.87}$$

This result is interesting, as the convergence of the complete economy to a steady state requires the same conditions as the convergence of the distribution of wealth. Or put

[89] In this context, it is interesting to point out that Marx assumed that decreasing returns to capital lead to a collapse of the capitalist system. As shown here, capitalism leads to a form of socialism - in the sense that all individuals hold the same level of wealth.

[90] As noted by McCloskey (2014) the extreme case where wealthy individuals have no descendants, however, leads to the fact that rich dynasties completely die out.

differently, in the analytical framework of a neoclassical growth model, a non-growing economy is characterized by an equally distributed wealth.

Note that in this case, we argue from a neo-classical perspective in which capital is paid according to its marginal product. Another way of formalizing this argument would be:

$$\frac{\partial \frac{\dot{k}}{k}}{\partial k} < 0. \tag{5.88}$$

In economic terms, it requires the growth-rate of capital to fall with the amount of invested capital.

The condition can also be presented as follows:

$$\lim_{k \to \infty} \frac{\dot{k}}{k} < 0. \tag{5.89}$$

To account for the second argument of decreasing savings ratios, we have to make an assumption about the consumption function. However, initially we have to describe the flow of capital. For an individual agent, it is given as follows:

$$\dot{k} = y - c = f(k) - c. \tag{5.90}$$

We normalize the equations and in particular capital in effective labor terms ($k = \frac{K}{AN}$). Input factors are assumed to be paid according to their marginal productivity:

$$y = f(k) = rk + w = f'(k)k + w. \tag{5.91}$$

Let us assume a simple linear consumption function of the type:[91]

$$c = \bar{c} + c_y y + c_w k. \tag{5.92}$$

Note that due to the existence of a subsistence level of consumption $\bar{c} \neq 0$ the savings ratio increases with income.[92] Defining $s = 1 - c_y$ this implies:

$$\dot{k} = s(f'(k)k + w) - \bar{c} - c_w k = srk + sw - \bar{c} - c_w k, \tag{5.93}$$

leading to:

$$\frac{\dot{k}}{k} = sr - c_w + \frac{sw - \bar{c}}{k}. \tag{5.94}$$

Condition 5.89 relates to the stock of capital. If we discuss the previous equation in this light, this makes a statement about the first two terms. We require:

$$\lim_{k \to \infty} \frac{\dot{k}}{k} = \lim_{k \to \infty} (sr - c_w) = \lim_{k \to \infty} (sf'(k) - c_w) < 0, \tag{5.95}$$

[91] The notation in this case is kept similar to the consumption function employed in our case. We however, employ small case letters in order to signify variables in effective labor terms.

[92] The latter part is more thoroughly discussed in section 4.3.

which holds for decreasing returns ($\lim_{k\to\infty} f'(k) = 0$) together with consumption out of capital:

$$c_w > 0. \tag{5.96}$$

If we have no scale effects for capital[93], implying that all individuals earn the same rate of return on capital r, long-run convergence requires:

$$sr < c_w. \tag{5.97}$$

As pointed out in Bertola (2000), consumption out of capital $c_w > 0$ in a standard Solow-type growth models is modeled via depreciation of capital $c_w = s\delta$ (Solow, 1956), for which δ is equal to the geometric rate of depreciation. Therefore, the latter condition is equal to the condition requiring a negative real interest rates net of depreciation $r - \delta < 0$, implying the case in which long-run capital converges to a finite level.

Following the example presented in Carroll (1998) we can make a very striking computation. Consider Bill Gates, who at the time of writing this text is claimed to be the richest single person in the world, as documented by the *Forbes List* with a net worth of approximately 82.5 billion US$ (Forbes-Magazine, 2014). If he would only consume out of his net worth and not of income ($s = 1$), a convergence of his income level in nominal terms would require $r = c_w$ (the knife-edge condition). For the case of an interest rate of 3% this would result in an amount of non-durable consumption of roughly 6.8 million US$ daily. Recall the fact that most luxury goods (e.g. jewelry or yachts) are durables and thereby only lead to a recomposition of the balance sheet and thus do not contribute to the attempted reduction of wealth. Yet, there are luxury goods that can be considered non-durables, such as vacations to exotic places or VIP-tickets to sports or cultural events. However, the *working rich* in contrast to the *idle rich* in the times of Keynes and Marx do not possess the necessary time to enjoy this sort of consumption. The new *caste* of entrepreneur rich rather than rentiers thereby also implies a stronger persistence of wealth inequality.

It might also be interesting in reconciling the convergence of wealth condition $rs < c_w$ with the results of the optimal control problem as presented in section 4.2. In this section, we showed that for an economy without income growth ($g = 0$) the result is easy to transform into a standard Keynesian case with $\bar{c} = c_w = 0$ and $c_y = \frac{\rho}{r}$. Inserting this result in the stability condition yields:

$$rs < c_w \leftrightarrow r(1 - c_y) = r\left(1 - \frac{\rho}{r}\right) < c_w = 0 \rightarrow r < \rho, \tag{5.98}$$

implying that equality of wealth only emerges for the current account deficit case - in which in the long-run agents consume an amount of zero and hold debt. This is also straightforward from intuition since it implies that - since all agents hold zero net worth - they are all identical in terms of wealth.[94]

[93] Formally this implies $f''(k) = 0$ and $f'(k) = r$ for all $k > 0$.

[94] It is, however, noticeable that all agents hold a heterogeneous level of debt identical to their human capital Y_i/r.

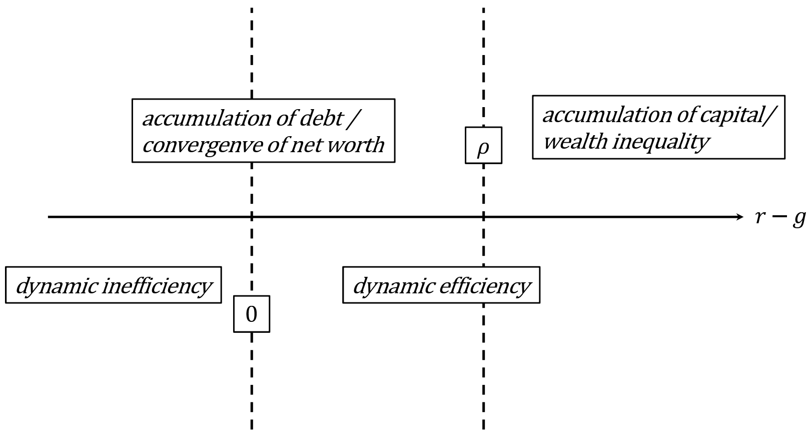


Figure 5.21.: Parameter region of dynamic (in)efficiency and wealth (in)equality

We can also discuss the case of a growing economy $g > 0$. (Deaton, 1991, p. 1237, eq. 26) argued that a convergence in wealth inequality ($Gini(W_\infty) < 1$) only emerges if the following condition holds:

$$\beta/RE(G^{-\gamma}) < 1. \tag{5.99}$$

In this case E represents the expectation operator[95] and γ the risk aversion of a CRRA-type utility function. One can reformulate this equation to make it apply to the case derived in section 4.2. As we assume that income growth does not contain a stochastic component ($E(g) = g$) , the following result can be derived:[96] [97]

$$r - \rho < \gamma g. \tag{5.100}$$

It is important to emphasize that this condition is less restrictive than the condition $g > r$ emphasized for the convergence of the capital ratio κ in the work of Piketty (2014) for all $\rho > 0$ and $\gamma \geq 1$.

For the special case of an isoelastic utility function ($\gamma = 1$) discussed in section 4.2 the condition reads as follows:

$$r - \rho < g. \tag{5.101}$$

[95] Note that this equation argues with certainty equivalents (the first moment) and does not account for noise in the income process also playing an important role in shaping the distribution of wealth.

[96] Furthermore, the derivation implies the standard log-linearization with $G = 1 + g$ and $\log(\beta) - log(R) - \gamma \log(G) < 0 \rightarrow -\rho + r - \gamma g < 0$.

[97] It is also important to note that the case without growth of income ($g = 0$) is a special case of this one requiring $r < \rho$.

We can compare this condition with the condition of dynamic efficiency requiring $r > g$. The convergence of wealth equation $r - \rho > g$ does not make a statement about dynamic (in)efficiency per se. Yet, a case with diverging wealth inequality ($r - g > \rho > 0$) is always dynamically efficient. In contrast, the case of wealth convergence $r - g < \rho$ *can* be dynamically inefficient. The relation between the two concepts is presented in figure 5.21.

As already shown in equation 4.64 there is a closed-form solution to the evolution of debt respectively capital.[98] Following Fischer (2014) we take the ratio of capital $z_t = \frac{K_{i,t}}{K_{j,t}}$ as a measurement of inequality between two agents with different labor income and inherited wealth. If this converges to one in the long-run, we have total equality ($\lim_{t\to\infty} z_t = 1$). Otherwise we can have a finite level of inequality ($\lim_{t\to\infty} z_t < \infty$) or total inequality ($\lim_{t\to\infty} z_t = \infty$), implying a Gini coefficient of 1.[99]

First, we discuss the condition emphasized by Deaton (1991) ($r - \rho < g$). In this case, we have:

$$\lim_{t\to\infty} z_t = \lim_{t\to\infty} \frac{\left(\frac{Y_i}{r-g} - D_i\right)\exp([r-\rho-g]t) - \frac{Y_i}{r-g}}{\left(\frac{Y_j}{r-g} - D_j\right)\exp([r-\rho-g]t) - \frac{Y_j}{r-g}} = \frac{-Y_i}{-Y_j}. \tag{5.102}$$

As a result, all agents have zero net worth and accumulated a level of debt that is equivalent to their human capital ($\frac{Y_i}{r-g}$ respectively $\frac{Y_j}{r-g}$).

The knife-edge case ($r - \rho = g$), emphasized by asset pricing theory (cf. section A.3), yields:

$$\lim_{t\to\infty} z_t = \frac{-D_i}{-D_j}. \tag{5.103}$$

In this case, the inequality only depends on the inherited capital or debt. Labor income does not matter since it is completely consumed as $c_y = \frac{\rho}{r-g} = 1$. This is the case emphasized by Piketty (2014). Or put differently, if there was no inheritance ($D_i = D_j = 0$), there would also be no wealth inequality for this case.

The third case - the converse condition of Deaton (1991) - $r - \rho > g$ leads to the following result:

$$\lim_{t\to\infty} z_t = \frac{\frac{Y_i}{r-g} - D_i}{\frac{Y_j}{r-g} - D_j}. \tag{5.104}$$

Once again there is a finite level of inequality, this time, however, depending on both an unequal flow of labor income and inherited stock of capital. Moreover, the level of inequality is growing in time ($\frac{dz_t}{dt} > 0$ since $r - \rho - g > 0$). Note that the case of the non-growing economy ($g = 0$) is nested within this case. The knife-edge case is $r = \rho$ and the cases of convergence (divergence) are $r < \rho$ (respectively $r > \rho$).

[98] It is important to point out that this closed-form solution only holds for the case of dynamic efficiency $r > g$. In the converse case, the growth rate of labor income would always exceed the interest level on capita, implying an infinite value of human capital posing no constraint on acquiring debt.

[99] To better grasp the ratio z_t as a measurement of inequality it is important to note that it is related to the Gini coefficient in the following manner: $Gini_t = \frac{z_t - 1}{z_t + 1}$ for $z_t > 1$ with $K_{i,t} > K_{j,t}$. Thereby, we have $Gini(z = \infty) = 1$ as well as $Gini(z = 1) = 0$.

In general, these results also confirms the general trade-off between efficiency and equality (Okun, 1975). In an economy with positive accumulated capital, agents are heterogeneous in terms of capital measurable in wealth inequality. On the other hand, an economy in which all agents are identical lacks accumulation of capital.

The stability condition is also in line with our Keynesian consumption function as presented in section 4.2. For the consumption function assumed for the growth case we require:[100]

$$r - g < \bar{c}_w, \tag{5.105}$$

for a convergence to total equality ($Gini(W_\infty) = 0$). For the computed value of $\bar{c}_w = \rho$, we have the well-known condition:

$$r - g < \rho. \tag{5.106}$$

We can also compare this result to the work of Piketty (2014) who argues that wealth diverges in the long-run if $r > g$. Implicitly he assumes that $\rho = 0$. This assumption is accompanied by the other implicit assumptions that there is no consumption of wealth ($\bar{c}_w = \rho = 0$) and the fact that all current income is saved ($s = 1 - \bar{c}_y = 1 - \frac{\rho}{r-g} = 1$). Thus, Piketty (2014) overestimates the region in which wealth inequality prevails (also cf. with figure 5.21). On the other hand, if one follows Piketty (2014) - making the assumption of $\rho = 0$ - in particular as its value is not directly observable in empirical data - the conditions of dynamic inefficiency and convergence of wealth inequality coincide in the equation $r < g$ (also cf. with figure 5.21).

To summarize, in all cases inequality converges to a finite level. For the case in which $r - g < \rho$ (deficit economy) in the long-run all agents hold zero wealth and thereby are identical. The level of debt, however, depends on their individual human capital. For the inverse case ($r - g > \rho$), wealth inequality converges to a finite level depending on both human capital and inherited wealth. For the knife-edge case in the growing economy ($r - \rho = g$), wealth inequality converges to a finite level only depending on initial conditions (i.e. inherited wealth). Note that we assumed a uniform rate of time preference ρ. As already shown in Becker (1980), if agents have heterogeneous time preferences in the long-run, all wealth is held by the single agent with the lowest time preference ρ_{min} having the highest growth rate of capital $g_K = r - \rho_{min} > 0$. As already shown a high rate of time preference ρ implies a high savings ratio leading to the fact that in a scenario with heterogeneous savings rates the richest agent is the agent with the highest savings ratio (Bernardo et al., 2014).

Piketty (2014) - seeing himself in a tradition of economists such as David Ricardo and Karl Marx - claims to have found a *fundamental contradiction* in the capitalist system - where the dominant scenario $r > g$ leads to a long-run divergence in wealth inequality. This bold claim sparked a large amount of comments by other economists trying to proof him wrong. Nevertheless, all commentators agreed with the fact that high levels of interest rate r and low growth rates g contribute to a high level of wealth inequality.

[100] This is easy to verify if we keep in mind that debt-income ratio ($\frac{D}{Y} \equiv -\frac{K}{Y}$) grows with $r - g$ while it is depleted by consumption out of wealth $\bar{c}_w$.

The simple relation $r > g$, however, holds only under very specific and unrealistic assumptions. In particular, he takes the unrealistic assumption of zero consumption out of wealth as well as current income ($c_w = c_y = 1 - s = 0$). In result the *fundamental law of capitalism* $r > g$ of Piketty (2014) does not inevitably lead to complete wealth inequality.[101]

Commentators referring to the Post-Keynesian literature - in particular to influential work of Kaldor, Kalecki, and Pasinetti - accuse Piketty (2014) of conducting a *fallacy of composition* error by equating the economic wide profit rate with the growth rate of individual wealth growth (Bernardo et al., 2014). The Post-Keynesian literature argue in a framework with two agents - the capitalists capturing total profits and the worker capturing the residual wage income - with heterogeneous saving rates out of flow income[102] ($s_c > s_w$ [103] with c for capitalists and w for workers). In this case the condition for ever increasing wealth inequality is only determined by the capitalist $s_c r > g$.[104] As in equation 5.97 this emphasizes the savings out of capital income. Nevertheless, for any realistic savings ratio $s_c < 1$ this is less restrictive than the case of $r < g$ brought forward by Piketty (2014).

In this part, we assumed consumption functions without subsistence consumption ($\bar{c} = 0$). As already put forward in the previous section, the latter, however, plays a crucial role in determining the distribution of wealth. To incorporate the subsistence consumption effect we have to consider condition 5.88 focusing on the flow of income. Using this condition we derive:

$$\frac{\partial \frac{\dot{k}}{k}}{\partial k} = -\frac{sw - \bar{c}}{k^2} < 0 \rightarrow sw > \bar{c} \tag{5.107}$$

This requires a positive degree of net savings. All households not accounting for this condition will converge to a steady state of zero capital. It is also important to point out that this condition is always satisfied for the standard Solow-case assuming no subsistence consumption $\bar{c} = 0$ and $0 < s < 1$. In fact, this condition refers to the last two terms of equation 5.94 and thereby accounts for the flow terms - in contrast to the first condition derived from equation 5.89 making a statement about the stock terms.

We can also apply this rationale to our model. Therefore, we consider our concrete flow equation (see equation 5.46) in the light of condition 5.88. Note that debt and capital

[101] If one reads the work of Piketty (2014) precisely (including online technical appendix), he himself is more modest. Nevertheless, the latter simple relation is easier to sell to a large audience and even works as a print on a t-shirt or a graffiti on a wall.

[102] Note that this literature does not capture consumption out of wealth implicitly assuming $c_w = 0$.

[103] This assumption is necessary to rule *euthanasia of the rentier* as coined in the last chapter of Keynes (1936) implying a long-run convergence of the capital share to $\alpha = 0$ (Taylor, 2014).

[104] Taylor (2014), furthermore, adds that r should be replaced by the effective rate of interest excluding taxes on capital and capital income as well as depreciations.

are mirror images $k = -D$ leading to the following flow equation for an exogenous rate of interest r:

$$\begin{aligned}\dot{k} = Ys - Hc_wY - \bar{c} - YH\frac{PE(\Delta p)}{\gamma} - r\left[-HY\frac{P}{\gamma} - k\left(s + \frac{P}{\gamma}\right)\right] \\ -k\left(c_w + \frac{PE(\Delta p)}{\gamma}\right).\end{aligned} \tag{5.108}$$

The condition is of the form:

$$\frac{\dot{k}}{k} = A + \frac{B}{k}, \tag{5.109}$$

for which the necessary condition for convergence are (resulting from condition 5.88:[105])

$$-\frac{\partial\frac{\dot{k}}{k}}{\partial k} = B > 0, \tag{5.110}$$

and (resulting from condition 5.89):

$$\lim_{k\to\infty}\frac{\dot{k}}{k} = A < 0. \tag{5.111}$$

The concrete values are:

$$A = r\left(s + \frac{P}{\gamma}\right) - c_w - \frac{PE(\Delta p)}{\gamma} = rs - c_w + \frac{P}{\gamma}[r - E(\Delta P)], \tag{5.112}$$

and:

$$B = -\bar{c} + Y\left[s + H\left(-c_w + \frac{P}{\gamma}[r - E(\Delta p)]\right)\right], \tag{5.113}$$

with the stability condition satisfied if:

$$B > 0 \to \frac{\bar{c}}{Y} < s + H\left(-c_w + \frac{P}{\gamma}[r - E(\Delta p)]\right), \tag{5.114}$$

and:

$$A = rs - c_w + \frac{P}{\gamma}[r - E(\Delta P)] < 0 \tag{5.115}$$

For the special case of no *Flight effect* or *Portfolio effects* ($\gamma \to \infty$) already assumed in the previous section, the first condition is the condition for a primary surplus (see equation 5.47):

$$\frac{\partial\frac{\dot{k}}{k}}{\partial k} < 0 \to \frac{\bar{c}}{Y} < s - Hc_w. \tag{5.116}$$

This implies that all households that earn beyond a specific level - guaranteeing them to accumulate a surplus - converge to the same final level of capital.

[105] This is the case since $\frac{\partial\dot{k}/k}{\partial k} = -\frac{B}{k^2} < 0$. with $B \neq f(k)$.

For infinite risk aversion ($\gamma \to \infty$) the second condition converges to the already derived condition:

$$\lim_{k\to\infty} \frac{\dot{k}}{k} < 0 \to rs < c_w. \tag{5.117}$$

In fact, these conditions are similar to the conditions 5.70 and 5.72 of the previous section. In this section we presumed that interest rate is exogenous and no flight effects prevail ($\gamma \to \infty$). The condition 5.70 leading to a negative capital ratio for low-income households ($k_0 < 0$) and an increasing capital to labor income ratio for increasing income ($k_y > 0$) is identical to the condition 5.117 requiring a convergence of net returns to a negative level for high incomes.

Condition 5.72 argues from a flow rather than a stock perspective and ensures that capital rather than debt is accumulated. This condition boils down to condition 5.116 for the case without relative consumption effects ($\bar{c} = 0$). The condition is necessary to assure decreasing returns to capital in an environment without conspicuous consumption effects ($\bar{c} = 0$) . If there is no relative consumption effect, all households maintain a sustainable amount of consumption without debt, the economy has a surplus, and all households converge to the same level of wealth as the savings and consumption out out wealth ratio is balanced (cf. with figure 5.18).

Non-linear Savings Functions

Up to this point, increasing savings ratios were modeled via a linear *Keynesian* mechanism of subsistence consumption. As already put forward in Stiglitz (1969), the behavior becomes more complex for a non-linear savings ratio. Let us assume a general non-linear savings function $s(k)$ depending on the level of capital k. The flow equation reads as follows:

$$\dot{k} = s(k) \leftrightarrow \frac{\dot{k}}{k} = \frac{s(k)}{k}. \tag{5.118}$$

Condition 5.88 [106] requires:

$$\frac{\partial \frac{\dot{k}}{k}}{\partial k} = \frac{s'(k)k - s(k)}{k^2} < 0 \to s'(k)k < s(k), \tag{5.119}$$

which is equivalent to $s''(k) < 0$, implying a concave savings function, respectively a convex consumption function.

However, we can also follow the modeling of Carroll (1997) assuming buffer stock savings. Following the rationale presented in section 4.4 we can model consumption as a function out of cash-on-hand being the sum of current (flow) income and current stock of wealth ($y + k$):

$$c = (y + k)^{\theta}. \tag{5.120}$$

[106] We can also compute the result using condition 5.89 requiring $\lim_{k\to\infty} \frac{\dot{k}}{k} = \lim_{k\to\infty} \frac{s(k)}{k} < 0$. Using the rule of l'Hôpital this leads to $\lim_{k\to\infty} s'(k) < 0$ identical to $s''(k) < 0$.

For $0 < \theta < 1$ this function is concave and the savings function is convex, implying that the savings ratio increases for high incomes. The savings function is given as follows:

$$\dot{k} = s(k) = y - c = y - (y + k)^\theta = f(k) - [f(k) + k]^\theta. \tag{5.121}$$

Using the condition for the second partial derivative with respect to capital yields:[107]

$$s''(k) \equiv \frac{\partial^2 s(k)}{\partial k^2} < 0 \rightarrow f''(k) \left(-\frac{[f(k) + k]^{2-\theta}}{\theta} + [f(k) + k] \right) > (1 - \theta)(f'(k) + 1). \tag{5.122}$$

For the simple case of a constant return to scale technology ($f''(k) = 0$) - as also prevailing in our model - this results in:

$$\theta > 1, \tag{5.123}$$

in diametrical result to the empirically confirmed convex savings function (cf. section 4.4). As a result, in a scenario were all agents earn the same uniform return on capital, a convex savings function leads to the fact that wealth inequality diverges. In the very long-run ($t \rightarrow \infty$), the distribution of wealth converges to a Gini of one.

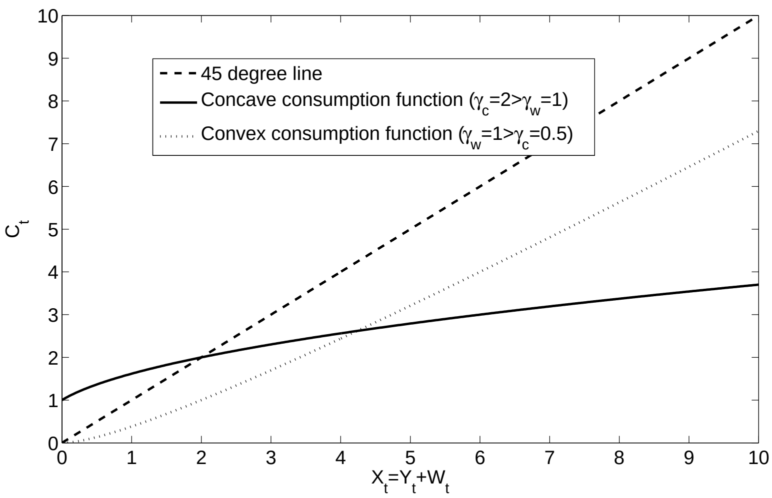


Figure 5.22.: Consumption C_t as a function of cash-on-hand ($X_t = Y_t + W_t$) for different degrees of risk aversion

In contrast, the condition $\theta < 1$ implies a savings ratio that decreases with income and wealth thereby contributing to a convergence of wealth to a Gini of zero. Consider the optimization problem presented in section 4.4 adopted from the work of Carroll (1997). It was assumed that utility cannot only be derived from current consumption C_t but also from future wealth W_{t+1}, leading to the following utility function:

$$U = u(C_t) + v(W_{t+1}) = \frac{C_t^{1-\gamma_c}}{1 - \gamma_c} + \frac{W_{t+1}^{1-\gamma_w}}{1 - \gamma_w}. \tag{5.124}$$

[107] The first partial derivative is $\frac{\partial s(k)}{\partial k} = f'(k) - \theta[f(k) + k]^{\theta-1}(f'(k) + 1)$.

The assumption $\gamma_c > \gamma_w$ made in section 4.4 implies that agents derive higher utility from wealth than from consumption. This is the case because - for a given level of consumption (or wealth) - the level of marginal utility diminishes if the risk aversion increases. The optimization problem is subject to a flow constraint:

$$W_{t+1} = (1+r)(W_t + Y_t - C_t), \tag{5.125}$$

resulting in the following optimality condition:

$$C_t^{\gamma_c} = \frac{(Y_t + W_t - C_t)^{\gamma_w}}{(1+r)^{\gamma_w - 1}}. \tag{5.126}$$

Let us now assume - conversely to the case presented in section 4.4 - that agents derive more utility from consumption than from wealth, with the coefficients being $\gamma_w = 1 > \gamma_c = 0.5$, leading to the following consumption function:

$$C_t = 0.5(1 + 2(W_t + Y_t) - \sqrt{1 + 4(W_t + Y_t)}). \tag{5.127}$$

The resulting consumption function is depicted in figure 5.22. Not only is the consumption level always lower than cash-on-hand ($X_t = Y_t + W_t$), depicted by the 45-degree-line in figure 5.22), but it is also a convex function of cash-on-hand. This implies that - in the case in which agents prefer consumption relative to wealth - the savings ratio decreases with income (and wealth). This is identical to a value of $\theta > 1$ and thereby satisfies the condition of a convergence of wealth. As a result, the presence of a love of wealth motive in the utility function *per se* does not imply higher savings ratios for high-income individuals. The latter is only the case if agents derive more utility from wealth than from consumption.[108]

Consider a more simple case, in which the savings ratio is given as follows:[109]

$$s(y) = y^{1/\theta}, \tag{5.128}$$

only depending on current income y. Assume that production technology evolves according to a standard Cobb-Douglas type specification ($y = f(k) = k^\alpha$); this leads to:

$$s(k) = k^{\alpha/\theta}, \tag{5.129}$$

with the necessary condition for stability:

$$\frac{\partial^2 s(k)}{\partial k^2} = \frac{\alpha}{\theta}\left(\frac{\alpha}{\theta} - 1\right) k^{\frac{\alpha}{\theta} - 2} < 0 \rightarrow \alpha < \theta. \tag{5.130}$$

For a scale-free technology ($\alpha = 1$) this condition holds for any $\theta > 1$, implying a concave savings function. A standard Cobb-Douglas type technology with decreasing

[108] It is also important to note that if agents assign the same risk aversion to consumption and wealth $\gamma_c = \gamma_w$, consumption is a linear function of cash-on-hand, even if heterogeneous weights are assigned to the different sources of utility.

[109] This rationale follows a similar argument presented in Fischer (2014).

returns to scale ($0 < \alpha < 1$) allows for a convex savings function $1 > \theta > \alpha$ up to a certain extent going along with an increasing savings ratio for high-income households. The opposed case with a positive scale technology $\alpha > 1$ only allows for concave savings function $1 < \alpha < \theta$. Note that in this case - opposed to the results presented earlier - the conditions for equality of wealth are weaker.

As already shown, the standard Solow-type framework with $\bar{c} = 0$ and $0 < s < 1$ leads to a convergence of wealth for decreasing returns. The impact of decreasing returns is elaborated more thoroughly in Fischer (2014).[110] The basic idea is the following. Assume that capital evolves according to the following production function:

$$k_{t+1} - k_t = y - c = sy = sf(k_t) = sk_t^\alpha. \tag{5.131}$$

For the case of $0 < \alpha < 1$, this is the standard Cobb-Douglas production technology with decreasing returns to scale. Assume two individuals, one rich individual (index r) with a high level of initial capital and a poor individual (index p) with a low level of capital ($k_{r,t} > k_{p,t}$). We rewrite the ratio of the two levels of capital as $z_t \equiv \frac{k_{r,t}}{k_{p,t}} > 1$ implying the following recursive production function:

$$z_{t+1} = z_t^\alpha, \tag{5.132}$$

which boils down to:

$$z_t = z_0^{\alpha^t}. \tag{5.133}$$

For $\alpha < 1$, in the long-run this converges to total equality:

$$\lim_{t\to\infty} z_t = \lim_{t\to\infty} z_0^{\alpha^t} = z_0^0 = 1 = \lim_{t\to\infty} \frac{k_{r,t}}{k_{p,t}}. \tag{5.134}$$

The economic rationale is also straightforward: bigger investments projects yield a lower return to investment, implying a lower growth of capital. Considering this idea from a long-run growth international growth perspective implies that for a given available production technology all countries will converge to a well-defined level of capital independent of the initial conditions. The latter would also lead to the fact that inequality in living standards between countries would disappear.

Portfolio Decision

So far we argued from a (standard) macro-perspective drawing on the properties of an aggregate production function to make statements about wealth inequality. In this section, we take the perspective of finance which explains the heterogeneous returns on capital as a result of (risky) portfolio compositions. In particular, we draw on the normative result of portfolio optimization as presented in Merton (1969). When deciding on the level of leverage $l > 0$ (equivalent to the ratio of risky assets in the portfolio) and therefore on the portfolio composition between risky assets $R \sim N(\mu_R, \sigma_R^2)$ with

[110] This paper features a far more sophisticated model also covering other important aspects. In particular, we discuss the N agent case rather than the simplified $N = 2$ agents case.

$\mu_R > r_f$ and risk-free asset $r_f \sim N(r_f, 0)$, - as already presented in section 5.1 - an optimal portfolio structure is given by:

$$l = -\frac{U'}{U''W}\frac{\mu_R - r_f}{\sigma_R^2}. \tag{5.135}$$

The portfolio return is given by:

$$r_p = l\mu_r + (1-l)r_f = r_f + l(\mu_r - r_f). \tag{5.136}$$

For the special case of CARA utility.[111] the following results can be derived:

$$l = \frac{1}{\eta \cdot W}\frac{\mu_R - r_f}{\sigma_R^2}, \tag{5.137}$$

implying that wealthy individuals have lower portfolio returns as emphasized in this section as necessary condition for convergence of wealth to total equality.[112] However - as already put forward in the original paper of Merton (1969) - it seems highly unrealistic that wealthy individuals hold less risky portfolios. Moreover, it is at odds with empirical evidence. Yitzhaki (1987) and more recently Bivens and Mishel (2013) show that wealthier investors have a higher return on investment.

In fact, positive scale effects for investments are not irrational. In contrast to very conservative investment products such as life insurances, riskier investments - for example hedge funds yielding a higher return - require a high level of minimum investment not available to low-income households. Once again level effects matter: wealthier individuals have a higher buffer and thereby can have a larger speculative share.

In this work, we take the standard assumption of CRRA utility. For the standard case of CRRA utility[113] this yields:

$$l = \frac{1}{\gamma}\frac{\mu_R - r_f}{\sigma_R^2}. \tag{5.138}$$

implying portfolio composition and thereby return is independent of wealth W. This is the case we assume in this work. Thus, the growth of the portfolio (i.e. the rate of return given the assumption that agents do not consume out of their portfolio) is identical for all agents.

We can discuss the evolution of debt in a similar manner as presented in the part starting from equation 5.132. For the CRRA case we have a constant scale technology $\alpha = 1$, implying that:

$$\lim_{t\to\infty} z_t = \lim_{t\to\infty} z_0^{\alpha^t} = z_0^1 = z_0. \tag{5.139}$$

[111] Formally, this can be captured by a utility function of the type $U(W) = \frac{-\exp(-\eta W)}{\eta}$ for which we have $-\frac{U'}{U''W} = \frac{1}{\eta W}$.

[112] Or to use the terminology of an aggregate production function, the portfolio composition derived from a CARA utility function can be described as a production function of the type for which $\frac{W_{t+1}-W_t}{W_t} \sim \frac{1}{W_t} \leftrightarrow W_{t+1} \sim \ln(W_t)$.

[113] Formally, for the case in which utility from wealth W is given by $U(W) = \frac{1}{1-\gamma}W^{1-\gamma}$, we have $-\frac{U'}{U''W} = \frac{1}{\gamma}$.

	Cash or cash-equivalents	**Bonds or stock market equity**	**Private business**	**Real estate**
Return and riskiness	Low	Medium	High	Medium
Liquidity	High	Medium	Low	Low
Tangibility	Medium	Low	Low	High
Inflation protection	Low	Medium	High	High
Function of age	Increasing	Increasing	Decreasing	Inverse u-shape
Function of wealth	Decreasing	Increasing	Increasing	Inverse u-shape

Table 5.1.: A qualitative analysis of the households portfolio composition

In this case, as a result of the portfolio structure wealth inequality does not increase in time. On the other hand, it also does not converge to equality. In fact, it remains at the very level of initial inequality. In this case, a social planner could influence the distribution of wealth by intervening a single time, namely at the point of heritage at which the initial endowment is determined. We discuss this result more formally in A.6 also showing that - in the presence of collateral constraints - the inequality of wealth is increases in times of increasing prices and vice versa.

It is important that we assumed that there is no idiosyncratic portfolio risk. Fernholz and Fernholz (2014) take a similar model in which agents have CRRA preferences, but are also subject to an idiosyncratic uninsurable risk. In this case, long-run wealth distribution converges to total inequality.[114] It is also important to point out that for CARA preferences the model would converge to a finite level of inequality, ($0 < Gini(W) < 1$) in which the level of inequality is a function of the level of idiosyncratic risk.[115]

The empirical literature also discusses the portfolio composition of households. Shorrocks (1982) conducts a thorough analysis of household portfolios under different characteristics. In general, the author finds a positive correlation between age and the level of wealth. We summarize these findings in table 5.1. Shorrocks (1982) identifies four major categories with different characteristics: (1) cash or cash-equivalents, (2) bonds or stock market equity, (3) private business, (4) real estate which also correlates with debt. One can summarize these findings that both poor and old agents hold liquid assets. High net worth agents hold (risky) assets yielding high returns - contributing to increased wealth inequality. With increasing age these agents, however, shift the portfolio weight to more liquid positions also yielding lower returns. In particular, as individuals get older they pass on their real estate and their private business to their offspring generation. Glover

[114]More technically, the presence of idiosyncratic risk leads to the fact that the variance - and thereby inequality - grows without bounds.

[115]The latter result is more thoroughly discussed in Fischer (2014).

et al. (2011) formally - based upon on an optimization problems - show that the share of risky assets should decline with age and confirm this with data.[116]

The rich tend to hold intangible assets, whereas low and middle-income agents prefer tangible assets. In our case, cash is rather tangible since it can be easily converted into consumption. Real estate, by means of serving as a collateral, can also be considered tangible. Moreover, self-used real estate serves the purpose of a pure consumption good making it highly tangible. Old and poor agents severely lose from inflation due to their strong cash holdings.

A more recent analysis drawing on the newly established *Household Finance and Consumption Survey* (HCFS) for the Euro area conducted under the umbrella of the European Central Bank is presented in Arrondel et al. (2014) especially discussing the key role of real estate. They find that real estate - in particular self-used real estate - is the major asset for private households. Different average participation rates in real estate can rationalize different median wealth levels. Especially, the low average participation rate in Germany is a candidate explanation for Germany being the Euro country with the lowest median wealth in the sample. Moreover, inheritance is important since households that received an inheritance also exhibit a higher probability of holding real estate. Interestingly, highly educated - in general also having a higher income and higher wealth - hold a lower share of their portfolio in real estate, potentially reflecting their higher mobility. In general, marriage status also has an important impact on the portfolio composition. Singles hold a higher share of risky financial assets compared to those married with children, for who real estate is of importance.

A Short Summary on Wealth Inequality

This section summarized the conditions for a convergence of wealth inequality. In particular, convergence depends on the relation between savings ratio and income respectively wealth as well as the return on savings resulting from a portfolio decision. In a constant return to scale framework - implicitly imposed by the assumption of the homothetic CRRA utility function - wealth converges in a non-growing economy if the condition $sr < c_w$ holds. In contrast to the framework of Piketty (2014) - for which equality of wealth is only ensured for $r < 0$ in a non-growing economy - the presence of consumption out of wealth $c_w \neq 0$ leads to the fact that wealth levels also converge. We also showed that for fairly general conditions the consumption ratio of net worth can be described by the time preference of agents ($c_w = \rho$). Note that the rate of interest is not constant and exogenous. As we will discuss more thoroughly in our simulation results, in our model the long-run return of interest decreases in time and with the amount of created credits. This is not the result of an exogenously assumed aggregate production function with negative scale effects, but follows from decreasing demand for debt by low-income agents due to binding collateral constraint in the presence of a constant supply of savings by high-income agents. As the rate of interest decreases, in the long-run the stability

[116] It is important to note that they categorize real estate - being the the key asset position of private households with a declining share as a function of age - as being risky.

condition $rs < c_w$ is satisfied for a given savings ratio s and consumption ratio out of wealth c_w.

In the presence of subsistence level effects $\bar{c} \neq 0$ - as assumed in our model - two groups of agents emerge: low-income agents being debtors and high agents being lenders contributing to a substantial net worth inequality. Each group of agents, however, converges to a constant wealth to income ratio. In the case of non-linear savings function (resulting from the fact that agents derive a high utility of wealth), also supported by empirical data (cf. section 4.4) wealth inequality also diverges.

6. Simulation Results

> The map is not the territory [...]
> The only usefulness of a map
> depends on similarity
> of structure between the empirical
> world and the map.
>
> (Korzybski, 1933, p. 6)

The previous section has already provided an insight into the workings of the model. To capture the full model dynamics, however, we, have to rely on numerical simulations. One of the major challenges when working with ABMs is the calibration of the models. Since the model involves several mutual interactions amongst different variables in a non-linear fashion, complex, even chaotic, dynamics may emerge. We assume that one simulation period equals one year. In particular, this implies that the computed rate of interest is an annual rate of interest.[1] Some parameters can be observed in business reality whilst other (especially the behavioral parameters) are hard to measure and therefore leave a lot of space for tuning. As already discussed, the dynamics, moreover, depend severely on the coping mechanism of the low-income class. Therefore, we present different simulations for the *Austerity*-case (section 6.1) and the *fire sales* case (section 6.3).

After comparing the model results with empirical evidence, we can also point to the underlying rationale for the stylized facts we are able to reproduce. Moreover, our model allows us to produce counterfactuals - in particular a variation of inequality - in order to quantify the macroeconomic impact of inequality. The model itself and its benchmark calibration presented here will be further employed to conduct some policy experiments as discussed in section 7.

6.1. Simulation Results of the Complex Model in the Austerity-Case

In this section we discuss the case in which constrained households react by cutting back on their consumption which we label the *Austerity*-case. Section 6.1.1 discusses the

[1] The model is run for $T = 500$ years. Note that agents do not represent single individuals but complete dynasties. There is no general consensus when modeling the discrete time steps. While some ABM-modelers interpret a discrete time step as being a year (Dosi et al., 2006), other interpret is as months (Deissenberg et al., 2008).

concrete modeling of this case, whereas section 6.1.2 presents the associated simulation results. Note that the robustness checks as discussed in section 6.2 are also based upon this version of the model.

6.1.1. Modeling the Austerity-Case

As already discussed, the reaction of the collateral constrained households has a decisive impact on the dynamics of the model. In this section, we start by presenting the *Austerity*-case for which collateral constrained households decide to decrease their consumption as opposed to their initial plan dictated by their consumption functions. We assume that agents lever up as much as possible in order not to fall short of their consumption plans:

$$\dot{D}_{i,t}^{max} = D_{t+1}^{max} - D_{i,t} = (1-m)P_{i,t}q_{i,t} - D_{i,t}. \tag{6.1}$$

If, however, this is not sufficient, agents start by refraining from taking positions in the market for durables ($d_{i,t} = 0$).[2] As presented in section 5.3, if households already accumulated debt, there might be forced savings or more precisely deleveraging (the group 0 as presented in the schematic figure 5.10). This eventually can lead to the unrealistic result of negative consumption. Since at this point we disregard the possibility of private bankruptcy, the notion of negative consumption and delevarging can ensure positive net worth households. While both cases - negative net worth and negative consumption - are unrealistic to a certain extent we prefer allowing negative net worth for households indicating delay in filing for bankruptcy. This is modeled as follows for the change in debt:

$$\dot{D}_{i,t} = max\{\dot{D}_{i,t}^{max}, 0\}, \tag{6.2}$$

and the consumption:

$$C_{i,t} = max\{\dot{D}_{i,t} + Y_{i,t} - r_t D_{i,t}, 0\}. \tag{6.3}$$

The latter equation implies that low-income agents facing their collateral constraint react by spending all disposable income - i.e. all available labor income less interest on outstanding debt.

Both modeling approaches introduce strong nonlinearities that feed back into the model economy, especially if a lot of agents belong to the last group. Moreover, if the economy is dominated by lower-class households, trading in the market for durables breaks down.[3]

6.1.2. Basic Simulations

In this section, we show simulation results of the complex model. Rather than performing an econometric calibration we decided to rely on a reasonable tuning of the

[2] It would be more reasonable to only refrain from taking long positions $d_{i,t} = min\{0, MPCD_t \cdot W_{i,t}\}$. This would lead to a downward pressure. However, at this point we control for this effect. In the latter in which we also discuss fire sales this will be of importance.

[3] For the extreme case we have $d \to 0$ leading to $P = P_0 \equiv F$.

model allowing it to generate realistic features. The model not only requires making assumptions about the value of parameters but is also sensitive to the assumed initial conditions. Our tuning heavily relies on the closed-form solutions conducted in section 5. The chosen parameters are summarized in table 6.1.

Category	**Symbol**	**Description**	**Value**
Distribution	$\bar{Y}$	Median labor income	5
	σ_y^2	Inequality of labor income	1
	H	Heritage	20
Durable market	β_F	Aggressiveness of fundamental traders	1
	β_C	Aggressiveness of chartist traders	1
	Γ	Rationality	1
	Λ	Memory	0.98
	γ	Risk aversion	20
	$\sigma(d_{noise})$	Standard deviation of noise trading	0.05
Consumption	j	Quantile of subsistence consumption	0.2
	c_y	Marginal propensity to consume out of income	0.5
	c_w	Marginal propensity to consume out of net worth	0.01
	ε	Curvature of the consumption curve	1
Collateral constraint	m	Equity requirement	0.2
Markets	μ	Market illiquidity for durables	0.01
	μ_r	Market illiquidity for durables	0.005
Initial conditions	r_0	Initial interest rate	0.02
	$D_{i,0}$	Initial debt for all agents	0
	P_0	Initial price level in durable market	$1 \equiv F$

Table 6.1.: Benchmark simulation parameters and initial conditions

For the distribution parameter we assume an arbitrary value of $\bar{Y} = 5$ and $\sigma_y^2 = 1.0$, leading to a Gini coefficient of roughly 0.5, broadly consistent with US data for income distribution before redistribution.[4] We run the simulation for $N = 1,000$ agents and $T = 500$ periods.

The behavioral parameters of the model for durable consumption are set according to several well-established models explaining the behavior of financial markets by means of a model with fundamental and chartist traders (cp. e.g., De Grauwe and Grimaldi (2006)). The noise demand operates with a fixed random seed to ensure reproducibility of the results. The assumed risk aversion $\gamma = 20 >> 1$ is rather high, but in line with the literature rationalizing the high equity premium respectively the low risk-free rate of interest. The value is actually the value assumed to rationalize the equity premium for the US by means of the Consumption Capital Asset Pricing Model (CCAPM) (Semmler, 2007, p. 123).

The most important parameters are those governing the consumption decision. We present and rationalize a feasible benchmark parametrization. In section 6.2.1, we furthermore conduct a numerical robustness analysis in a *ceteris paribus* fashion. First of all, we set the level of the minimum consumption quantile to $j = 0.2$. For the given distribution of income, this implies a ratio of $\frac{\bar{c}}{E(y)} \approx 0.25$. For heritage we assume a rather high value of $H = 20$, implying that each agent is initially endowed with an equivalent of 20 years of annual income. Thereby the share of self-earned wealth amounts to approx. 71 %.[5]

As discussed in section 5.5.1 and 5.5.2 a model that aims at deriving a realistic pattern of capital and debt accumulation as well as a long-run convergence of wealth inequality has to obey some restrictions. The first restriction is $c_w H < s$, which is satisfied for the chosen calibration. The second condition requires $rs < c_w$. This condition is not satisfied for the initial case with $r = r_0$ and holds as an identity $rs = c_w$, implying a *knife-edge* calibration. Yet, the interest rate is a model endogenous variable changing in time. Once the interest rate decreases relative to the initial condition $r_t < r_0$, the condition $rs < c_w$ also holds. As we will discuss in the following, this will emerge for our simulations. Moreover, the latter condition implies that in a model with a zero lower bound on the rate of interest - as implicitly assumed in our model - a positive degree of consumption of capital ($c_w > 0$) is necessary - especially to guarantee a convergence in the distribution of wealth.

The equity ratio is set to $m = 0.2$ being lower than the value of $m \approx 0.47$ depicted in figure 5.17 being the minimum equity ratio for the representative household with mean income. This implies that in the simulations the Ponzi case is not totally ruled out by assumption.

[4]Note that if we conduct ceteris paribus analyses in the following we presume the exact same distribution of income. This is made in order to achieve comparability as we only have a finite draw of $N = 1,000$ elements of the distribution.

[5]As presented in section 5.5.1, this ratio is derived by $\frac{1}{Hr+1}$ implicitly assuming infinitely living agents making labor income - and thereby human wealth $\frac{Y}{r}$. Our model even overstates the true ratio of self-made income amounting to approx. 60% (Davies and Shorrocks, 2000).

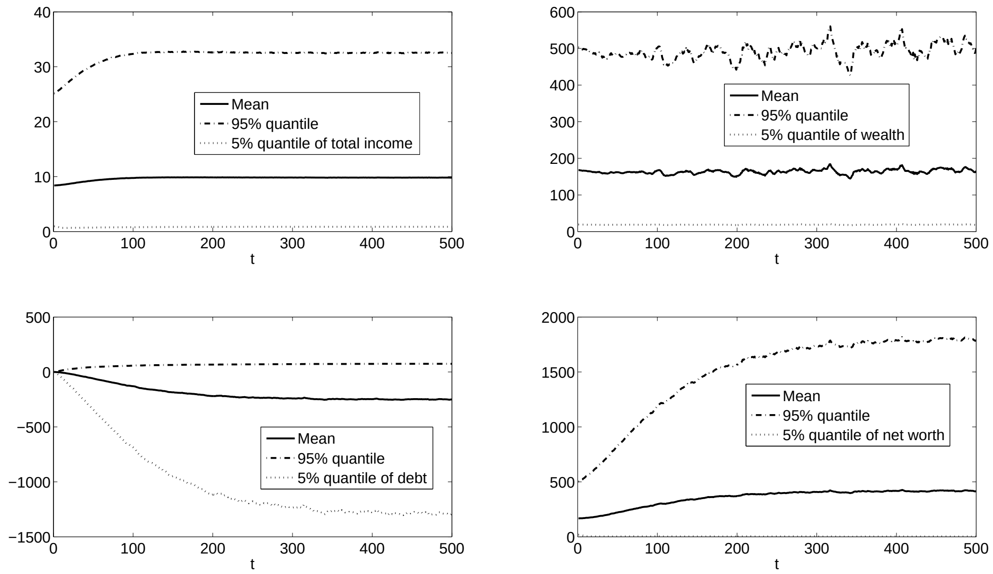


Figure 6.1.: Mean as well as 5% and 95% quantile of total income, wealth, debt, and net worth

First of all, we can look at the quantiles of different quantities as presented in figure 6.1.[6] The distribution of total labor income is assumed to be identical in time. As the agents, however, accumulate debt and claims, there is capital income, allowing top labor income individuals to increase their total income and vice versa for low-income individuals. The increase for the top quantile is more pronounced as they also export savings abroad and receive interest payments by foreigners. As the level of capital $K = -D$ converges to a well defined steady-state, so does the income out of capital rK and thereby also total income.

Meanwhile, wealth is volatile in time due to changing expected prospects for the durable market ($MPCD_t$). In units of prices the volatility is higher for wealthier individuals. This, however, is due to the fact that demand in unit of assets is scaled by the value of net worth. While some agents hold debt - which is fixed at a rather constant level owing to the initial endowment of pledgeable collateral and the constant required equity ratio - other agents hold a substantial amount of capital ($D < 0$). This, moreover, varies in time as the agent (de)levers to go long (short) in the durable market. The combination of wealth and debt creates net worth. While the net worth of the lowest quantile is basically zero, top quantile agents hold a substantial amount of net worth. If we compare the three forms of stock - assets, debt, and net worth - the level of net

[6] Note that these are not the quantiles of the cumulative density function as e.g. presented for empirical data in figure 2.2 or 2.6, but the quantiles of the density function. Hence, the value of a certain percentile is presented but not its overall share of the aggregate. Note also that all reported figures are in price rather than quantity units.

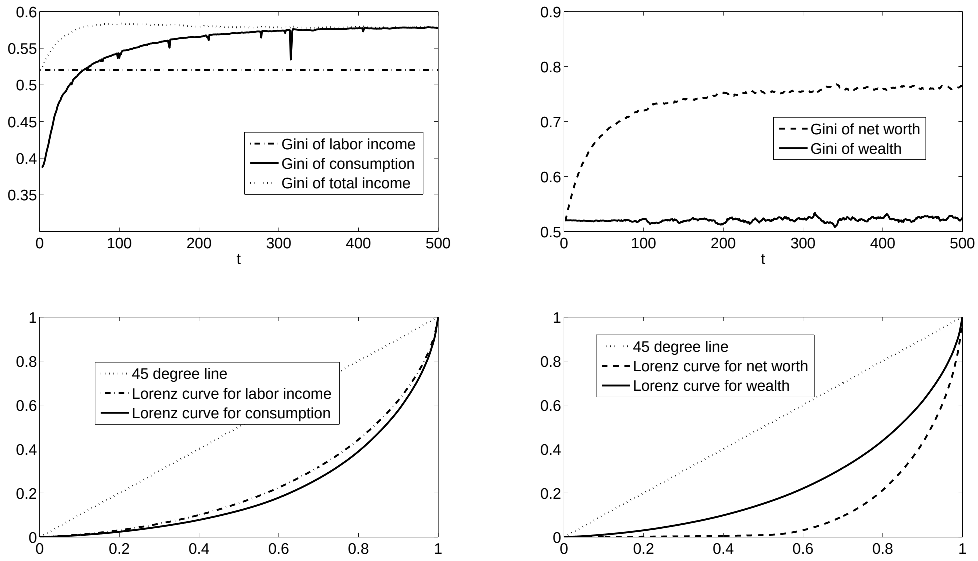


Figure 6.2.: Gini coefficient in time and Lorenz curve in last period ($t = 500$) for labor income, consumption, wealth and net worth

worth shows the smallest variation in time as the increase in debt is diminished by an increase in asset holding and vice versa.

In our model, the labor income as initial source of income inequality is constant in time (see figure 6.2). Wealth inequality varies in time, on mean, however, takes the same level as the Gini of income. This can be linked to the fact that - in a CRRA world - portfolio structuring is independent of the level of wealth together with our assumption that the initial distribution of wealth is directly related to the distribution of income.[7] [8] Yet, there is a positive correlation between the overall price level and wealth inequality as presented in figure 6.3, implying that booms increase inequality of wealth. A formal proof for the latter is presented in A.6. In a nutshell, the underlying rationale is that high-income individuals - not being in debt - have a higher equity ratio allowing for a stronger participation in booms. Low-income individuals face a binding collateral constraint not allowing them lever up. Only near the peak of the cycle, when prices are already high and the collateral constraint is very lax, low-income individuals are able to go long in the asset market. However, by being amongst the last to jump the

[7] The exact proof for this relation is presented in appendix A.6.

[8] By our assumption of a perfect correlation between income and inherited capital $P_0 q_0 = HY$ we implicitly assume that both follow the same distribution. The empirical literature, however, confirms that inherited wealth is more unequally distributed than self-made wealth (Davies and Shorrocks, 2000). The latter can (amongst others) be attributed to the existence of primogeniture (Stiglitz, 1969).

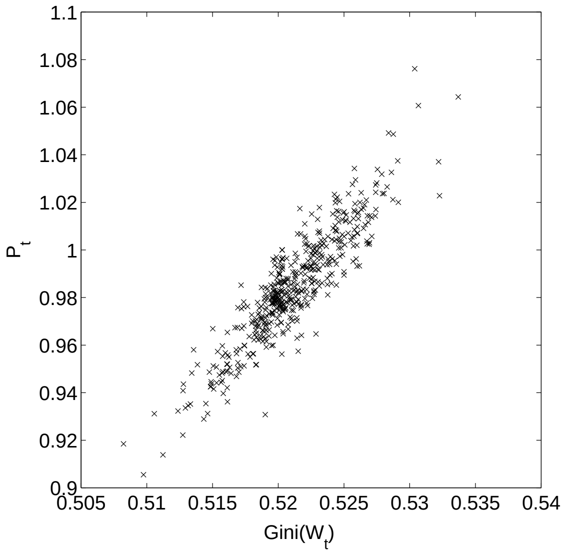


Figure 6.3.: Inequality of wealth and prices are positively correlated

bandwagon they also suffer the most from reevaluation losses in the subsequent bust.[9] This is in line with the empirical evidence of Roine et al. (2009) showing that inequality increases in times of high asset prices and vice versa.

Due to the initial condition of zero debt, the Gini for net worth starts at the identical level as wealth, however, increases to a higher level due to the mutual accumulation of debt and claims. As suggested by empirical findings, the inequality of net worth (stock) is higher than the inequality in income (flow) (cp. e.g., Davies and Shorrocks (2000)). In fact, in the recent crisis the Gini of net worth for the USA increased to a level of 0.87 (Wolff, 2013), implying that our model even understates the true level of inequality (cf. figure 6.2). The evolution of net worth inequality is an emergent behavior in the model propagated by debt. In line with empirical evidence (e.g. (Davies and Shorrocks, 2000)), the financial assets in contrast to the real assets are very unequally distributed explaining a vast amount of the inequality of net worth. If we have a closer look at the Lorenz curve, the curve for net worth exhibits a slope close to zero in the lower end suggesting a large amount of agents with net worth close to zero.[10]

We can also make a statement about non-durable consumption inequality. The Gini of consumption C is always below the Gini of total income X, in line with empirical

[9] Glover et al. (2011) argue that the financial crisis mostly hurt older individuals holding a large amount of wealth in housing. In a calibrated OLG-model they show that while young agents were also subject to the negative shocks in the labor market, they could partly neutralize these welfare losses by means of buying housing assets at fire sale prices from older individuals. As a policy result they conclude that therefore programs that shift welfare weight from young to old individuals - such as increased government debt - might be favorable.

[10] It is also interesting to emphasize that the Lorenz curves do not intersect allowing us to make the statement of Lorenz-dominance between labor income and consumption as well as wealth and net worth.

evidence (Krueger and Perri, 2006), in the long-run still both measurements of inequality converge to the same value given by the inequality of total income (cf. figure 6.2). This is the case since low-income individuals face a binding collateral constraint and thereby only consume *hand to mouth* as suggested by equation 6.3, while the subsistence level rarely matters for the other agents. In the early simulation phase the Gini of consumption is even below the Gini of labor income. This basically can be attributed to the presence of the subsistence level of consumption ($\bar{c} \neq 0$). If we assumed $c_w = 0$, the Gini of consumption C eventually would always be below the Gini of labor income Y in the presence of conspicuous consumption:[11] [12]

$$\begin{aligned} Gini(Y) - Gini(C) &= \\ \frac{2}{n}\left[\frac{\sum_{i=1}^{N} Y_i \cdot i}{nE(Y)} - \left(\frac{c_y \sum_{i=1}^{N} Y_i \cdot i + 0.5\bar{c}(N^2+N)}{N(c_y E(Y) + \bar{c})}\right)\right] &= \\ \frac{\bar{c}}{n}\left(\frac{2\sum_{i=1}^{N} Y_i \cdot i - E(Y)(N^2+N)}{NE(Y)(c_y(E(Y)+\bar{c})}\right) &> 0. \end{aligned} \tag{6.4}$$

Or put differently, if there was no subsistence level of consumption ($\bar{c} = 0$) income and consumption inequality would be identical. Yet, in this case net worth inequality diverges (i.e. it does not converge to a finite level of inequality $\lim_{t\to\infty} Gini(W_t) < 1$) as the condition $rs < c_w = 0$ is never fulfilled for positive rates of interest. Moreover, the interest rate would converge to a very low, yet always positive level.[13]

As net worth now, however, also affects consumption, this leads to the fact that consumption inequality in a dynamic process increases to a level above income inequality. One could say that a convergence in net worth inequality comes at the cost of a consumption inequality higher than labor income inequality. The latter can also be formally proven using a similar argument as the preceding one. The difference between net worth and income inequality can be computed as follows:

$$\begin{aligned} Gini(W) - Gini(Y) &= \\ \frac{2}{N}\left[-\frac{\sum_{i=1}^{N} Y_i \cdot i}{NE(Y)} + \left(\frac{\Omega_y \sum_{i=1}^{N} Y_i \cdot i + 0.5k_0(N^2+N)}{N(\Omega_y E(Y) + k_0)}\right)\right] &= \\ -\frac{k_0}{N}\left(\frac{2\sum_{i=1}^{N} Y_i \cdot i - E(Y)(N^2+N)}{NE(Y)(\Omega_y(E(Y)+k_0)}\right) &> 0, \end{aligned} \tag{6.5}$$

with $W = \Omega_y Y + k_0$.[14] For the case of no relative consumption $\bar{c} = 0$ we would have $k_0 = 0$ and thereby $Gini(W) = Gini(Y)$. As a result, the presence of the relative

[11] Keep in mind that the Gini can be computed by: $Gini(x) = \frac{2}{n}\left(\frac{\sum_{i=1}^{N} x_i \cdot i}{\sum_{i=1}^{N} x_i} - \frac{N+1}{2}\right)$ or for a finite sample (especially important for small samples in which the upper bound would otherwise be given by $\frac{n-1}{n}$): $Gini(x) = \frac{2}{n-1}\left(\frac{\sum_{i=1}^{N} x_i \cdot i}{\sum_{i=1}^{N} x_i} - \frac{N+1}{2}\right)$

[12] Also note that this computation does not account for possibly binding collateral constraints presenting a non-linearity.

[13] For a simulation treatment of the special case with $c_w = 0$, the reader is referred to Fischer (2013).

[14] Following from section 5.5.1, the values are given as follows: $\Omega_y = \frac{s(1-rH)}{c_w - rs} > 0$ and $k_0 = \frac{\bar{c}}{rs - c_w} < 0$.

consumption motive $\bar{c} \neq 0$ contributes to low consumption inequality at the expense of high net worth inequality.

One can also take a broad long-run perspective on figure 6.2 and rank different forms of inequality. The exogenously assumed distribution of labor income Y has the lowest inequality. By the assumption of perfect heritage ($P_0 q_0 = HY$) the distribution of wealth Pq exhibits the same level of inequality. The inequality of total income X is higher than labor income inequality due to more unequally distributed capital income. The inequality of consumption C is always below the inequality of total income. Yet, this gap narrows in the long-run. The stock quantity of net worth W exhibits the highest level of inequality.

Despite the long-run convergence of consumption inequality to income inequality, consumption inequality shows negative seasonal peaks. The latter can be attributed to asset price booms (compare figure 6.2 and figure 6.4) relaxing the credit constraints for low-income households allowing for debt-financed consumption of non-durables. A similar result showing that consumption inequality is countercyclical (i.e. low in booms and vice versa) is presented in the theoretical work of Airaudo and Bossi (2014). The necessary building blocks of their model are the presence of limited asset market participation (in our case in form of the collateral constraint) and trickle-down consumption (here: $\bar{c} \neq 0$).[15]

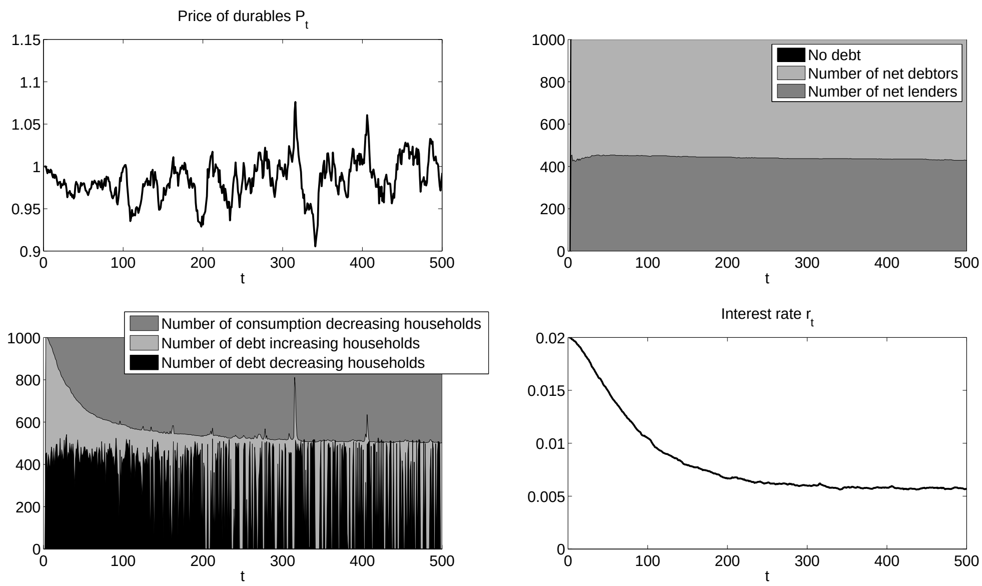


Figure 6.4.: Market conditions for savings/debt market and durable market and composition of household groups

[15] They also argue that these two factors lead to the effect that monetary policy becomes more effective, since - technically - the Phillips-curve becomes flatter shifting more weight to the output rather than the inflation objective.

The real rate of interest in the simulations is approximately 0.7% (see figure 6.4). In standard models without aggregate growth (as in our case) the rate is $r = \rho$ or in *Bewley*-types it is even $r < \rho$. As presented in section 4.2 in the Keynesian model we use one can approximate this (opaque) measurement of time preference by $\rho \approx c_w$, which for the concrete parameter setting is $c_w = 1\% > 0.7\%$ coming to a comparable result as the *Bewley*-type literature.

We now change our focus from the distribution and the individual to the total outcome. In the aggregate, there are two central effects (see figure 6.4): firstly and as already discussed in section 5.4.1, the increased debt holdings lead to a current account surplus accompanied by a convergence of the interest rate in the savings market to a low, but positive, stable level. Note that in our model the interest rate is also the mechanism required to balance the current account as we (implicitly) assume a fixed exchange rate regime.[16] Once the interest rate converged to a stable level the (excess) savings - presented as a grey area in the upper panel of figure 6.5 - also disappear.

Moreover, since the other part of the wealth is invested in the market for durables, the price volatility in this market increases in time. The increasing current account surplus is accompanied by an increasing mean net worth (see also figure 6.5), which, as documented in equation 5.60, feeds back into higher price volatility for durables. The number of lenders and debtors (stock) is relatively constant while lenders represent the majority contributing to the current account surplus. Formally, in the long-run for which demand for durables is zero ($E(MPCD) = 0$), lenders are all households that earn a lower labor income than:

$$Y < Y^* = \frac{\bar{c}}{s - Hc_w}. \tag{6.6}$$

For the given tuning, we have $\frac{Y^*}{E(Y)} \approx 0.833$.

Secondly, the number of collateral constrained and unconstrained agents varies throughout time (cf. figure 6.4). This can be linked to the conditions in the market for durables. Price booms in the market for durables relax the collateral constraint, thereby reducing the number of low-class agents and vice versa in the case of busts. Yet, the composition of high and middle-class households varies even stronger in time. This can be attributed to the fact that purchases in the market for durables can be conducted with credit. As suggested in figure 5.8, an asset price boom leads to the fact that the upper-class supplying debt temporary disappears. While all agents increase their debt, for net lenders this only means that - in stock terms - they lower their level of claims.

Figure 6.5 decomposes GDP (flow) and aggregate balance sheets (stock) for which once again values are given in the unit of prices. While total labor income is exogenous

[16] In a floating exchange rate regime, the exchange rate leads to a convergence of the current account to zero, for which surplus countries face a devaluation of the currency and vice versa. In a fixed rate regime, a central bank intervention can influence the exchange rate and thereby the current account. For a deficit country, this requires selling foreign reserves to strengthen the domestic currency. This, however, is constrained by the available amount of accumulated reserves. In contrast to that, a current account surplus (if desired) can be easily eliminated by issuing domestic currency and thereby devaluating the domestic currency.

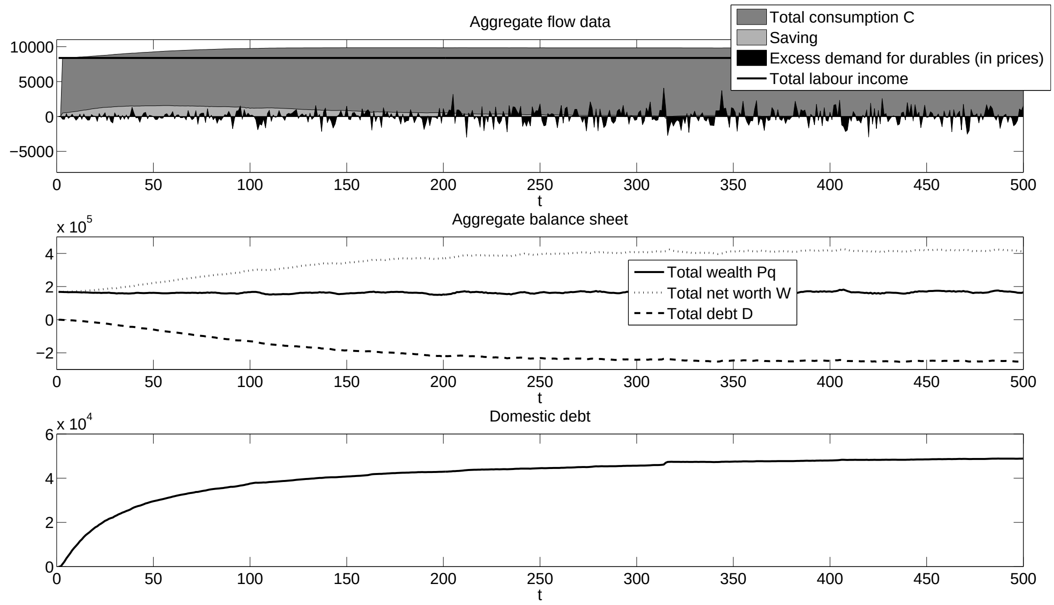


Figure 6.5.: Decomposition of GDP (aggregate flow) and aggregate stock

and assumed to be constant, its composition varies massively in time.[17] Firstly, the increased volatility in demand for durables becomes obvious. Due to the fact that income is constant, consumption only shows little variance which can be attributed to temporary binding collateral constraints. The sum of consumption and savings exceeds GDP since the economy is net lender to foreigners and receives interest payments from abroad. It is important to acknowledge that, in our consumer-only economy, capital income can only emerge if net claims to foreigners are established. In a closed economy, there would be no capital income since every capital income is accompanied by interest on debt of the same height resulting in the fact that the economy in terms of capital is zero-sum in the aggregate.

Total wealth - as already proved in appendix A.6 - oscillates around a steady state (cf. figure 6.5). Note that even though we have a current account surplus, domestic debt level increases in time.[18] Since we assume an initial condition of zero debt, households lever up until they face their collateral constraint leading to a decreasing growth rate of domestic debt. We can interpret the simulation starting with zero debt (and therefore an equity ratio of $m = 1$) to $m = 0.2$ as the result of financial liberalization. The concave shape of the evolution of domestic debt can be attributed to the decreasing number of middle-class households. As shown in figure 6.4, the unconstrained agents (the middle-class) disappear after having levered up to the maximum resulting in a

[17] The black area cover the gains or losses (above or below the zero line) from the trades in the durable market. The mirror position (to make it zero sum) occurs to an unmodeled market maker entity.

[18] To only compute net positive debt the following approach is used: $DomesticDebt_t = 0.5(\sum_{i=1}^{N} |D_{i,t}| + \sum_{i=1}^{N} D_{i,t})$.

bipolar society. In the steady state, the (flow quantity of) current account surplus measured by excess savings vanishes. As figure 6.5 shows, in terms of stock this leads to a constant negative debt level representing a claim to foreigners. As the foreigners, however, pay an interest on their debt in the long-run there is an inflow of capital income of $\frac{-r\sum_{i=1}^{N} D_i}{\sum_{i=1}^{N} X_i} = \frac{\sum_{i=1}^{N} X_i - \sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} X_i} \approx 14.5\%$.

We can also make a statement about the ratio between stock and flow - in the concrete form of total earned income ($X \equiv \sum_{i=1}^{N} X_i = \sum_{i=1}^{N} [Y_i + rD_i]$) and total net worth ($W \equiv \sum_{i=1}^{N} W_i = \sum_{i=1}^{N} Pq_i - D_i$) amounting to approx. 49 in the long-run. As discussed in Piketty (2014) the empirical literature presents a much lower value of 4-6 for developed economies.[19] By our assumption of $H = 20$ we already overstretched this ratio. Future work might try to find a more reasonable calibration.

The strength of the given model is that it - not only - can make a statement about aggregates, but also about individual levels. In fact, the only underlying - yet persistent - inequality we assumed is inequality in labor income. In the following, we map other outcomes to the underlying driver of labor income.

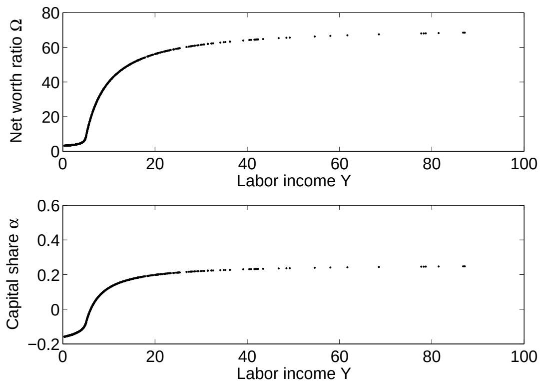


Figure 6.6.: Net worth to labor income ratio Ω and capital share of total income α as a function of labor income Y (time averages)

We start with the individual net worth to labor income ratio $\Omega_i = \frac{W_i}{Y_i}$ presented in figure 6.6. This ratio shows an s-shape with a positive slope, implying that the net worth ratio grows with underlying labor income. Hence, individuals with a high labor income not only have a high level of capital in absolute terms but also in relative terms measured by the ratio between wealth and labor income. As discussed in section 5.5.1, the increasing ratio can be attributed to the diminishing effect of minimum consumption $\bar{c}$. In fact, high-income households have a higher value than the average value of approx. 42.[20] The left end of the curve can be rationalized by the existence of the collateral constraint. Low-income households only hold the minimum required equity ratio $\frac{W_{min}}{Y} = \frac{mHY}{Y} = mH = 4$.

[19] Also refer to figure 5.19.

[20] As shown in section 5.5.1, their value is eventually given by $\Omega = \frac{s(1-rH)}{c_w - rs} = 60$ for $r = 0.5\%$.

In line with the empirical evidence of Piketty (2014), the individual capital share of total income ($\alpha_i = \frac{-rD_i}{Y_i - rD_i}$) increases with (labor) income (refer to the lower panel of figure 6.6).[21] [22] Once again we have an s-shaped pattern in which low-income individuals eventually have a negative value of α_i due to the fact that they are indebted. The inflection point ($\alpha = 0$) emerges for $Y = Y^*$ being the level that separates lenders and debtors.

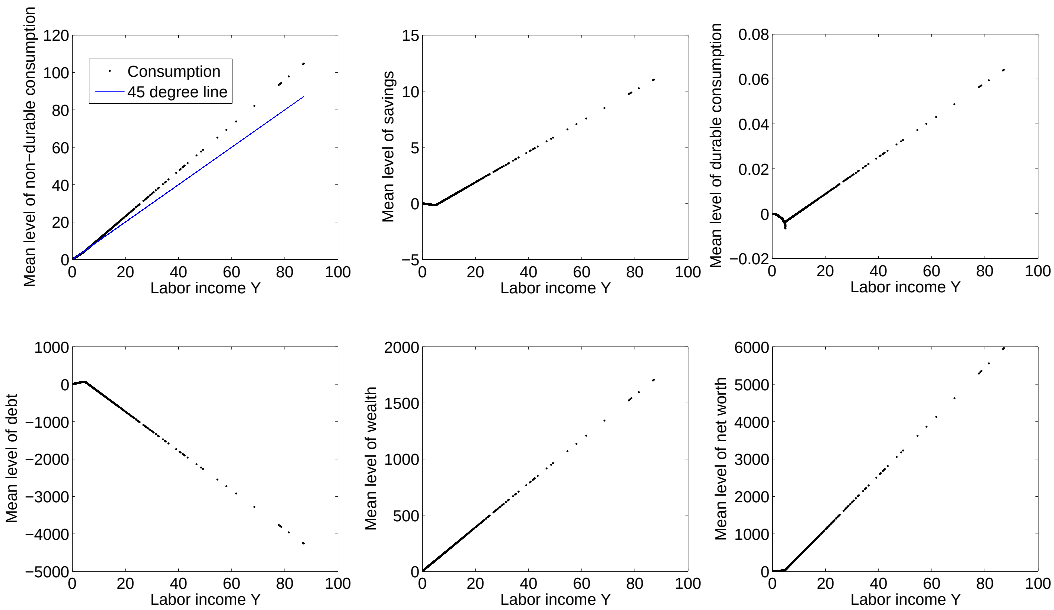


Figure 6.7.: Mean of non-durable consumption, savings, durable consumption, debt level, wealth and net worth as function of mean income (time averages)

We can also relate other measurements to the underlying heterogeneity of income. Figure 6.7 shows that the highest amount of debt (stock) is held by middle-class households corresponding with the lowest (negative) savings (flow). This is broadly in line with empirical evidence. Doepke and Schneider (2006) argue that debt - in particular in the form of mortgages - is a middle-income phenomenon. Low-income households hold lower levels of debt and furthermore have other types of debt - namely consumer debt rather than mortgages. The theoretic rationale presented in this paper is that low-income households have little wealth to borrow against constraining their ability to lever

[21] Note that $Y_i \sim X_i = Y_i - rD_i = Y_i + r(k_0 + k_y Y_i)$ with $r > 0$ and $k_y > 0$ (cf. section 5.5.1). Hence, there is a simple and positive relation between labor and total income allowing us to state that the capital income ratio α not only increases with labor income Y (as presented in figure 6.6) but also with total income X.

[22] We, however, fail to reproduce the high levels of capital income share for the top income which Piketty (2014) quantifies as 60% respectively 70% for the USA and France in the recent years. Our value converges to $\alpha = \frac{r(s - c_w H)}{c_w(1 - Hr)} = 16\%$ for $r = 0.5\%$. This is the case since - as already put forward - in our model aggregate capital can only be created by creating claims against foreigners.

up. The latter is also confirmed by the analysis of the new dataset for 15 Euro countries labeled *Household Finance and Consumption Survey* (HFCS) (Bover et al., 2014). Moreover, the authors show that while consumption debt is high for low age individuals (16 to 34 years), mortgage debt is highest amongst middle aged (34 to 44 years) and a middle level of education (being correlated with income). Moreover, Bover et al. (2014) document a wide disparity of the overall level of debt in the different countries covered in the study. The highest levels are reported for Spain, Portugal and the Netherlands. In their econometric analysis they are able to link this to a lower repossession period - being the time the creditor can call the collateral. In our model this could be captured by a lower value of m.

In the model, wealth remains constant on mean, in which the ratio of wealth and labor income is given by the initially assumed value for heritage H (cf. figure 6.7).[23] Total net worth ($W_i = Pq_i - D_i$) is a hockey-stick shaped functional form of income. Eventually the lower end is not flat (as in a real hockey stick function), but has a slope of Hm indicating that low and middle-class households only hold market required equity ratios. High-income households hold no debt at all but are net claim holders.

Interestingly, the consumption is above the 45-degree line and exhibits a convex shape (cf. figure 6.7). Note that on the abscissa we only plot labor income. However, agents above a certain labor income level hold claims rather than debt, thereby additionally receiving capital income. In the presence of positive interest rates, this makes a convex relation between consumption C and labor income Y.[24] In the aggregate, the difference between consumption and labor income for high-income households are in fact transfers from foreigners in the form of interest payments on debt that are employed for non-durable consumption. This behavior is also presented in figure 6.5 in which the total consumption and savings exceed the level of labor income Y.

Another intriguing result emerges in the market for durables for which high-income households on mean provide long positions ($d > 0$) while low and middle-income agents go short (once again refer to figure 6.7). The strongest downward pressure is applied by medium-income households. The rationale for this is that by selling their durables, households can enhance their consumption possibilities for non-durables. The range for going short is strongest amongst the middle-class (as opposed to low-class households) due to their higher initial endowment. It is important to point out that in this model the *short-bias* for low and medium-income households is not incorporated via a deleveraging mechanism but by the fact that these agents do not participate in the boom since they cannot due to their binding collateral constraint and therefore have to refrain from the market in this part of the cycle. In fact - and as already discussed - low-income households tend to jump on the bandwagon near the end of the cycle exposing them to higher reevaluation losses.

In the model description in section 5.1, we have been vague about the nature of the durable asset. However, it can be considered as real estate rather than stock market shares. As presented in Wolff (2013), trading in stock markets is an activity mostly only

[23] Or put differently, the slope of the curve equals $H = 20$.

[24] The proof for this condition is presented in appendix A.9.

observable for high-income households. The latter behavior is also documented in table 5.1 showing that while low-income agents hold cash, medium-income agents prefer real estate, whereas high-income agents hold financial assets and private business.

Glaeser and Nathanson (2014) summarize other important features of housing markets differentiating this asset class from financial market products. Houses are non-standardized and very heterogeneous and the most important asset for private households especially families (in contrast to singles holding large amount of their net worth in financial market products). In fact, the market is dominated by amateurs. While it is only rational to hold a bond or a stock shares is to cash on annual interest payments or dividends (an equivalent case for housing would be the rental feed), housing entails maintenance costs but also provides direct utility being a consumption good. Factors that hinder efficient price discovery such as short sale constraints or limits to arbitrage are highly prevalent for the housing market making it prone to price bubbles.[25] Moreover, housing markets are very opaque and entail high transaction costs (e.g. fees for real estate agents). Furthermore, housing is illiquid, indivisible or lumpy, and infrequently traded (Davis and Nieuwerburgh, 2014). As documented in Glaeser and Nathanson (2014) house prices exhibit a short-run momentum combined with a long-run mean reversion of prices, which can be well replicated in fundamentalist-chartist models.[26]

Finally - and very important for our model - real estate trading is usually conducted using a strong leverage resulting from several of the above mentioned features (in particular the lumpiness and the strong participation of medium-income individuals). In contrast to that trading in the stock market mostly conducted by unconstrained high-income households) does not involve (strong) leveraging. Wolff (2013) shows that the increase in net worth inequality between 2007 and 2010 can be attributed to reevaluation effects of real estate despite the fact that both real estate and stock markets experienced similar plummeting. This can be tracked to the massive usage of leverage in real estate trades.

6.2. Robustness Checks

So far we presented a benchmark simulation that was able to generate a large number of stylized facts and already hinted at some interesting policy implications. As, however, already emphasized the parameters presented in table 6.1 were not chosen based on an econometric calibration, but result from the closed-form solutions of a simplified model we discussed in section 5. The following section aims at providing robustness checks of the results as well as quantifying the exact impact of the parameters on several outcomes.

[25] On the other hand - and in contrast to financial market products - the supply of assets is rather elastic. This feature is not captured in our model only modeling the demand and thereby (implicitly) assuming a fixed supply of housing assets. When modeling financial markets the latter is definitely a reasonable assumption, yet a strong caveat when considering housing markets.

[26] Glaeser and Nathanson (2014) describe other - more standard - approaches to rationalize real estate bubbles being search frictions, underpricing of default options, agency problems, and cognitive limitations such as spatial benchmarking or usage of rule of thumbs.

We proceed by conducting a *ceteris paribus* analysis - basically assuming the parameter values as presented in table 6.1 but varying a single parameter to quantify its marginal effect. As most model outcomes are two-dimensional - N agents and T time periods - we have to aggregate to present readable results. Therefore, in most of the cases we not only aggregate among the dimension of agents but also compute the mean in time.[27] Moreover, we only conduct simulations for surplus economies, for which in particular the condition $s > c_w H$ holds. As we will discuss more thoroughly in the following, the second necessary condition $c_w > r_t s$ - being time varying due to the endogenous nature of the interest rate r_t - also holds most of the time. Thereby, we also have realistic distributions of net worth and debt (cf. section 5.5.1) and a convergence of wealth inequality to a finite level (cf. section 5.5.2). Formally, the deficit economy is not sustainable. In this case, all agents would lever up to the maximum resulting in the fact that debt is provided by foreigners. The interest rate would increase in time. In the medium-run, all agents belong to the lower-class, employing their labor income for paying the interest on their debt and (if available) using the residual for consumption of non-durables. In this case, the market for durables completely breaks down.

Firstly, we discuss the parameters of our *Keynesian* consumption function in section 6.2.1 to later go to the parameters of risk aversion and especially heritage as well as inequality (section 6.2.2), which also yield some very interesting policy results.

6.2.1. The Role of the Consumption Function

The consumption function is of key importance in our model. In fact, our *macroeconomic model* consists of little else but the consumption and savings decision of heterogeneous households neglecting other important building blocks such as the investment decision by firms or the labor market dimension. Moreover, the interest rate on debt r_t is jointly determined by the consumption function and the distribution of income amongst agents in a complex manner.

We start by varying the marginal propensity to consume out of (disposable) income c_y. Note that the consumption function itself is uniform for all agents. The input factors - disposable income and net worth - however are heterogeneous for all agents. In general, an increase in c_y is accompanied by an increase in consumption. Or to speak in the terms of figure 5.7, the slope of the consumption function increases. To finance this consumption a higher aggregate amount of debt is employed. As depicted in figure 6.8, the number of debtors, the amount of aggregate debt as well as the interest rate on debt thereby increases. It is important to point out that we remain in the case for which the aggregate debt is negative and the economy accumulates credit (rather than debt) to foreigners. This current account surplus case is guaranteed since we only simulate the cases in which the condition $c_y < 1 - c_w H = 0.8$ is satisfied. The other condition - related to the flow quantity - requires $rs < c_w \leftrightarrow c_y > 1 - \frac{c_w}{r} \leftrightarrow r < \frac{c_w}{1-c_y} = \frac{0.01}{1-c_y}$. The latter condition requires a higher interest rate for higher values of c_y ranging from 0.01

[27]For instance, we take the mean in time of total durable consumption being defined as $E(c_{tot}) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} c_{i,t}$.

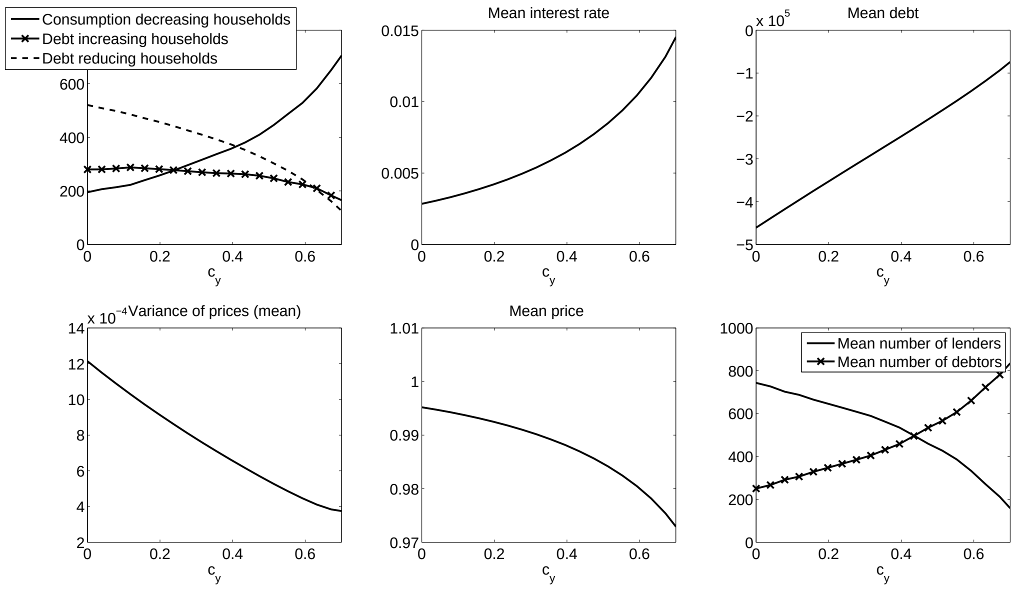


Figure 6.8.: Household composition, market for durables and savings for parameter variation of MPC out of income c_y

to 0.05 as c_y varies from 0 to 0.8. As becomes obvious in the panel of figure 6.8 depicting the interest rate as opposed to the exogenous factor c_y, this condition is always satisfied.

Note also that there is a negative relation between the mean price in the asset market and the rate of return (see figure 6.8). Technically, this follows from the portfolio formation of the agents as given in equation 5.9, for which low interest increase the demand for durables and vice versa. We also labeled this effect the *flight effect*.

If we consider the *class distribution* of agents (cf. figure 6.8), a higher propensity to consume leads to fewer *high-class* agents - supplying debt (or debt reducing as labeled in the figure) - and more lower-class agents - levering up to the maximum (labeled as consumption decreasing in the figure). The amount of *middle-class* agents - increasing their debt without being subject to a binding collateral constraint (labeled as debt increasing in the figure) - is ambiguous. They witness both an inflow from former upper class agents as well as an outflow to lower class agents. In the aggregate, the middle-class reaction to a variation of c_y is an inverse u-shape. This result, however, should not be confused with an optimal medium level of c_y leading to a maximum number of middle-class agents, as the increase in c_y leads to the fact that all agents are worse off *class wise* - being a Pareto inferior situation.

On the other hand, as most income is now employed for durable consumption less is available for speculating on the durable market, accompanied by a lower price volatility in the durable market (cf. the panel of price volatility in figure 6.8). Thus, as already emphasized in section 5.4, higher consumption leads to the fact that stability of debt decreases, yet the stability of durable markets increases. Moreover, as the interest rate discounting cash flow of durables increases, the price level of durables decreases.

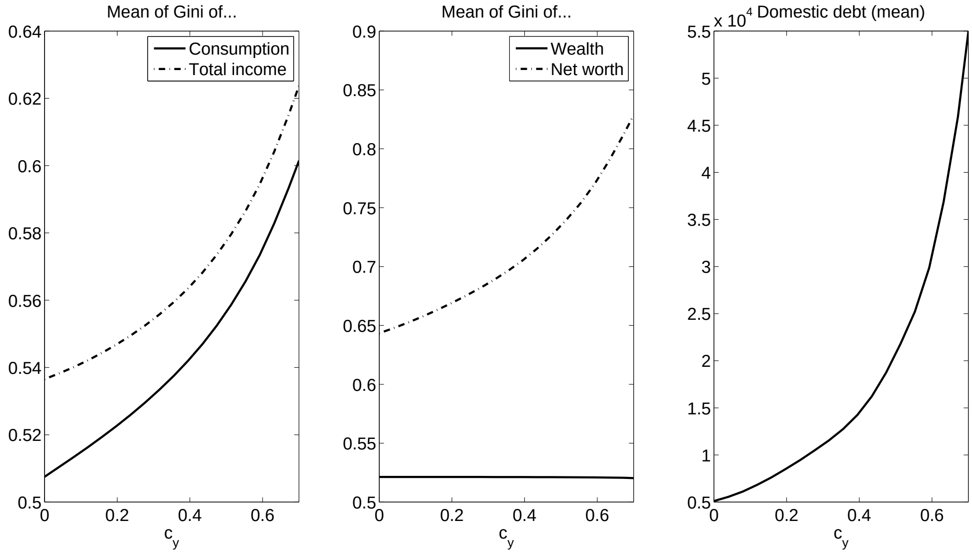


Figure 6.9.: Inequality for parameter variation of MPC out of income c_y

The aggregation of debt and claims has important effects on the distribution of income and assets. As presented in figure 6.9 the increased domestic debt is accompanied by increased net worth inequality as some agents hold debt ($D_i > 0$) and others hold claims ($D_j < 0$). This also changes the total income on the flow level, since some agents receive capital income ($rD_j < 0$) while others pay interest on debt ($rD_i > 0$). Thereby, an increase in the (general) marginal propensity to consume is accompanied by higher inequality.

Lastly, we consider the aggregate impact on total consumption and the functional distribution of income (cf. figure 6.10). First of all, the stronger use of debt decreases the aggregate capital to income ratio. The total capital share, however, reacts in the form of an inverse u-shaped function to a variation of c_y. We dwell more thoroughly on this point since in one of his key points, Piketty (2014) comes to a different result.

For Piketty (2014) the relation between the functional distribution (in particular the capital share $\alpha = \frac{rK}{X}$ with $X = rK + Y$ being total capital and labor income) and the capital ratio is just the result of an accounting identity for which:[28]

$$\alpha \equiv r \cdot \frac{K}{X} = r \cdot \kappa, \tag{6.7}$$

holds. Hence, an increase in the capital ratio κ should go along with an increase in the capital share α and vice versa. As our simulations - presented in figure 6.10 - however show the relation is non-monotonous. Despite the unambiguous decrease in the net worth ratio Ω, the functional form of the capital share α is given by an inverse u-shape. The key rationale lying behind is the reaction of the interest rate. Due to a higher amount of debt, the interest rate rises for an increase of c_y (cf. figure 6.8). Thus, the

[28]This point is more thoroughly discussed in section 4.2.

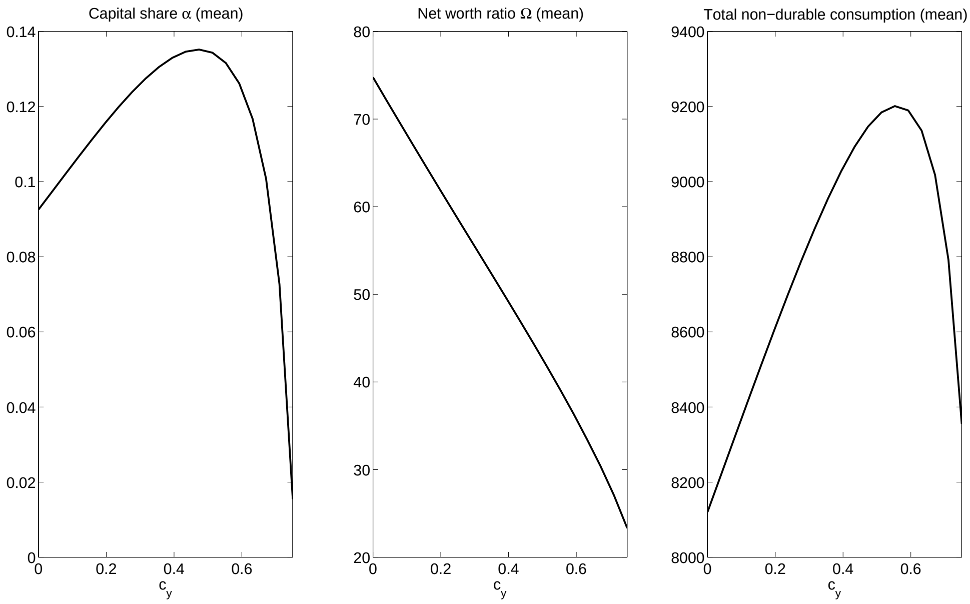


Figure 6.10.: Capital share, net worth ratio, and non-durable consumption for parameter variation of MPC out of income c_y

capital share is determined by two opposing forces: for low values of c_y the *price effect* dominates for which a higher interest on capital increases its share, while for higher values a *volume effect* dominates for which the total share of capital decreases due to the lower volume of capital.

In the standard (neo) classical literature the equilibrium price of capital - being the interest rate - is determined by the marginal product of capital. Hence, we have to define a production function. The partial derivative with respect to capital in this case represents the equilibrium interest rate. Moreover, the assumption of negative scale effects ($f_{kk} < 0$) rationalizes a negative demand function for capital. A very general production function is the so-called Constant Elasticity of Substitution (CES) production function:

$$Y = \left(\bar{\alpha}K^{-\rho} + (1-\bar{\alpha})N^{-\rho}\right)^{-\frac{1}{\rho}}, \tag{6.8}$$

with the inputs capital K and labor N as well as a pseudo-capital share of $0 < \bar{\alpha} < 1$. We can normalize this function with capital and income per capita ($k = \frac{K}{N}$ respectively $y = \frac{Y}{N}$) leading to:

$$y \equiv f = \left(\bar{\alpha}k^{-\rho} + (1-\bar{\alpha})\right)^{-\frac{1}{\rho}}, \tag{6.9}$$

for which $\rho > -1$ can be transformed to the elasticity of substitution $\sigma = \frac{1}{\rho+1} > 0 \leftrightarrow \rho = \frac{1-\sigma}{\sigma}$. The capital share is determined as follows:

$$\alpha = \frac{kf'(k)}{f}, \tag{6.10}$$

which - using the Constant Elasticity of Substitution (CES) production function - results in:

$$\alpha = \frac{\bar{\alpha}}{\bar{\alpha} + (1 - \bar{\alpha})k^{\rho}}. \tag{6.11}$$

We can now compute the change of the functional distribution α with respect to the level of capital k:

$$\frac{\partial \alpha}{\partial k} = \frac{\bar{\alpha}(\bar{\alpha} - 1)\rho k^{\rho - 1}}{(\bar{\alpha} + (1 - \bar{\alpha})k^{\rho})^2}. \tag{6.12}$$

The pleasant feature of the CES function is that several - more standard - cases are nested within this function. In particular for $\rho = 0$ ($\sigma = 1$), the function boils down to the standard Cobb-Douglas production function ($Y = K^{\bar{\alpha}} N^{1-\bar{\alpha}}$), for which the capital share is constant ($\alpha = \bar{\alpha}$) and does not vary with the level of capital ($\frac{\partial \alpha}{\partial k} = 0$) (Cobb and Douglas, 1928).[29] This is the workhorse model in basically any macroeconomic model and implies the strong assumption that the functional distribution is a *stylized fact* not varying in time or with changes of the level of capital.

Piketty (2014) (implicitly) assumes a case in which the elasticity of substitution is larger than one ($\sigma > 1$ respectively $-1 < \rho < 0$) - implying an elastic demand function. In this case, the volume effect dominates and an increase in capital leads to an increase in the capital share ($\frac{\partial \alpha}{\partial k} > 0$). In fact, Karabarbounis and Neiman (2013) - discussing the evidence of a long-run decrease in the labor share - estimate a level of $\sigma = 1.25 > 1$. This case is sometimes also referred to as the *capital-biased* case (Giovannoni, 2014) for which capital and labor are substitutes. Moreover, this type of production function can rationalize the general increase in the capital share in an environment of increasing capital as measured in the data and e.g. presented in figure 4.5.

The inverse case would emerge for $0 < \sigma < 1$ ($0 < \rho < \infty$) presenting an inelastic demand function for which the price effect prevails and increase in the level of capital would decrease the functional share of capital ($\frac{\partial \alpha}{\partial k} < 0$). This case can be labeled the *labor-biased* case for which capital and labor are complements (Giovannoni, 2014). Most of the empirical literature confirms this case (Acemoglu, 2003). In fact, the Cobb-Douglas case is just the extreme iso-elastic case ($\sigma = 1$) for which price and volume effects exactly offset each other.

The CES production derives its name from the property of assuming that the elasticity of substitution is independent of the level of k and thereby constant. In effect, the sign of the partial derivative of the capital share with respect to capital ($\frac{\partial \alpha}{\partial k}$) is independent of the level of k. This, however, is not the case in our model as we have an inverse u-shaped function. Consider a simple *ad-hoc* demand function of capital:

$$r = r_0 - r_k k, \tag{6.13}$$

with $r_0 > 0$ and $r_k > 0$. This function is elastic ($\sigma > 1$) for a low level of $k < \frac{r_0}{2r_k}$ and inelastic ($\sigma < 1$) for a high level of $k > \frac{r_0}{2r_k}$. In fact, we have a capital share that is

[29] For the other extreme case of $\sigma \to \infty$ ($\rho = -1$) the function boils down to a linear production function $Y = \bar{\alpha}K + (1 - \bar{\alpha})N$. In this case there are no scale effects and the return to capital is constant. Moreover, the capital share increases with capital ($\frac{\partial \alpha}{\partial k} > 0$).

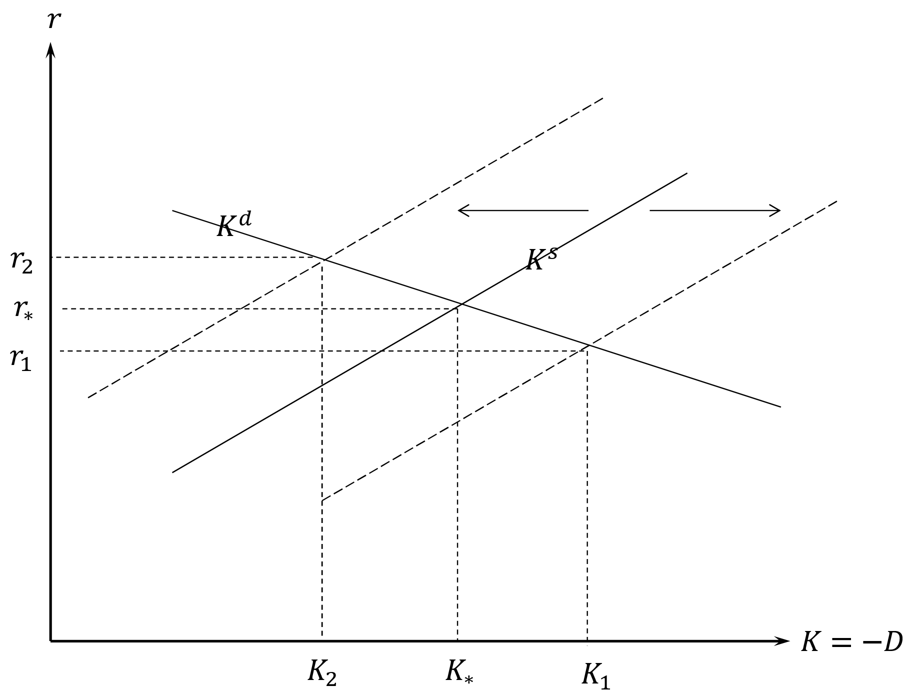


Figure 6.11.: Schematic determination of the equilibrium rate of interest in our model

an inverse u-shaped function of the level of capital k with a local maximum at $k = \frac{r_0}{2r_k}$ going along with $\sigma = 1$.

As a result, the relation between the level of capital (proxied by the net worth ratio Ω) and the capital share is ambiguous. The standard assumption of a CES production function presumes a clear-cut relation being either positive or negative. In our model, for low values of capital there is a positive relation (as predicted by Piketty (2014) for the capital-biased case), whereas the labor-biased case persists for high values of capital.

In our model, the equilibrium rate of interest is determined by the interaction of the (uniform) consumption function and the distribution of income and wealth rather than by the chimera of an aggregate production function.[30] In fact, the market is mainly driven by the supply: if the equilibrium level of capital decreases, the equilibrium rate

[30] The assumption of the aggregate production function - explaining the functional distribution of income by means of an exogenous technology - was also one of the key points criticized by the member of Cambridge, UK in the *Cambridge capital controversy*. For a thorough depiction of the debate the reader is e.g. referred to Burmeister (2000).

of interest increases (also see the schematic figure 6.11 [31]). The demand of capital is given by a combination of the consumption function - in particular the minimum consumption level $\bar{c}$ -, the initial endowment with collateral (driven by heritage H) and the equity requirement m which is fixed in the long-run. Meanwhile, the number of well-endowed individuals - supplying the capital - has a strong impact on the equilibrium level of interest. Thus, the top income households - despite only representing a small share of total agents - have a strong macroeconomic impact on the equilibrium interest rate. In the past literature discussing inequality focused on the low-income households focusing on issues such as poverty traps. As already shown in section 2.2, the current inequality in the developed countries in particular results from the increased role of very high-income individuals. This, however, of macroeconomic importance since they have a major impact on the market for savings.

The total non-durable consumption also shows a similar behavior as the capital share - i.e. an inverse u-shaped - to a variation of c_y (see figure 6.10). The rationale is also similar. For the higher values of α the total income $\sum_{i=1}^{N} X_i = X = Y - rD$ increases, as there is a higher inflow from foreigners.[32] The non-durable consumption, however, peaks at higher values of c_y. The latter is due to the direct impact of c_y increasing non-durable consumption. Yet, for higher values the indirect effect (decreased current account) prevails leading to the presented inverse u-shaped function.

We proceed by varying the relative consumption level $\bar{c}$. As already put forward in section 5.1 as well as 5.2, we define the relative consumption level as a function of the income distribution $\bar{c} = quantile_j(c)$. To get a variation, we therefore change the level of j.[33] In contrast to c_y, there is no threshold value to derive a surplus.[34] We chose to simulate for values of $0 < j < 0.5$, implying that the relative consumption level is always below the (labor) income level of the median household.[35] [36]

First of all, and as depicted in figure 6.12, comparable to the case of c_y the increased consumption results in a higher level of debt, higher interest rates, as well as lower volatility and price levels in the durable market. The *class distribution* of agents looks slightly different. Once again, we have fewer lower-class agents and fewer higher-class agents. In this case, the middle-class, however, also decreases. In contrast to the in-

[31] To put our model in a context of the history of economic thought it can be related to the *Loanable Funds Theory* dating back to the late Dennis Robertson (Tsiang, 2008) arguing that the rate of interest is determined by demand and supply of *loanable funds* emphasizing the flow aspect. The equilibrium is mainly determined by investment opportunities, depreciations, the relation between current (flow) income and consumption expenditures, and liquidity considerations.

[32] Keep in mind that $\alpha > 0$ in the aggregate in our model can only emerge for a surplus economy for which $\sum_{i=1}^{N} X_i > \sum_{i=1}^{N} Y_i$.

[33] Note that in the figures we refer to this as c_{min}.

[34] The only unambiguous case is $j = 1$, for which the minimum consumption level would be equal to the labor income of the individual with the single highest income. Hence, all (but one) would require debt to execute their consumption desires resulting in a current account deficit.

[35] As put forward in section 7.2.2 the minimum consumption level is also frequently defined as a ratio ($< 100\%$) of median income.

[36] As we will argue more precisely in section 7.2.4 a moderate income tax has the same effect as an increase in the minimum consumption.

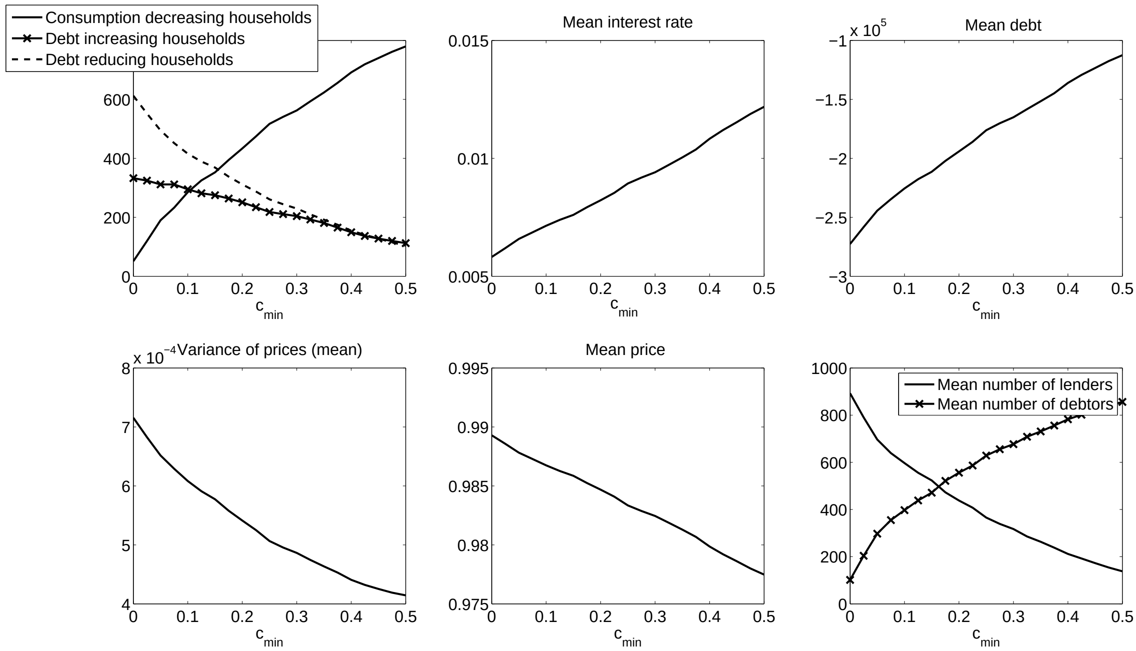


Figure 6.12.: Household composition, market for durables and savings for parameter variation of minimum consumption quantile c_{min}

creases of c_y affecting all agents equivalently, the increases of j in particular hits the middle-class and contributes to their disappearance. The underlying rationale is that the minimum consumption level is of particular importance in defining whether an agent is low or middle-class.

Once again, the accumulation of debt increases the inequality amongst agents regarding total income and net worth (cf. figure 6.13). It is important to point out that the gap between total income and consumption inequality increases with $\bar{c}$ [37] implying that consumption inequality grows at a slower pace than income inequality. The variation of minimum consumption $\bar{c}$ on capital share α, the net worth ratio Ω and the total durable consumption are qualitatively identical to the case in which c_y is varied (cf. figure 6.14). Total capital decreases and total non-durable consumption as well as total capital share exhibit an inverse u-shaped behavior. Compared to the variation of the marginal propensity to consume, aggregate results are less sensitive to the variation of $\bar{c}$.

[37] This also becomes clear from our computations presented in equation 6.4, in which the gap directly depends on the level of $\bar{c}$.

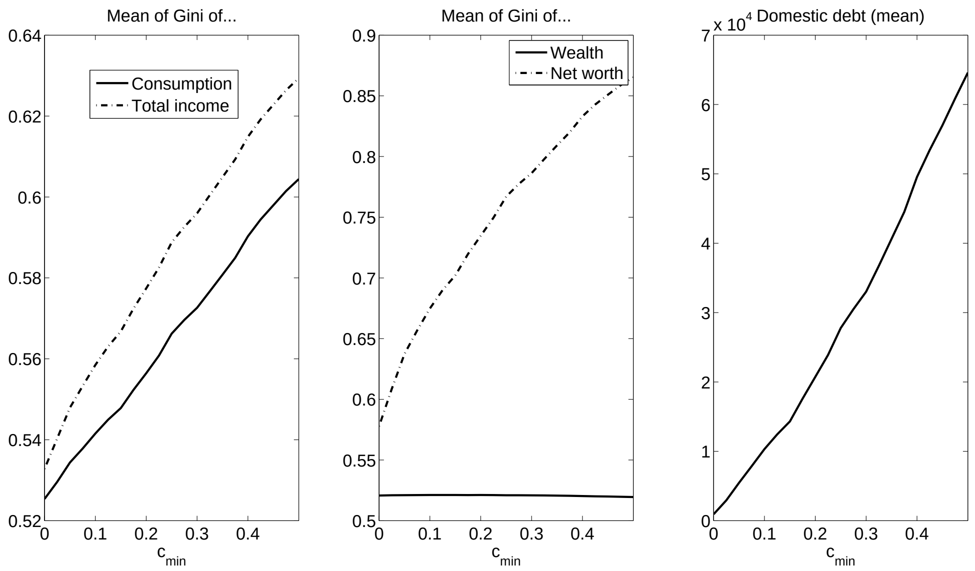


Figure 6.13.: Inequality for parameter variation of minimum consumption quantile c_{min}

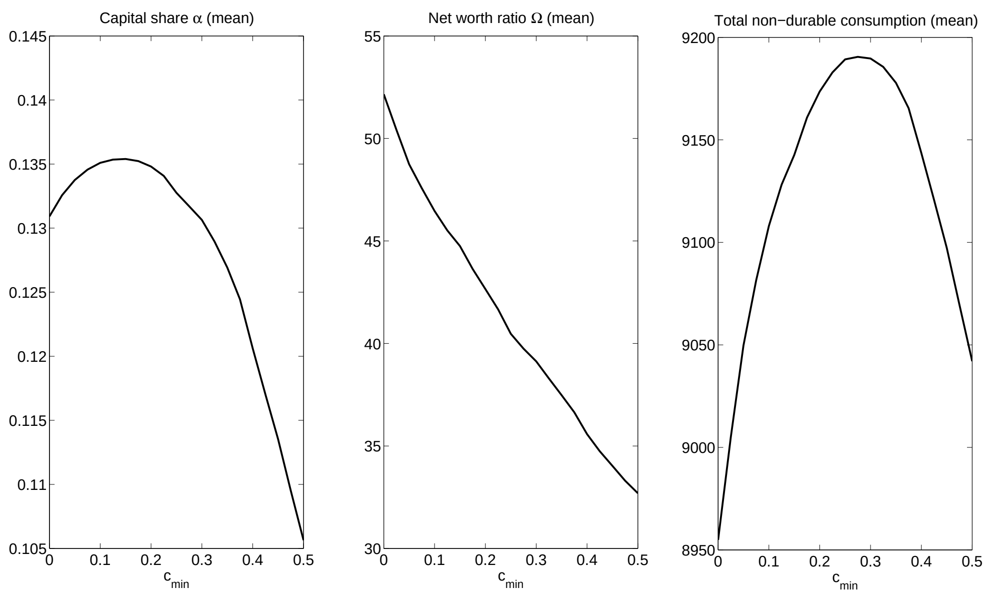


Figure 6.14.: Capital share, net worth ratio, and non-durable consumption for parameter variation of of minimum consumption quantile c_{min}

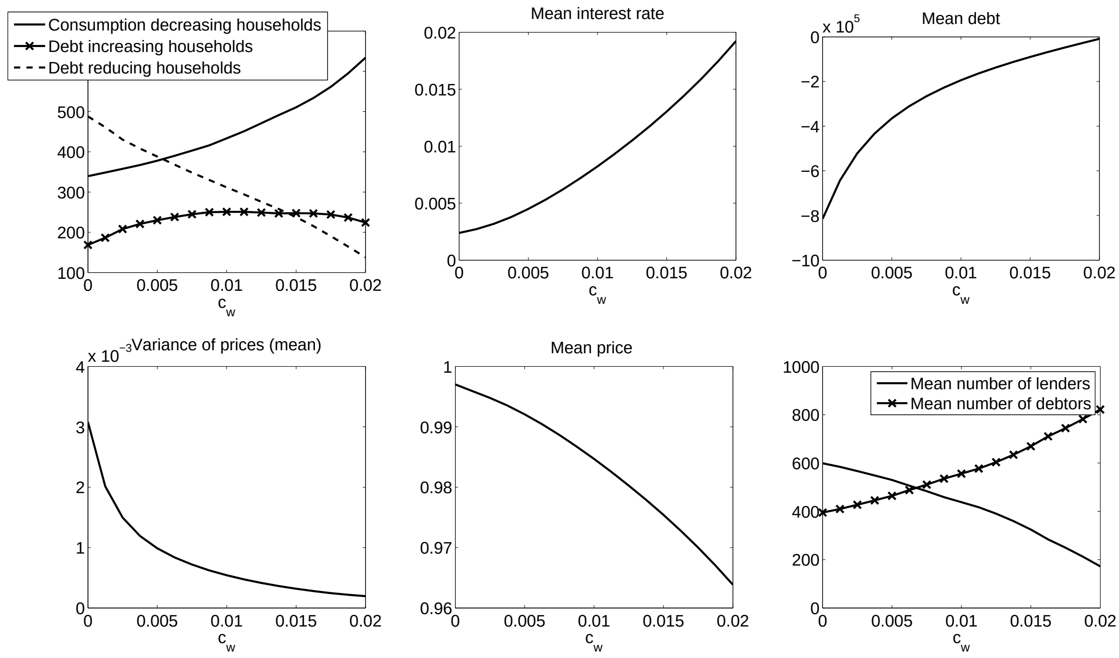


Figure 6.15.: Household composition, market for durables and savings for parameter variation of MPC out of net worth c_w

Next we vary the MPC out of net worth c_w.[38] Qualitatively, the results are identical to the results with a variation of c_y (cf. figure 6.15). The debt increases, the interest rate increases, volatility and price of durables decrease. We have more lower-class and fewer upper-class households, whereas the number of middle-class households exhibits an inverse u-shaped reaction to a variation of c_w.

The inequality increases with c_w due to the extended use of debt (cf. figure 6.16).[39] The capital share α, the net worth ratio Ω, and the total non-durable consumption also react in a comparable manner as with the variation of c_y (cf. figure 6.17).

[38] Note that once again we only simulated the surplus case for which $c_w < \frac{s}{H} = 0.025$ holds. The second (time-varying) condition reads $c_w > r_t s = 0.5s \leftrightarrow r_t < 2c_w$. As shown in figure 6.15 in the panel in which c_w and r are related this condition does not bind for low values of $c_w \lessapprox 0.0025$. This figure is, however, slightly misleading as it only plots the time average of r_t. As detailed simulations for the case $c_w < 0.0025$ confirmed the condition $c_w > rs$ is always satisfied in the long-run in which the interest rate converges to small enough values to satisfy this condition.

[39] The figure suggests that consumption inequality is very low for low values of $c_w \lessapprox 0.0025$. In particular, it is lower than inequality of labor income (exogenously given with $Gini(Y) \approx 0.52$). As already put forward in the previous footnote this is deceptive and is reversed in the long-run in which the condition $c_w > rs$ holds.

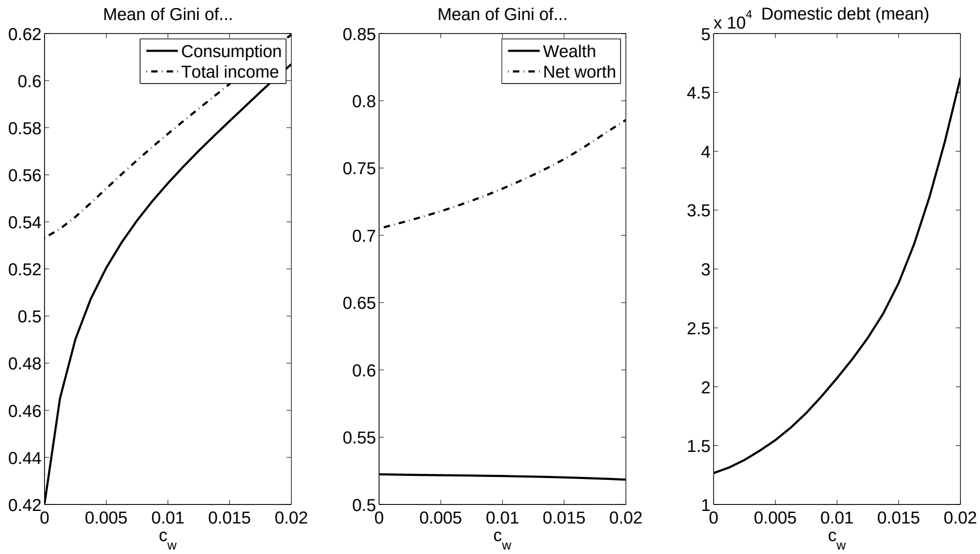


Figure 6.16.: Inequality for parameter variation of MPC out of net worth c_w

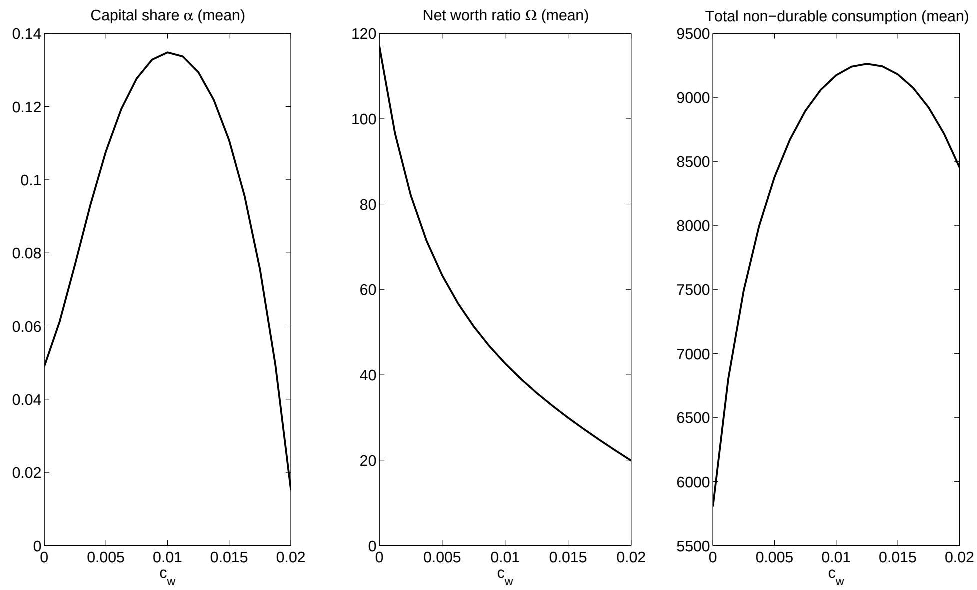


Figure 6.17.: Capital share, net worth ratio, and non-durable consumption for parameter variation of MPC out of net worth c_w

Our basic simulation argued based upon a linear consumption function implicitly assuming $\varepsilon = 1$. As already presented in section 4.4 empirical evidence points to the fact that the consumption function is concave, implying that wealthy individuals have a higher savings ratio. As moreover already discussed in section 5.5.2, this property leads to the fact that wealth inequality does not converge in the long-run.

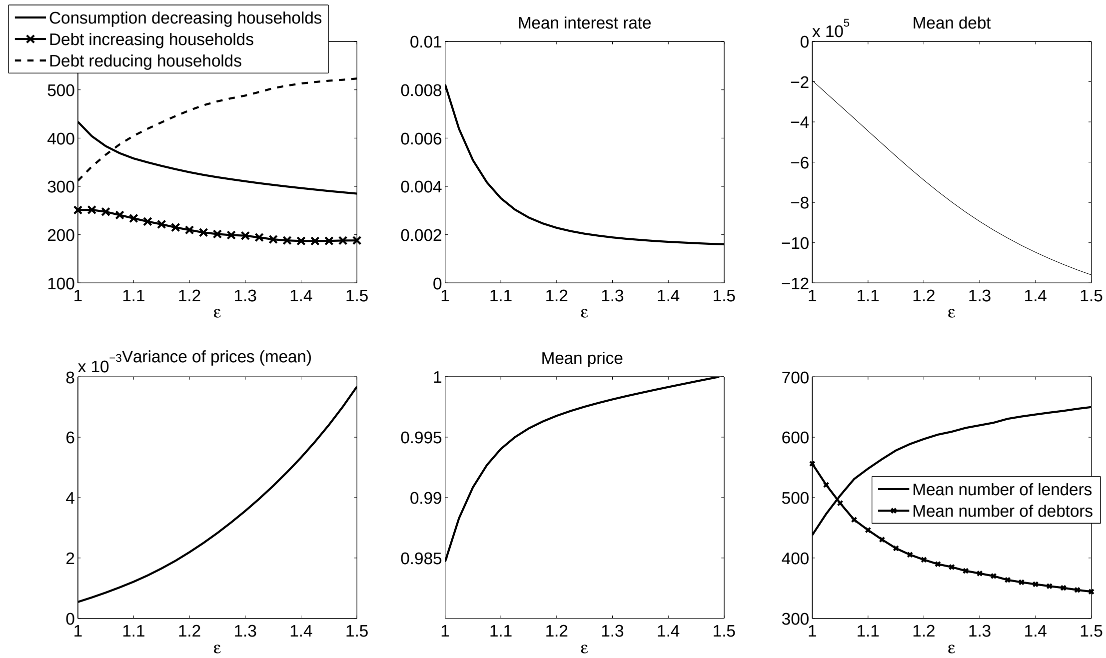


Figure 6.18.: Household composition, market for durables and savings for parameter variation of curvature of consumption function ε

We vary the level of $1 < \varepsilon < 1.5$, implying a concave consumption function. In fact, the results can be compared to a decrease in the minimum consumption level. As the supply of capital increases, interest rates fall, volatility and prices of durable markets increase. We have fewer lower-class agents, but more higher-class agents. The increase in the high-class agents, however, dominates, resulting in the fact that the middle-class decreases. This is the crucial difference to the case of a variation of minimum consumption. While a higher level of minimum consumption decreases the middle-class at the expense of the lower-class, a higher ε - implying higher savings ratios of the wealthy - contributes to a lower middle-class since more agents turn high-class. In contrast to all situations in which the consumption was increased (increase in c_y, $\bar{c}$, and c_w), all agents are better off *class-wise* for a higher level of ε. In fact, an increase in ε does little else but increase the (non-linear) savings propensity. This, however, comes at the cost of increased financial instability.

As the overall level of debt decreases the level of inequality also decreases (see figure 6.19). In particular, the inequality of consumption decreases substantially. This is the case since the consumption function is now concave, decreasing the inequality of

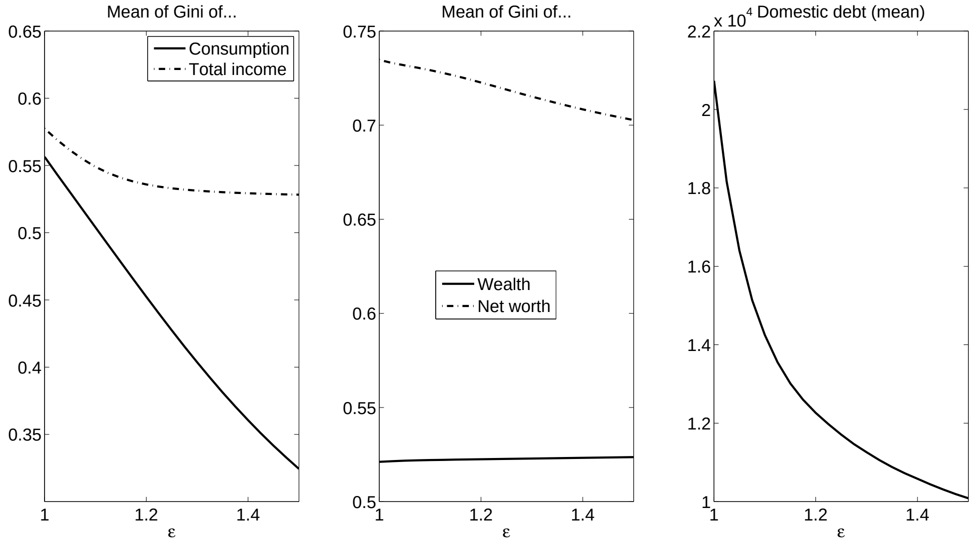


Figure 6.19.: Inequality for parameter variation of curvature of consumption function ε

consumption for a given level of labor income inequality.[40] Somewhat surprisingly, the inequality of net worth also seems to decrease. This, however, is deceptive. For the finite run ($T = 500$) the effect of the decrease in debt dominates. The inequality of wealth, however, increases in time. Thus, in the long-run - as already presented in section 5.5.2 - wealth inequality diverges ($\lim_{T\to\infty} Gini(W_T) = 1$).

We can also discuss the effect on the other aggregate variables as presented in figure 6.20. Due to the high savings the aggregate net worth ratio increases. Nevertheless, the aggregate capital share decreases, as the effect of the decreased interest rate (cf. figure 6.18) dominates. Or to speak in the words of the CES production function, this is the case of an ineleastic capital demand function. In contrast to the case assumed by Piketty (2014) the increase in capital does not increase the capital share in the economy.

One could interpret the increase in ε as a depiction of the *global savings glut hypothesis* in which the excess supply of savings of countries such as China and Germany increased their current accounts, resulting in debt for other countries. Moreover, this put downward pressure on the world interest rate and increased the fragility of durable markets. In our stylized model, investor only can invest in domestic durables. In reality, however, capital was frequently invested in foreign countries. In fact, countries with current account deficits (such as the US or Spain and Ireland) witnessed real estate booms and busts financed with foreign capital.

Finally, we wrap up the main findings. Factors that increase consumption make agents worse off class-wise. The increased use of debt leads to higher inequality. The high level of debt also makes the emergence of Ponzi schemes more likely. Meanwhile, the stability of durable markets increases. In contrast to that a higher desire to save decreases interest rates and destabilizes durable markets.

[40] A formal proof for the latter is presented in the appendix A.9.

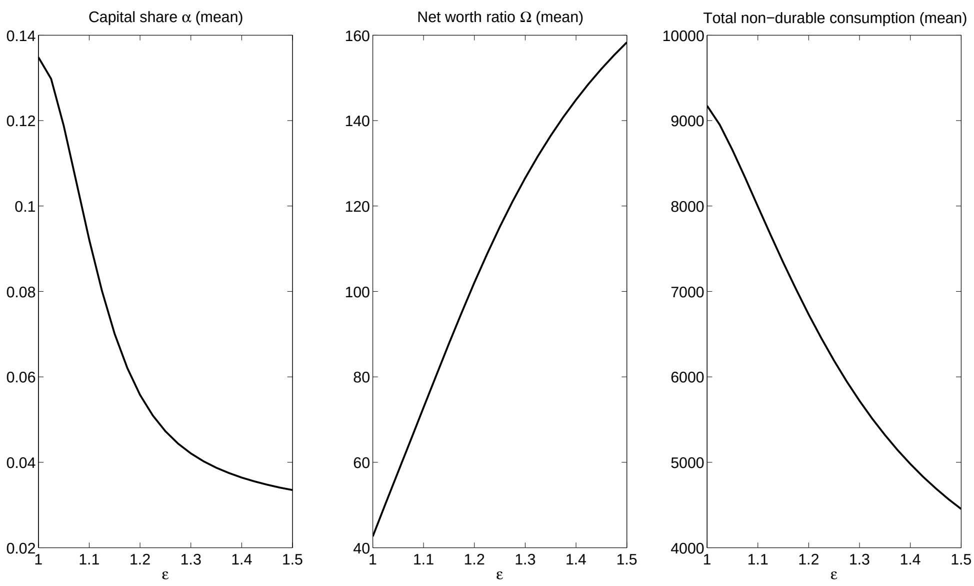


Figure 6.20.: Capital share, net worth ratio, and non-durable consumption for parameter variation of curvature of consumption function ε

Moreover, there seems to be an optimal level of the parameters in the consumption function to ensure a maximum level of total consumption. If the propensity to consume is too low, there is too little consumption, whereas for the inverse case too much is consumed and too little capital is accumulated. In the latter case, aggregate consumption could be increased by refraining from consumption in the short-run and lending resources to foreigners. These claims yield future interest payments that enhance total income $X = Y + rK$. When the overall level of income increases, so does aggregate consumption in the long-run. It is important to note the similarities to the classic case of dynamic efficient economy (Phelps, 1961) as discussed in section 4.2. In this popular theory, there exists an optimum savings level $s = 1 - c_y = s^*$. For the case of too little savings ($s < s^*$, respectively too much current consumption), aggregate capital is below its steady state $k < k^*$ and the rate of interest is above the rate of growth $r > g$. Such an economy is labeled a dynamic efficient economy. As already shown, this prevails in the data (also cf. figure 4.5). The converse case is considered dynamically inefficient.

Speaking in terms of classes as introduced in our model, a large middle-class contributes to high aggregate consumption. Note that due to the subsistence level of consumption the average propensity to consume decreases with wealth and income. However, the agents with low income and especially wealth face a binding collateral constraint hindering them from consuming. As a result, the middle-class is the group with the strongest consumption propensity. We only considered aggregate consumption. It is, however, important to note that the specification of the consumption changes the consumption level of different classes. A low value of marginal propensity to consume flattens the convex consumption function of labor income as presented in figure 6.7 in the long-run, implying that consumption inequality decreases.[41] Hence, lower values of marginal propensity to consume lead to less consumption inequality and more consumption of middle-class households. For the inverse case, the bulk of aggregate consumption results from the consumption of high-class individuals.

Opposed to that, a large number of high-class agents lead to the fact that financial stability in the market for durables decreases. As these agents - rather than consuming their high-income - use it for creating claims to foreigners, the net worth of the economy increases. This high net worth favors speculation in the market for durables and thereby increases financial fragility.

Finally, one might point to the fact that the (nominal) capital in our model presenting a debt to another agent by assumption is a *bad* rather than a *good* as it is solely employed for financing unsustainable debt resulting from the relative consumption motive or conducting speculative activity as we do not consider productive capital (usually modeled by means of a production function). Thereby, any decrease in capital is welfare-enhancing in our model. This is definitely a very gloomy picture of capital. However, we think that the role of capital lies between the one presented here and the very enthusiastic picture presented in the standard growth literature.

6.2.2. Risk Aversion and Inequality

In the following we vary other important parameters that cannot be classified as belonging to the consumption function.

We start by the general risk aversion γ which scales the demand for durables - leading to a higher demand for durables for low values and vice versa.[42] As already extensively discussed in section 5.4.1, a low value of γ promotes the flight effect stabilizing the market for debt. On the other hand, this decreases the stability of the durable market (cf. section 5.4.2). The latter behavior becomes clearly visible in the panel of figure 6.21 that depicts the variance of durable prices. Other variables show little reaction to a variation of the risk aversion. However, the risk aversion impacts on the distribution of agents. In general, a lower risk aversion increases the number of middle-class households due to the already presented flight effect. There is an inflow of both low and high-income agents in the middle-class. In particular in booms, all agents are middle-class: former lower-class agents do not face a binding collateral constraint and can lever up, while even high-class agents conduct levered purchases with debt.

[41] The fact that consumption inequality is higher than labor income inequality is due to the fact that consumption as a function of labor is convex. The inverse case would emerge for a concave function. For a simple linear function, both consumption and labor income inequality would take identical levels. The convexity itself emerges as a result of the presence of the collateral constraint. For details the reader is referred to appendix A.9.

[42] We simulate a value ranging from 5 to 50. The formal stability condition (as derived in equation 5.59) requires $\gamma > 0.5\mu_r E(W)\beta$ which for the given values boils down to $\gamma > 0.5$. Note that the condition is only a lower bound by not including non-linear effects. Thus, our search halts at parameter of $\gamma = 5 > 0.5$.

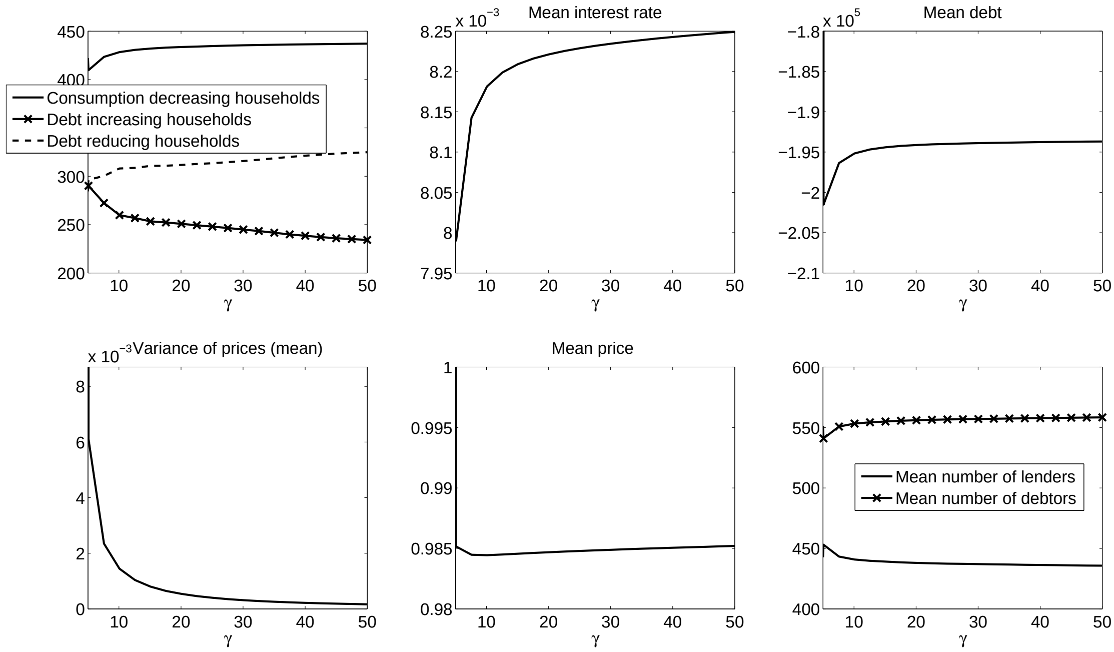


Figure 6.21.: Household composition, market for durables and savings for parameter variation of risk aversion γ

We also did a *ceteris paribus* variation of m. However, it turned out that quantitatively it has little impact.[43] In a qualitative manner all expected effects are observed. In particular, high levels of equity requirement impede the access of households to credit resulting in a higher number of lower class households. In effect, the equilibrium level of debt and interest rate is lower.[44] Moreover, the inequality of net worth is reduced, also reducing the inequality of total income and consumption. Interestingly - and conversely to theoretical ideas brought forward in section 5.4.3 - the price volatility of durables also decreases as the leveraged speculation in durable markets is also discouraged. As a result, higher equity requirement always increase financial stability.

We can also vary the level of heritage H. Note that a higher level of heritage leads to the fact that the share of self-earned wealth decreases. We ran the simulation for values of $H < \frac{s}{c_w} = 50$ satisfying the surplus economy case. In fact, the role of heritage is very similar to the role of MPC out of net worth c_w since - as detailed in section 5.5.1 - they always come as twins.

A higher level of heritage increases the level of assets and thereby total net worth available for consumption. The higher consumption increases debt, interest rates and the price level in the durable market (see figure 6.22). Class-wise we have more lower-class and fewer higher-class agents. The middle-class exhibits an inverse u-shaped reaction

[43] For the latter reason, we spare the depiction of the concrete simulation results. They, however, are available on request.

[44] In the logic of figure 6.11 the demand function of capital shifts to south-west.

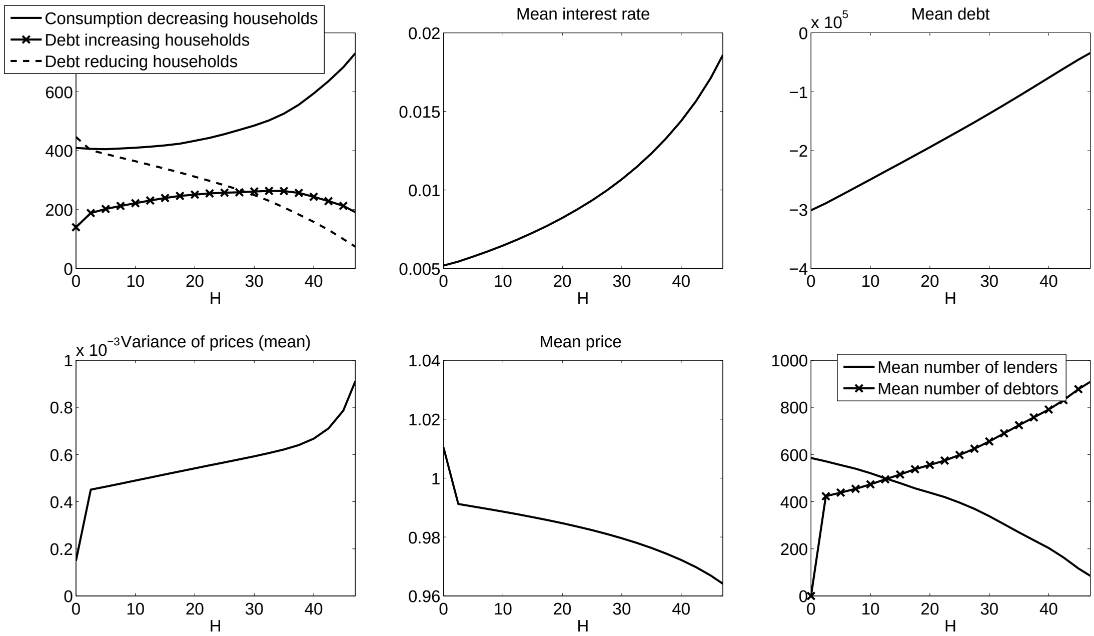


Figure 6.22.: Household composition, market for durables and savings for parameter variation of heritage H

to an increase in heritage H. The latter is more pronounced than in the case of c_w and can be attributed to the second property of heritage - namely the *collateral effect*. By initially endowing agents with more net worth available as collateral to borrow against fewer agents face a binding collateral constraint and therefore there are less agents qualified lower-class.

The key difference between the aggregate reaction to an increase in the MPC out of net worth c_w and the increase in heritage H, however, lies in the market for durables. Strong heritage increases net worth and thereby increases the available amount in the market for durable scaling up the price variance in this market. As a result, strong heritage increase the instability in durable markets (see figure 6.22).

As heritage increases debt, inequality increases (cf. figure 6.23). In this case, it is important to point out the special case with zero heritage ($H = 0$). In this case, all agents are born without an initial endowment of assets serving as a potential collateral. As a result, there are no domestic agents in debt and thus there is no domestic debt at all (cf. figure 6.22 as well 6.23). Moreover, the inequality of wealth is identical to the inequality of net worth. There is little activity in the durable market accompanied by the fact that the durables in fact are overvalued on mean ($E(P) > F = 1$).[45] In the case without heritage ($H = 0$), the middle-class are high-class households that

[45] The more precise rationale for the latter is that agents temporarily create a positive amount of durable assets at an initial price of $P = F = 1$. When selling they, however, face a short-selling constraint of leading to the fact that the lower bound of prices is always $P = F = 1$ and thereby $E(P) > 1$.

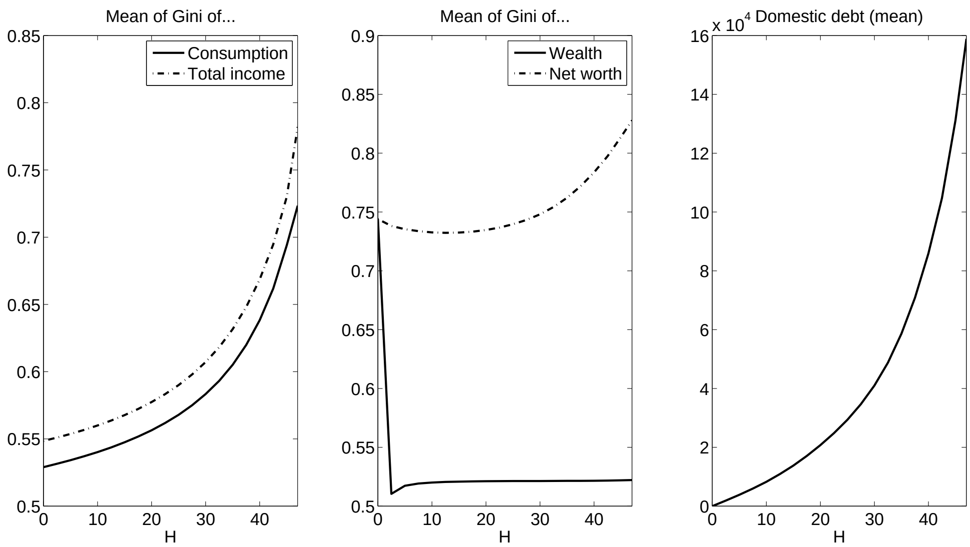


Figure 6.23.: Inequality for parameter variation of heritage H

temporarily lever up in order to finance durable purchases. In the long-run, there is a clear bisection in lower and higher-class households presenting agents that cannot realize their consumption desires and agents that are net savers. Note that for zero heritage, the number of lenders and debtors does not add up to the total number of 1,000 agents as now there is a third group of agents holding neither assets nor debt. The latter is also the explanation of the increased wealth inequality depicted in figure 6.23. We will treat this issue more thoroughly in the section 6.3.2 devoted to fire sales in which similar behavior emerges.

Figure 6.24 presents the effect of a variation of heritage H on the variables capital share α, net worth ratio Ω and total consumption. First and foremost, it is noticeable that the net worth ratio increases. This is somewhat surprising as we argued that an increase in heritage H can be considered equivalent to an increase in the MPC out of net worth c_w suggesting that the net worth ratio would actually decrease. The effect of increased consumption requiring more debt and therefore lowering the aggregate capital exists. However, it is dominated by a price effect. As presented in section 5.5.1 the net worth ratio (for the simplified case without relative consumption) is given by:

$$\Omega = \frac{W}{Y} = \frac{s(1 - rH)}{c_w - rs} = h(s[+], r[+], c_w[-], H[-]), \tag{6.14}$$

implying that it decreases with H. However, an increase in r - which emerges for higher values of H as documented in figure 6.22 - leads to an increase in the net worth ratio.[46] In the aggregate, the latter effect dominates leading to a (slight) increase in the net worth ratio for an increase in heritage H.

[46]Formally, the latter is the case since $\frac{\partial \Omega}{\partial r} = \frac{s(s-Hc_w)}{(c_w-rs)^2} > 0$, since in the surplus case we have $s > Hc_w$.

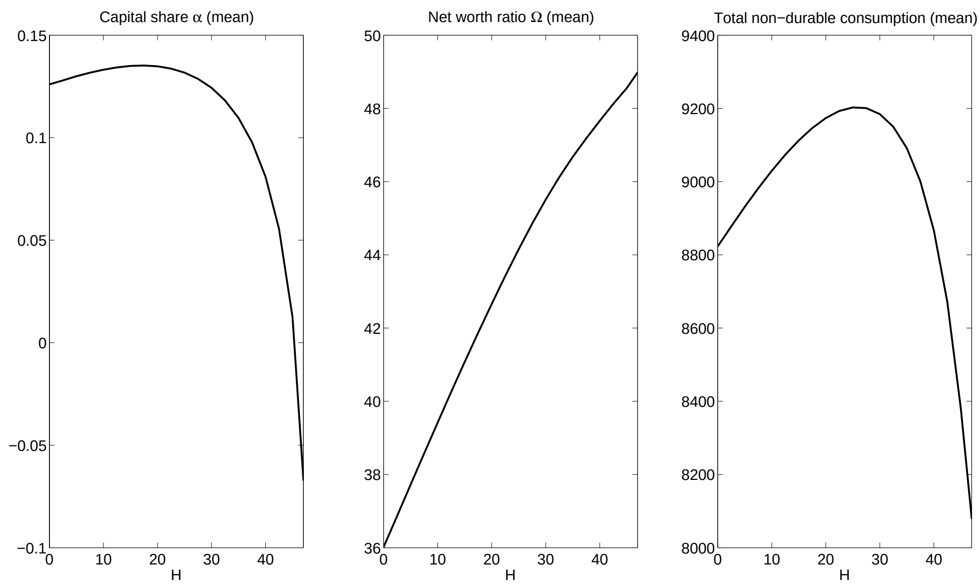


Figure 6.24.: Capital share, net worth ratio, and non-durable consumption for parameter variation of heritage H

The aggregate capital share α once again exhibits an inverse u-shaped behavior owing to the trade-off of more debt (less capital) and higher interest rates (cf. figure 6.22).[47] The inverse u-shaped behavior of the total non-durable consumption - similar to the case of c_w - reflects the trade-off between higher consumption propensity and lower aggregate capital available for consumption.

One also might consider H as a policy variable. In particular, a lower level of H can be achieved by imposing a tax on the stock level of wealth. In fact, a simple lowering of H would constitute an extreme form of a tax policy with total leakage - implying that proceedings from the tax are not redistributed to low-income individuals but completely destroyed. Note also that heritage H determines the value of real assets. An example of a destruction of real assets would be the world wars in the 20th century which - as empirically shown in the work Piketty and Zucman (2014) - led to massive reduction of the stock value of wealth. Eventually, in the context of the model could be considered welfare enhancing as both total income and net worth inequality would be reduced. Moreover, all agents would be better of class-wise. Furthermore, the financial stability would be enhanced as a lower amount of real assets - serving as collateral - bind the ability to create (artificial) financial assets that can contribute to financial instability. A

[47] For high values of heritage H the aggregate capital share is eventually negative. When disaggregating in the time dimension we find that this results from a temporary primary deficit that, however, in the long-run is reversed, implying $\lim_{t\to\infty} \alpha_t > 0$.

very high tax, however, would have an adverse impact on non-durable consumption.[48] Finally, the net worth ratio would be lowered.

As a result, our policy maker faces a trade-off. While a society might not be prone to financial instability or issues of inequality, it is poor in wealth. This result can be compared to the left arm of the Kuznets curve for which inequality (as well as its related problems) and development are conflicting goals.

The most interesting variation, however, concerns the role of inequality. As intensively discussed in section 5.2, we assume an exogenously given distribution of labor income that can be completely controlled by its standard deviation σ_y controlling for the degree of inequality.[49] We can take our model to simulate an exogenous change to inequality - in a *ceteris paribus* manner - and discuss its aggregate macroeconomic effects. In empirical discussions (cf. section 2.2) quantifying the effect of inequality is very difficult as it is interspersed with a variety of different factors that feed back in various directions. A natural experiment - satisfying the *ceteris paribus* assumption - is also hard to imagine in this domain, making the theoretic discussions still the most valuable tool for analyzing the effect of inequality.

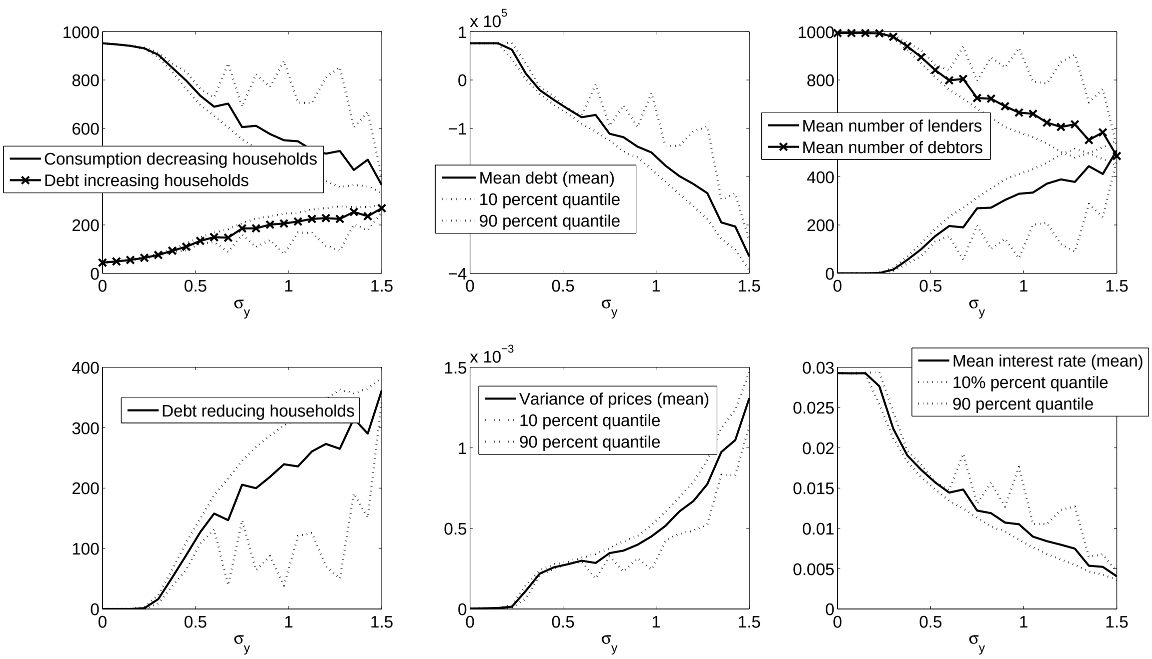


Figure 6.25.: Household composition, market for durables and savings for parameter variation of labor inequality σ_y (80% quantiles)

[48] Regarding non-durable consumption lowering heritage H in our model is an implicit form of lowering the value of consumption out of net worth c_w as the two always come as twins in the non-durable consumption function.

[49] Recall, that as presented in section 5.2 and derived in section A.4, there is a positive relation between the level of σ_y and the Gini. For small values it is furthermore well approximated by $Gini \approx 0.5\sigma_y$.

It is important to note that we only have a finite sample of $N = 1,000$ agents. As already put forward the draw from the distribution can vary severely. To control for this effect, we simulate each case various times and also aggregate along this dimension.[50]

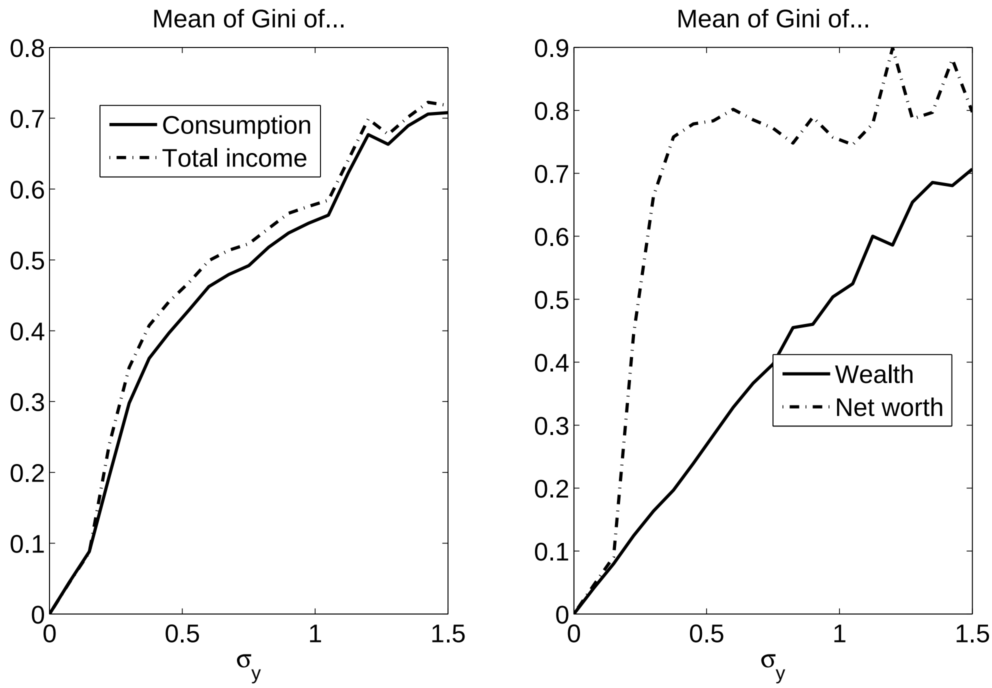


Figure 6.26.: Inequality for parameter variation of labor inequality σ_y

First of all, higher inequality changes the distribution of agents with respect to class (see figure 6.25). More inequality leads to more higher-class agents. Somewhat surprisingly, we have fewer lower-class agents and more middle-class agents. The latter factor can, however, be explained by the changing value of the relative consumption level. As depicted in figure 5.3, a higher inequality decreases the relative consumption level if $j < 0.5$. Since in our case we assume $j = 0.2$ this level decreases making the effect of an increase in inequality similar to a decrease in minimum consumption already discussed earlier.

In effect, less debt is accumulated, interest rates decrease, but the prices in the market for durables increase and - this is a key result of this thesis - the instability in durable markets increases. The accumulation of capital available for speculative purposes due to a strong share of high-income individuals is the driving force behind this result. Note also that we find a direct negative relation between inequality and the equilibrium level of interest rates. The rationale - as already presented in figure 6.11 - is that the market

[50]We take $R = 20$ repetitions. The reported values in this case not only aggregates in the time and agent dimension but also in the dimension of repetitions (e.g total durable consumption now is $E(C_{tot}) = \frac{1}{R}\frac{1}{T}\sum_{r=1}^{R}\sum_{t=1}^{T}\sum_{i=1}^{N} C_{r,i,t}$. We, however, also provide the 80% confidence interval. Another approach would be to just increase the number of agents to come close to a law of large numbers.

for debt is mainly driven by supply which increases with higher inequality. Note that for very low values of inequality - for the given consumption function - we eventually end up in a deficit economy. In this case, we have very little high-class individuals. Their social role is supplying the debt. As they disappear for low inequality, debt has to be imported from foreigners. More technically, it should also be noted that for high levels of inequality the confidence bands of the simulation increase. In effect, the simulation becomes very fragile for high inequality, implying that while a certain (finite) draw of the given income distribution might still lead to a stable aggregate result, another draw is accompanied by high instability.

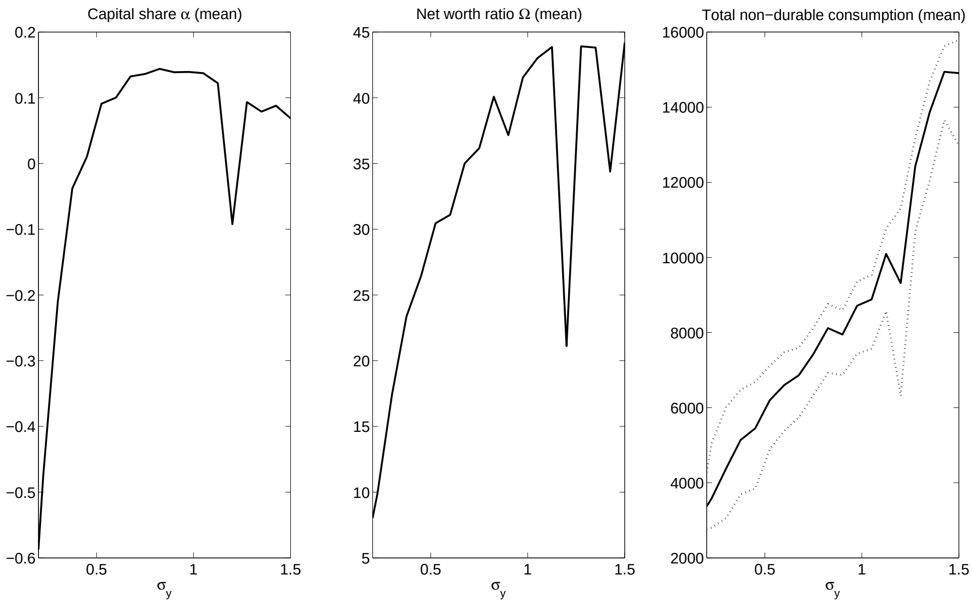


Figure 6.27.: Capital share, net worth ratio, and non-durable consumption for parameter variation of labor inequality σ_y

We assume an exogenous increase in the inequality of labor income. As initial asset endowment is assumed to be perfectly correlated the inequality of wealth equals the inequality of labor income. As predicted in section 5.2 the relation between controlling parameter σ_y and the inequality is well-described by a linear function $Gini(Y) \equiv Gini(Pq) \approx 0.5\sigma_y$. Other forms of inequality, however, grow at a different pace. As agents accumulate debt and claims net worth and total income inequality increase to a higher level (cf. figure 6.26). This also feeds back to inequality of consumption which, however, is always (slightly) below inequality of total income. We can rank different economic figures according to their inequality:

$$Gini(W) > Gini(X) > Gini(C) > Gini(Y) = Gini(Pq), \tag{6.15}$$

with net worth being the most unequal measurement and the (exogenously assumed) factor of wealth and labor income are most equally distributed.

Comparable to the cases of decreasing MPC of disposable income as well as net worth (c_y respectively c_w) and especially the relative consumption level $\bar{c}$, an increase in inequality leads to an inverse u-shaped reaction of the capital share (see figure 6.27 [51]). While the the net worth ratio increases for higher inequality (since we have both an increase in the mean income[52] and a decrease in $\bar{c}$), the interest rate decreases (see figure 6.25). Both effects work against each other leading to the inverse u-shape of the aggregate capital share. As a result - and as already argued in section 5.5.1 following a more theoretical rationale - higher inequality in the personal distribution of labor income does not have to go along with a higher functional share of capital. The simulation results, however, become very noisy for high values of inequality. Due to the increase in the general level of income, aggregate non-durable consumption unambiguously increases for inequality.

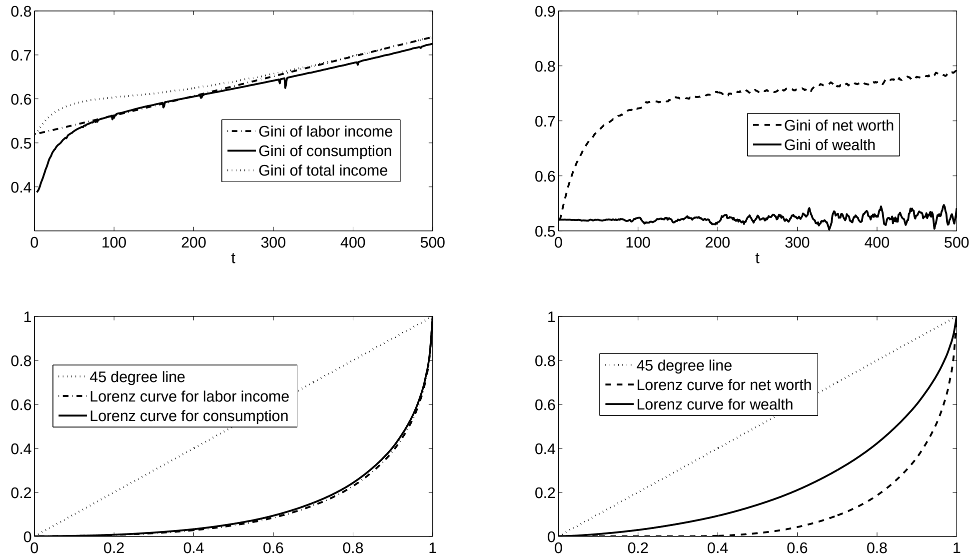


Figure 6.28.: Gini coefficient in time and Lorenz curve in last period ($t = 500$) for labor income, consumption, wealth and net worth for growing labor inequality ($g_{ineq} = 10^{-3}$)

The general model so far is a static model as we have no aggregate growth. In section 5.2, we already presented an approach to model an economy in which both inequality and total labor income increase in time. In fact, we model the left part of the *Kunznets curve*. Figure 6.28 shows the Gini coefficients for different economic variables. By construction, the inequality of labor income increases in a linear manner.[53] Due to the well-described

[51] Note that we only present the confidence intervals for the case of total durable consumption. The other factors are highly sensitive for high values of inequality.

[52] This can be clearly shown by the mathematical identity for which $Y_{tot} = nE(Y)$ unambiguously increases with σ_y as: $E(Y) = \bar{Y}exp(0.5\sigma_y^2)$.

[53] For the underlying rationale the reader might want to refer to appendix A.5.

argument the inequality of net worth oscillates around a constant steady state equal to the initial distribution of labor income. Following the upward trend of labor income inequality and the accumulation of debt and claims, net worth inequality, total income inequality and consumption inequality, however, all increase in time.

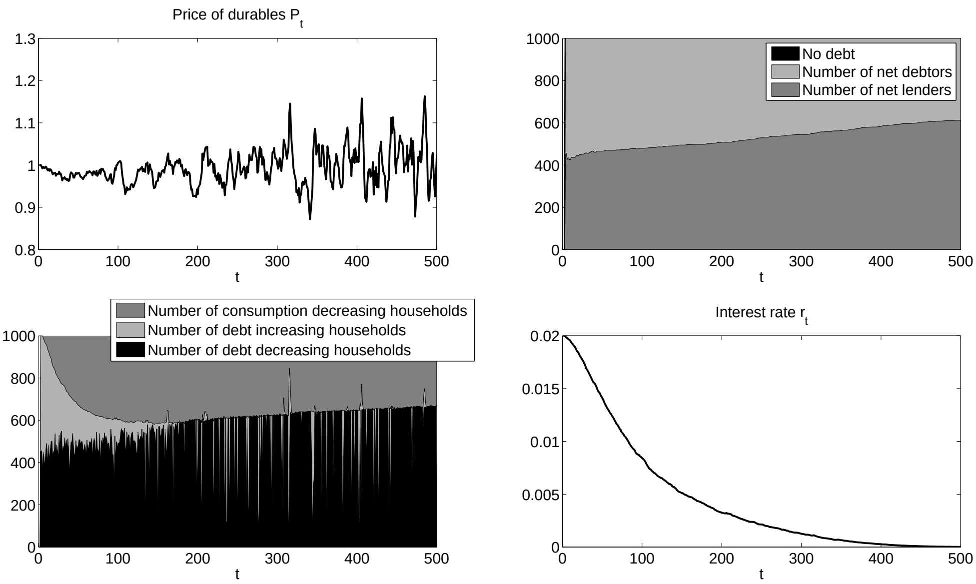


Figure 6.29.: Market conditions for savings/debt market and durable market and composition of household groups for growing labor inequality ($g_{ineq} = 10^{-3}$)

The higher inequality in time also has an interesting impact on the class-wise distribution of agents (see figure 6.29). As the relative consumption level falls and, moreover, some very high-income agents emerge the number of debtors decreases in favor of the number of lenders. Flow wise we have more higher-class agents and fewer lower-class agents. As usual the middle-class disappears in the long-run. The high supply of claims also applies downward pressure on the interest rate.

As already presented in section 5.2 (in particular in figure 5.6) the growing inequality - by assumption - is accompanied by an exponential growing output. In total, this is accompanied by higher aggregate consumption (both of non-durables and durables) and an increasing total balance sheet that - in the surplus economy - mainly consists of claims against foreigners (see figure 6.30). Interestingly, in later simulation periods - with higher values of inequality - the level of domestic debt falls. This can be rationalized by the fact that the lowering level of relative consumption $\bar{c}$ requires a lower level of debt and thereby allows low-income agents to (slightly) deleverage.

In effect, higher inequality eventually seems desirable from this aggregate standpoint. Not only does the standard of living - measured by total consumption - increase, but also the class-wise distribution of agent (debtors vs. lenders) is improving as the number of indebted households decreases. Thus, the problem of financial instability in the market

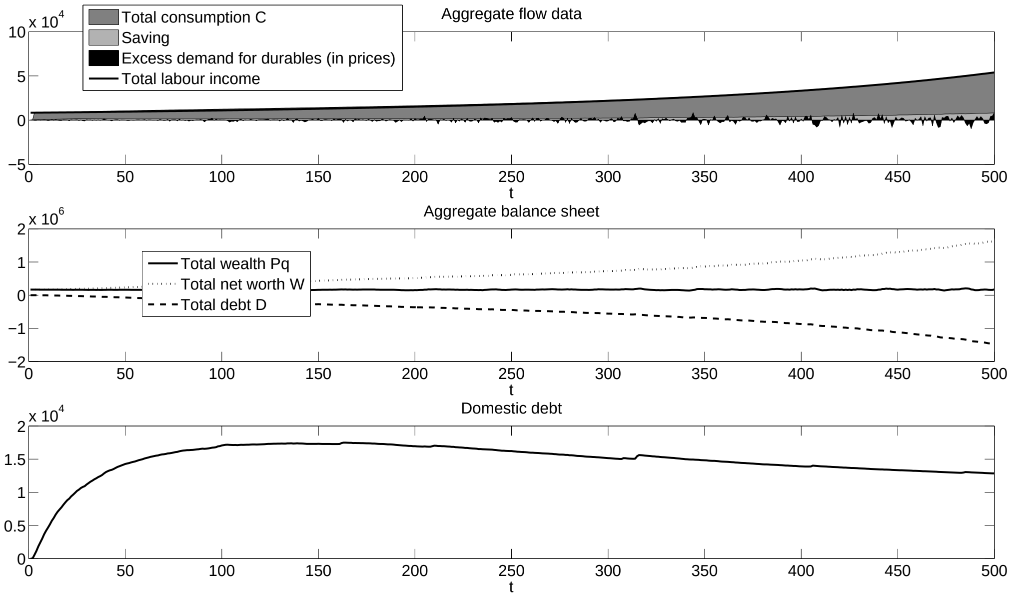


Figure 6.30.: Decomposition of GDP (aggregate flow) and aggregate stock for growing labor inequality ($g_{ineq} = 10^{-3}$)

for debt - as extensively discussed in section 5.4.1 - resulting from Ponzi games is reduced. This *calmness*, however, is deceptive.

We redo the simulations with a higher growth rate of inequality ($g_{ineq} = 2 \cdot 10^{-3}$). Figure 6.31 can be considered as an exemplary anatomy of a financial crisis. As more and more net worth is accumulated, the stability condition in the market for durables (detailed in section 5.4.2) given as $\gamma > 0.5E(W)\mu\beta_C$) ceases to hold. Due to some negative noise trading shock amplified by chartists trading an extreme strong flight to risky assets accompanied by a high demand for debt is created leading to an extreme increase in interest rates.[54] Thus, for a short period of time, all agents become debtors. As no domestic agents are willing to supply any debt, the interest increases to an extreme level.[55] Eventually, this is the situation presented in the illustrative figure 5.8. Now, however, all agents are over indebted and fall into the group of low-class agents. Trading in the market for durables completely halts and all labor income is employed primarily for paying interest on foreign debt. A potentially existing residual is taken for non-

[54]The reader should note the scale of the price diagram ranging up to $5 \cdot 10^{14}$.
[55]In the mathematical model it diverges to infinity.

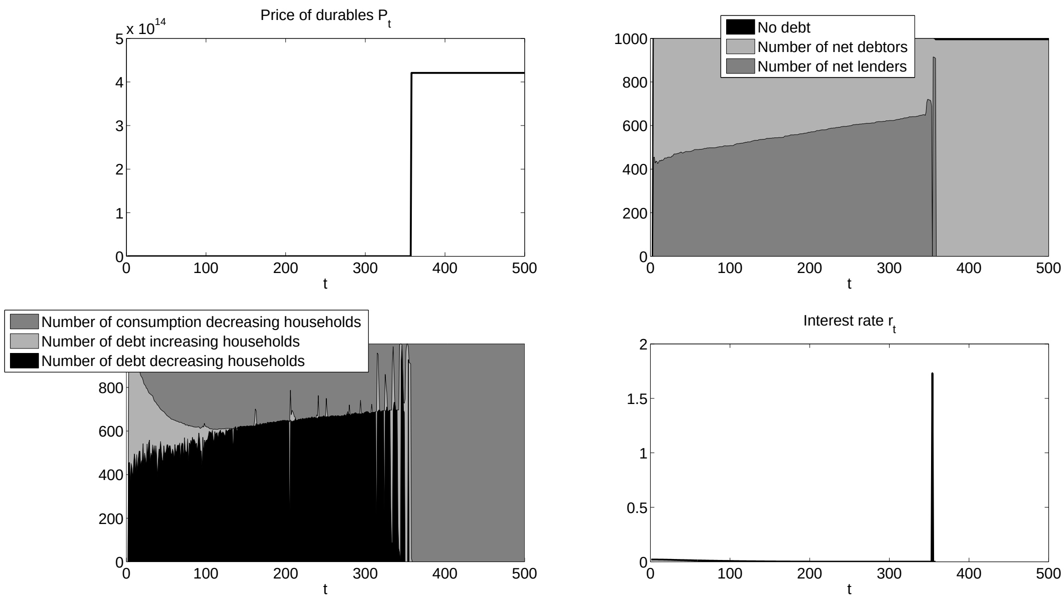


Figure 6.31.: Market conditions for savings/debt market and durable market and composition of household groups for fast growing labor inequality ($g_{ineq} = 2 \cdot 10^{-3}$)

durable consumption.[56] One might want to interpret this as a *Minsky moment* (Minsky, 1986).

It might also be interesting to compare this breakdown with the well-discussed breakdown known as a *sudden stop* (Calvo, 1996). The latter problem occurred in Latin American economies in the 1980s as well as in the Asian Crisis in the 1990s in particular in emerging economies. Basically, foreign investors that financed the economic expansion - due to some noisy unfavorable signal about future prospects - decided to *suddenly stop* their inflow of capital into the country also accompanied by a reverse of a massive current account deficit into a (slight) surplus and a depreciation of the domestic currency. As the emerging country, however, held debt nominated in foreign currency this devaluation made paying interest on foreign debt (merely) impossible leading to the breakdown of the economy.

Our case bears some similarities but, however, also some large differences. First of all, the episode of the sudden stop only occurs during a very limited time as the simulated economy was in a large current account surplus for a long period of time. This accumulated net worth, however, favored speculative activity in asset markets. Due to some noisy signal about good future prospects in the asset market, all agents want to

[56] This paragraph presented a more narrative approach. Technically, once the instability property is crossed prices grow without bound. Formally, the movement of prices (respectively interest rates) is described by a sine wave (as the eigenvalue has an imaginary part capturing the length of the cycle) as agents quickly reshift their portfolio between the asset classes. The envelope of this curve is given by a growing exponential process (since the real part of the eigenvalue is positive). This behavior is independent of whether the initial noise trading shock was positive or negative.

participate in the asset market, implying that no domestic financing is available leading to a massive increase in the interest rate. As we have no exchange rate mechanism this is similar to the depreciation of domestic currency increasing the effective rate of interest on foreign debt witnessed in the classic sudden stops. Note that in our case the foreigners are completely passive. The *sudden stop* of supply of capital in our case thus does not fall within the responsibility of the foreign agents but is due to the behavior of the (rich) domestic agents *suddenly* turning from lenders to debtors.

This result can also be compared to the empirical work of Atkinson and Morelli (2011), arguing that the change of inequality does not have an impact on financial stability. This result is confirmed by our simulations. Rather than the change of inequality, the absolute level of inequality matters. In the model, there is a threshold level of inequality going along with a specific accumulated level of claims leading to a breakdown in the market for durables. The growth rate of inequality only matters by leading to a faster convergence to this threshold level of inequality. In effect, the *financial meltdown* would also occur in the model with the lower growth rate of inequality $g_{ineq} = 10^{-3}$, however, at a later period of time ($t > T = 500$).

6.3. Simulation Result for the Case with Fire Sales

As already put forward in section 5.3, *Austerity* can be very painful for low-income households which have to suppress their consumption level for a prolonged time. In the previous section, we made the unrealistic assumption that agents refrain from trading in the market for durables and thereby hold their stock level of wealth constant. However, this behavior cannot be observed in real data. Selling their assets provides a cash inflow for collateral constrained households and therefore can enhance their consumption possibilities at least in the short-run. Yet, this has the effect of a negative externality on other agents by depressing the prices for durables serving as collateral and thereby increasing the number of collateral constrained households. This very argument is at the heart of *debt-deflation* argument by Fisher (1933) and the financial accelerator literature dating back to Kiyotaki and Moore (1997). We discuss this effect in the framework of our model in the following.

Comparable to section 6.1, we proceed by first presenting the concrete modeling of this case (section 6.3.1) to afterwards discuss the simulation results in section 6.3.2.

6.3.1. Modeling the Case with Fire Sales

To model this effect we have to make a statement about the behavior of the *lower class* households. We assume that these households generate the desired cash-flow to fulfill their consumption desire by selling a part of their assets:

$$\begin{aligned} P_t d_{i,t} = \dot{D}_{i,t}^{max} - \dot{D}_{i,t} = \dot{D}_{i,t}^{max} - (-Y_{i,t} + C_{i,t} + P_t d_{i,t} + r_t D_t) \Rightarrow \\ d_{i,t} = \frac{1}{2P_t}\left(\dot{D}_{i,t}^{max} - (-Y_{i,t} + C_{i,t} + r_t D_t)\right) \\ = \frac{1}{2P_t}\left((1-m)P_t q_{i,t} + Y_{i,t} - C_{i,t} - (1+r_t)D_t\right) < 0 \end{aligned} \tag{6.16}$$

It is important to note that in an inter-period context, agents first show collateral to the financial markets in order to lever up as much as possible and afterwards sell part of this collateral to generate further cash-flow. The fire sale, moreover, has a double rent in the short-run:[57] it not only provides cash-flow but also reduces demand for durables. This possibility nevertheless is limited with the short-sale constraint. Once agents have reached their short-sale constraint ($-d_{t,i} \leq q_{t,i}$) agents are once again forced to decrease their consumption (the *Austerity*-case).

We assume that these agents exhaust their shorting potential to the maximum by hitting the short sale constraint $d_{i,t} = -q_{i,t}$. This time we furthermore require delevaraging of over indebted agents.[58] Yet, we still do not allow negative consumption. Formally, the consumption of consumption-decreasing agents is modeled as follows :

$$C_{i,t} = \max\left\{Y_{i,t} - r_t D_{i,t} + \dot{D}_{i,t}^{max}, 0\right\}. \tag{6.17}$$

6.3.2. Basic Simulations

In this section, we present simulation results when allowing low-income agents employing a fire-sale heuristic. Apart from this change of rules we keep all parameters including random seeds identical to the case with wealth consumption presented in section 6.1.2 allowing for a direct comparison between the two results.

The effect of the fire-sale heuristic directly sets in at the beginning of the simulation. In particular, low-income individuals start by fire selling their assets in order to (temporarily) increase their consumption (see figure 6.32). This downward pressure in the market for durables is picked up by chartist agents further emphasizing this trend (see figure 6.33). Low-income agents sell their assets until short-selling constraints bind - implying that they have an amount of zero assets available. Furthermore, fundamental traders begin to dominate the market and bring the prices back to fundamentals (see figure 6.33). The upturn, however, essentially occurs to the benefit of the high-income agents who hold the sufficient amount of net worth to lever up. Therefore, the bust affects all agents, while in boom times high-income agents disproportionately profit. In

[57] Hence, the factor 2 in the previous equation.

[58] Formally, we spare the condition $\dot{D}_{i,t} = \max\left\{\dot{D}_{i,t}^{max}, 0\right\}$ introduced in section 6.1.

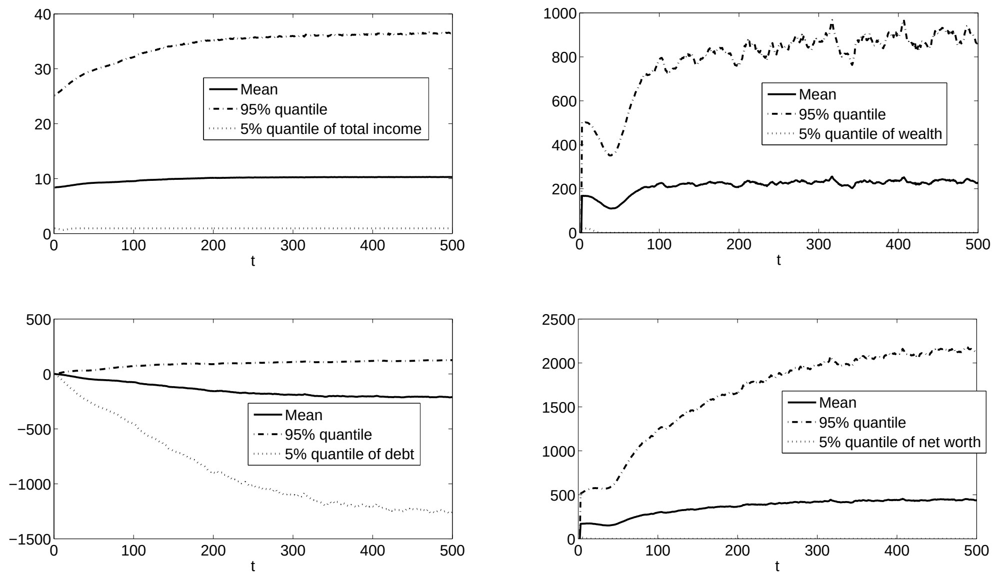


Figure 6.32.: Mean as well as 5% and 95% quantile of total income, wealth, debt, and net worth for the case with fire sales

effect, assets are redistributed within the first 100 simulation periods from low-income individuals to high-income agents severely altering the wealth distribution - in the sense of making it more unequal.

A strong caveat with regards to the simple version presented in section 6.1 was that the distribution of wealth was assumed to be exogenous. Moreover, by assumption it had the same degree of inequality as the distribution of labor incomes. This is not the case in the model with fire sales. The higher inequality of wealth feeds back into higher inequality of net worth W, total income X, and consumption C (see figure 6.34). Moreover - and in line with empirical evidence (e.g. Wolff (2013)) - the different economic variables can be put into a realistic ranking with respect to inequality:

$$Gini(W) > Gini(Pq) > Gini(X) > Gini(C) > Gini(Y). \tag{6.18}$$

Most importantly, stock variables are more unequally distributed than flow variables as this time $Gini(Pq) > Gini(X)$ and not the other way round as in the previous simulations. Moreover, the exogenously assumed distribution of labor income Y is eventually the resource which is distributed in the most equal manner. All variables evolving in the course of the simulation are distributed in a more unequal manner.

As also presented in figure 6.33, besides net debtors and lenders a strong group of agents emerges representing no debt agents. Since these agents sold all of their initial endowments, they lack collateral to borrow against, excluding them from capital markets. By allowing for fire sales the model endogenously creates agents that not only have little income (flow) as a result of underemployment but also cannot provide assets (stock) and

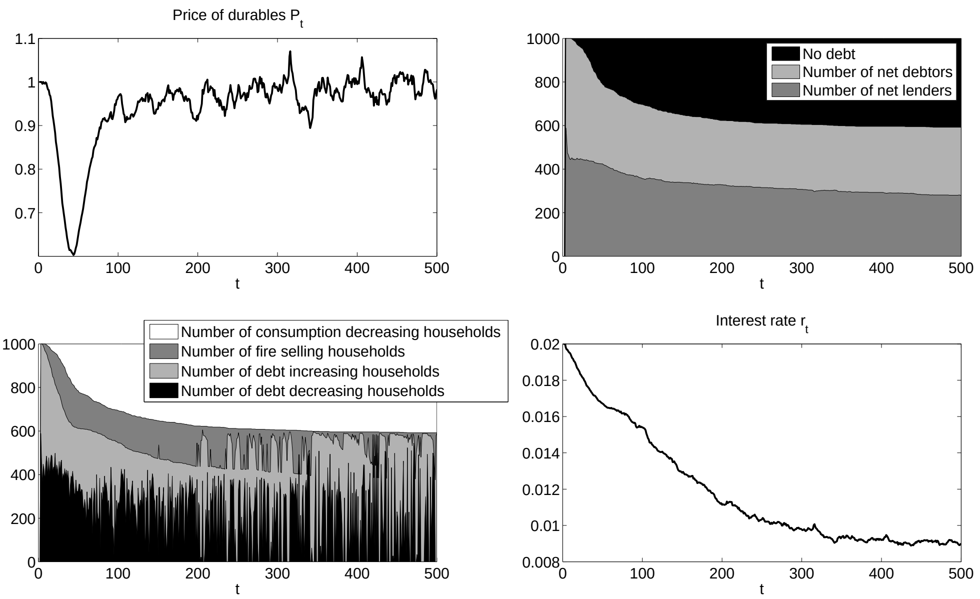


Figure 6.33.: Market conditions for savings/debt market and durable market and composition of household groups for the case with fire sales

therefore can be titled NINJAs (No Income No Job No Assets) as seen in the recent financial crisis. In effect, a low-income agent with zero wealth is always also an agent with zero net worth as it does not own assets available as collateral to borrow against. Financing the durable asset purchases by means of flow income is also not possible due to the fact that the prior-ranked consumption of non-durables exceeds the current income. The zero wealth agents also become observable in the Lorenz curve of wealth which now - and in contrast to the *Austerity*-cases presented in figure 6.2 - exhibits a zero slope at the lower end (see figure 6.34).

As also shown fire sales still persist in the latter part of the simulations. This is in particular the case in downturns in which they further exacerbate the trend as proposed by the *Debt deflation* theory (Fisher, 1933). More technically, they introduce a further non-linearity in the durable trading, adding a further instability.

This non-linear behavior also becomes obvious in the market for domestic debt as presented in figure 6.35. In contrast to the consumption decrease case, which exhibited a constant growth behavior in domestic debt, in the fire-sale case the market shows up- and downward movements. This behavior can be related to the situation in the market for durables, allowing for strong absolute debt holding in upturns or even requiring deleveraging in downturns.

We can also consider the individual capital share α_i as well as the individual net worth ratio Ω_i as depicted in figure 6.36. Regarding the net worth ratio, we see that very low-income agents have a ratio of zero, implying that they do not own any capital. As they, however, can also not borrow up they also do not hold debt. Hence, they neither have

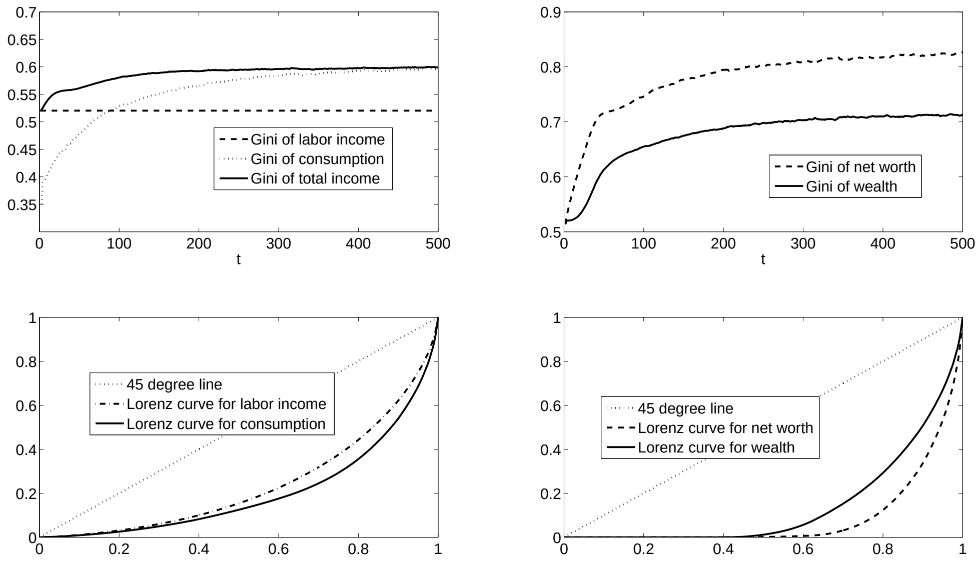


Figure 6.34.: Gini coefficient in time and Lorenz curve in last simulation period ($t = 500$) for labor income, consumption, wealth and net worth for the case with fire sales

to pay interest on debt nor receive capital income, implying that their capital share of income α_i is zero. In contrast to that, agents with a medium size labor income have a negative capital share, implying that they pay interest on debt. As the wealth is now distributed more unequally both the capital share and the net worth ratio for the individuals with the highest labor income also increase compared to the case without fire sales (cf. figure 6.6).

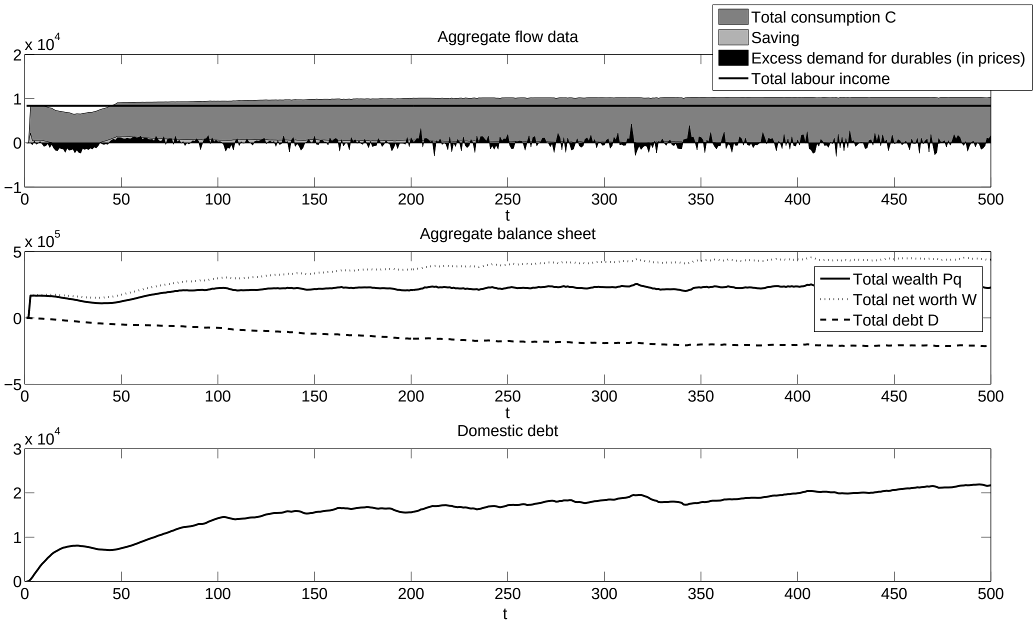


Figure 6.35.: Decomposition of GDP (aggregate flow) and aggregate stock for the case with fire sales

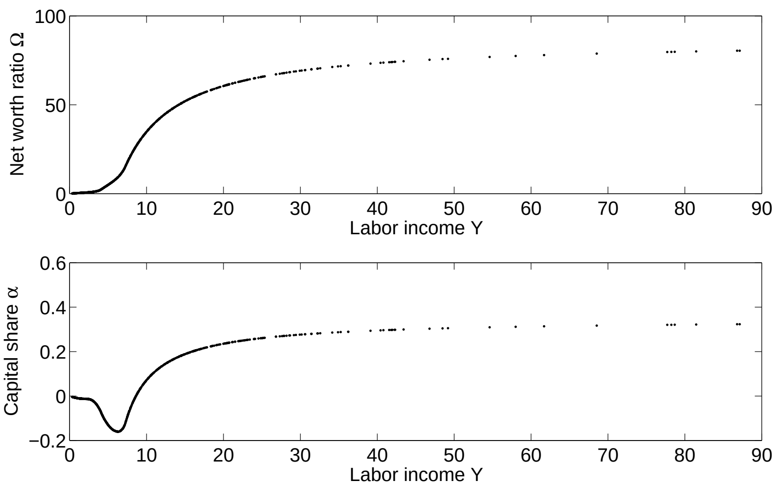


Figure 6.36.: Net worth to labor income ratio Ω and capital share of total income α as a function of labor income Y (time averages) for the case with fire sales

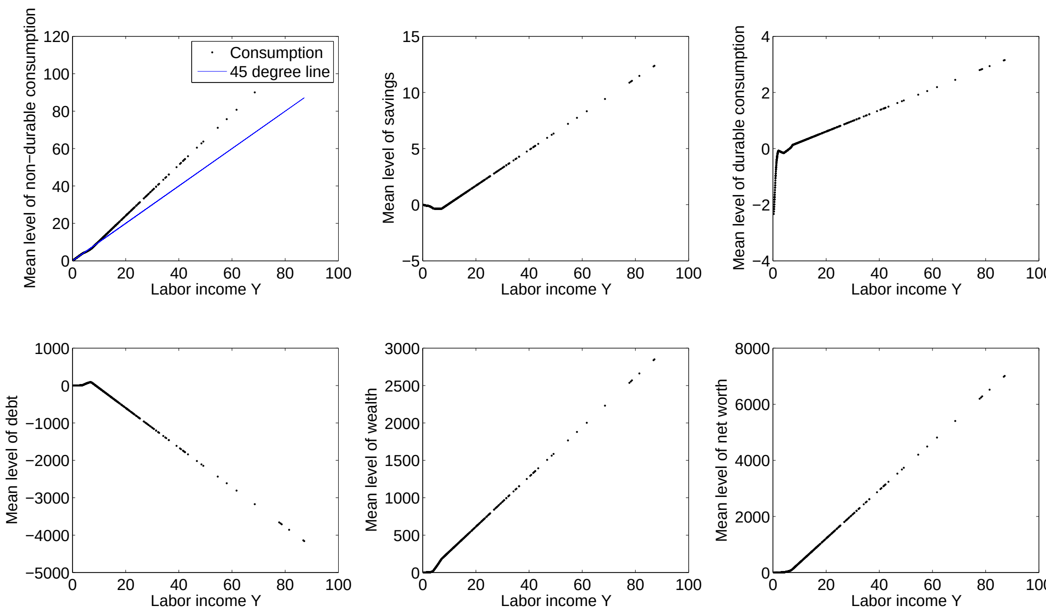


Figure 6.37.: Mean of non-durable consumption, savings, durable consumption, debt level, wealth and net worth as function of mean income for the case with fire sales (time averages)

It is important to discuss the income function as shown in figure 6.37. The very low-income individuals do not save, and hold neither wealth nor net worth nor debt. On the other side, the demand for durables shows a negative spike for the lowest income households due to their fire selling activities. A second negative spike once again emerges at a medium size income as also presented in the *Austerity*-case from figure 6.2 due to the fact that these households caused by their collateral constraint only tend to participate in bear markets.

For this case we refrain from presenting the variation of the parameters as executed in section 6.2 for the *Austerity*-case. Eventually, similar results as in the latter case emerge. However, the fire-sales promote higher fragility making the graphs less clear-cut. Small changes in the exact form of the income distribution as well as in the stochastic process of the noise trading alter the results of the durable market massively. Due to the collateral constraint (for the low income) as well as the portfolio composition (for the high income) this feeds back into the market for (domestic) debt. Besides that, the fire-selling pressure for a given income distribution leads to more lower-class individuals without wealth holdings. This feeds back into higher wealth inequality as well as consumption inequality.

So far we presented the model's results and showed how the model reacts to changes of the underlying parameters. As the parameters are exogenously determined, this analysis had the character of a robustness check. In the following section we slightly extend the model introducing parameters that can be subject to exogenous policy variations. In particular, we investigate policies aiming at redistributing flow and its aggregate effects.

7. Redistribution in the Model Economy

> I think the government solution to a problem is usually as bad as the problem and very often makes the problem worse.
>
> (Friedman, 1975, p. 6)

In the previous sections, we presented a theoretical rationale linking income inequality and subsequent net worth inequality creating the danger of financial instability. Since the latter is an undesirable result, redistribution could lead to a welfare improvement. This is a factor not widely discussed in the literature. However, there are several other factors that motivate redistribution while others speak against it. We review these arguments and counterarguments in section 7.1. We contrast a simple flat tax system with a minimum income with a *true* progressive system and show its aggregate and distributional consequences in the context of the model.

7.1. The Role of Redistribution as a Policy Goal

The want to redistribute economic resources arises from an initial discontent with the prevailing distribution. A normative question that emerges in such a context would be: what is the *best* distribution of economic resources? Normally, this question is considered as being beyond the usual scope of economics. However, the basic tools of economics already give some very far-reaching answers to this important question.

One can consider the case of a social planner trying to maximize the aggregate utility in an economy defined as the sum of individual utilities for homogeneous preferences (i.e. all agents have the same underlying utility function). For any standard utility function with decreasing marginal utility - also indicating risk aversion - total equality is the optimal distribution of income. This very strong result is easy to grasp if we consider the fact that - due to the decreasing marginal utility ($U'' < 0$) - each unit of resources the social planner takes away from a highly endowed individual and gives to an agent with lower amount of resources, increases the total welfare defined as the sum of individual utilities. In contrast to that, for an underlying convex utility function

($U'' > 0$) - implying risk-loving individuals - total inequality would eventually be socially desirable.[1] [2] [3]

On the other hand, in a standard case, any redistribution constitutes a Pareto inferior situation as at least one agent (the high income facing the taxes) is worse off than under a scenario without redistribution. More practically, any form of redistribution undermines the (legal) concept of property rights allowing each agent to keep the fruit of her own work. Another standard point put forward against (a complete) redistribution is that within the process of redistribution goods efficiency losses emerge - implying that not all that is taken away from wealthy individuals eventually reemerges for low-income individuals. This argument is often referred to as the *leaky bucket* argument (Okun, 1975). In the presence of a leaky bucket, the optimal distribution emerges as a trade-off between the *leakiness* and the risk aversion γ of the society. Note that low risk aversion is accompanied by a strong love of consumption and vice versa. Higher leakiness and lower risk aversion go along with a more unequal equilibrium distribution of resources. As already put forward in Arrow (1973) for the case of maximum risk aversion $\gamma \to \infty$, implying no marginal utility of increased consumption ($U' = 0$), total equality is the optimal distribution.[4] The latter is also the result of the so-called *maxi-min principle* of Rawls (1974) arguing that the welfare of the worst-off individual should be maximized. This concept can be understood under the *veil of ignorance*, describing a fiction in which agents can choose a society they want to live in before being born.

The key theoretical argument against redistribution is a loss of efficiency and a negative incentive argument. We will treat both more thoroughly and differentiated in section 7.2 in which we show how different concrete forms of redistribution distort individual decisions.

The presented argument so far was highly abstract. However, there are also more concrete arguments in favor of redistribution. As low-income individuals are assumed to have a higher marginal propensity to consume a redistribution to the bottom should result in higher aggregate consumption. This argument is frequently emphasized in the Post-Keynesian literature.[5] As put forward in Brunnermeier and Sannikov (2012), redistribution can also be welfare enhancing if a high amount of debt is already accumulated. In line with the classic *Fisherian debt-deflation*, the deleveraging of the low-income agents can have negative aggregate consequences. In this case, a redistribution can eventually be a Pareto improvement for all agents - including those high-income agents

[1] It is also interesting to point out the parallels to the work of Becker et al. (2005), who, following the work of Rosen (1997), introduce a status concern that leads to increasing marginal utility of consumption and, therefore, to risk-loving individuals that create an equilibrium demand for lotteries and lead to the fact that an unequal distribution can eventually be socially optimal.

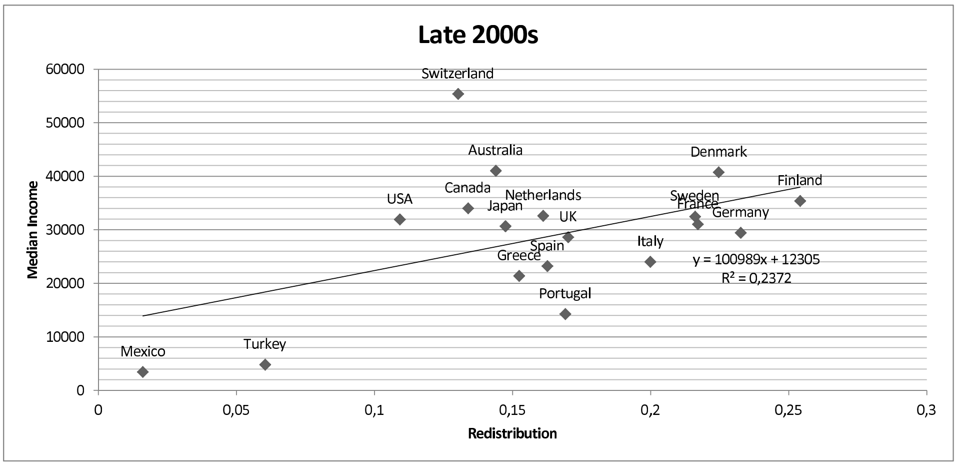
[2] For the special case of risk neutrality ($U'' = 0$), the social planner would be indifferent to any form of distribution. This is the case since redistribution does not change total welfare.

[3] The argument is also presented based on a more formalized reasoning in the appendix A.7.

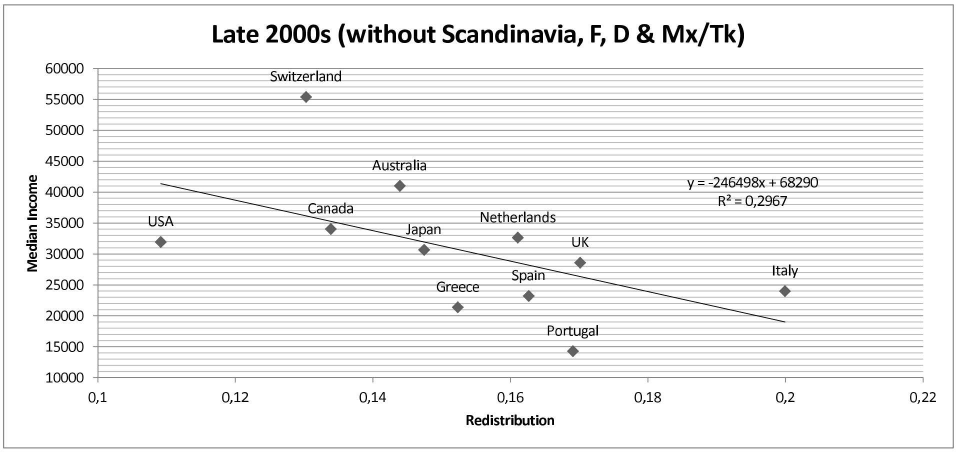
[4] The latter is presented based upon a formal argument in appendix A.7.

[5] For a detailed review of the arguments the reader is referred to Kurz and Salvadori (2010). The argument is also presented in section 4.3.

that lose in the short-run - since an unfavorable equilibrium is ruled out.[6] In our discussions of the simulations results (cf. section 7.2.4), we will also consider the aggregate consumption effect.



(a)



(b)

Figure 7.1.: Income level and income redistribution (Data source: OECD (2012))

In section 2.2, we already presented empirical evidence about the state of redistribution. In particular, in figure 2.4 we reported a trend of lower redistribution in Anglo-

[6]The major counterargument - not included in our analysis, but put forward in Brunnermeier and Sannikov (2012) - is the problem of time-inconsistency, in which ex-ante the policy maker has to commit not to redistribute in order to rule out moral hazard problems only to afterwards engage in redistribution.

Saxon countries (especially the US) and Scandinavia.[7] [8] Sometimes this is referred to as a paradigm shift of more deregulation and less redistribution (Piketty et al., 2014).

Using OECD data, Fitoussi and Saraceno (2010) relate the phenomenon of low redistribution to a decrease in maximum tax rate in Europe and the US as well as the decrease in labor employment protection as measured in the employment protection legislation (EPL) index.[9] The decrease in redistribution can be explained in a *beggar-thy-neighbor* setting in which countries compete for the mobile factors labor and (especially) capital by means of taxes leading to a race to the bottom in taxes. Therefore, increased inequality not only has exogenous causes but also is supported by a lower sovereign will to redistribute.

A simple linear regression on the presented countries suggests a positive relation between median income and redistribution (see figure 7.1(a)). The rationale for this is, that, if the overall *size of the pie* is larger, a society is more generous in sharing it. In the theoretical literature it is even argued that redistribution is a luxury good (Bénabou, 1996). However, this result is severely dependent on the developing countries in the sample (i.e. Turkey and Mexico) and the European countries with a strong favor for redistribution (Scandinavia, Germany and France). If these countries are removed from the sample, a negative correlation emerges predicting lower redistribution for high-income countries (see figure 7.1(b)).

Our simple univariate regression does not consider other underlying causes and especially does not rule out reverse causality issues. In a recent paper by the International Monetary Fund (IMF), Ostry et al. (2014) - using recent data for OECD countries - dwell thoroughly on the old growth vs. equality controversy emphasizing the transmittance effect between the two by redistribution. One can argue that higher inequality leads to higher redistribution thereby decreasing growth by decreasing incentives to work[10] and imposing an inefficient government system (Okun, 1975). In this case, high inequality leads to high redistribution resulting in low growth. On the other side - as frequently emphasized in the literature of growth (e.g. Galor and Zeira (1993)) - redistribution can promote growth by supporting the accumulation of human capital. Decomposing the data, Ostry et al. (2014) show that while in a direct effect inequality decreases growth,

[7] The latter, however, are still at a far higher level of redistribution than the Anglo-Saxon countries by means of our proxy.

[8] As emphasized by Bénabou (1996), redistribution does not have to impact on the difference between pre and post transfer Gini. Other forms of *redistributive* measures include minimum wages, land reform, education subsidies to build up human capital, trade protection (protecting low-income workers from foreign competition), and (very important in our context) stronger regulation of financial markets (Bénabou, 1996).

[9] The employment protection in Anglo-Saxon countries USA, UK, and Ireland as measured by the EPL eventually leveled or even increased. However, the overall level is far below the level of European countries. Once again, this suggests that increased inequality in Europe is more a problem of the lower living conditions for the poor while in the Anglo-Saxon countries it is driven by an increased share of the top income class.

[10] The problem is sometimes also referred to as the *Samaritan's dilemma* (Kumar, 2014), in which the presence of redistribution leads to moral hazard problem resulting in the problem that agents have little incentive to provide effort in the first place and to accumulate (human) capital.

the resulting redistribution (partly) alleviates these negative effects making stronger redistribution eventually favorable for growth. However, they also argue that there are limits to redistribution, implying that for an already high degree of redistribution further redistribution might be harmful to growth. Based upon their data analysis they identify (amongst others) France, Germany, and the Netherlands as being part of the latter group.

The dominating literature in political economy predicts a positive relation between inequality and redistribution (Bénabou, 1996). As the median voter becomes poorer in unequal societies, governments opt for more redistribution. However, this is only the case under populist (left-wing) regimes. In fact, the reverse could emerge. In elitist or *wealth biased* (right-wing) regimes the rich have more power, opting for lower redistribution (Bénabou, 1996). This can take the form of lobbying, lower democratic participation by low-income individuals, or even the buying of votes. The elitist case is popularized by Piketty (2014), presenting a dystopia of a society in which economic success and political power go hand in hand. The rent seeking of the rich (Tullock, 2008) is especially strong if property rights are not well enforced (Bénabou, 1996). Eventually, the *wing* of a political party is identical to the side of the income distribution (left or right of the median) they aim to politically represent. On the other hand, even the rich have an incentive to redistribute to the poor in order to avoid political turmoils.[11]

While the latter results from a rather misanthropic *Weltanschauung*, we can also take a more positive perspective arguing that individuals are altruistic. In this case, transferring goods not only provides utility to the recipient but also to the donor leading to a Pareto improvement (Boadway and Keen, 2000). However, rather than installing a forced system of redistribution by means of taxation, a subsidy of gifts and charity should be advisable based upon this underlying concept (Boadway and Keen, 2000).

As already presented in section 2.1, a large number of theoretical models (e.g. Galor and Zeira (1993) or Banerjee and Newman (1993)) presenting persisting inequality also accompanied by negative effects on aggregate growth rely on the underlying notion of imperfect credit markets. In these models, private insurance markets (partly) do not exist due to imperfect information resulting in moral hazard and adverse selection. Publicly provided mandatory insurance markets can (partly) compensate for that. More extremely, the models in the *Bewley*-tradition (cf. section 3.3), even require the presence of uninsurable idiosyncratic risk - being an imperfection of the financial market to generate inequality in the first place. Following from that one could argue that financial liberalization reducing restrictions on financial markets can eventually be favorable. In fact, the 1980s saw a paradigm change in which the public redistribution was cut back and replaced by financial liberalization (Rajan, 2010).

To some extent credit is a private and voluntary form of temporary redistribution from high-income individuals to low-income individuals with consumption wishes that exceed their current income. While tax payers do not have any entitlement to a service in return,

[11]The latter argument is sometimes labeled as *Disraelian Conservatism* named after the strategy of British politician Benjamin Disraeli (Boadway and Keen, 2000).

a credit is an explicit contract that requires a repayment with interest.[12] Thereby, and as extensively presented in the previous sections, this short-term solution, however, may lead to negative long-term effects. Or to put it more metaphorically, the credit acts like a painkiller treating the symptoms, being high consumption inequality due to income inequality. In the short-run, the presence of credit reduces consumption inequality, however, leading to increased net worth inequality that might lead to financial instability. In essence, this *painkiller* erases the pain in the short-run and makes the patient fell well-off and overstretch her resources (being the budget constraint). The underlying disease (income inequality) still remains the same. In contrast to this, redistribution can try to envision this underlying problem directly. While credit is a form of a private solution, redistribution by means of taxation can be considered the government solution. However, the aggregate result, crucially depends on the concrete design of the tax system, which will be subject of the subsequent section.

7.2. Taxation in Unequal Societies

In this section, we discuss the effect of the tax system as a form of redistribution. The taxation model and its implications are presented in the following section while its effects on the dynamics are discussed based upon simulation results in section 7.2.4. Before we do so, however, we review the very wide literature on taxation.

7.2.1. Literature Review

The first question put forward by the literature on taxation is whether taxation is needed in the first place. If a perfect efficient market exists with the *First Welfare Theorem* holding - stating that every market outcome is Pareto efficient - the presence of taxation would only lead to distortions away from efficient outcome. Hence, to rationalize taxation, some form of market failure is necessary. In particular, financial markets are considered to be inefficient by not allowing to (fully) insure against idiosyncratic risks due to factors such as credit constraints. Thereby, the tax system and especially the social insurance system provides an insurance against income risks resulting from adverse shocks out of health, disability, and longevity not available on the private market. A prediction resulting from this theoretical thought is that countries with less evolved financial markets in which individuals exhibit a strong risk aversion, should provide a larger social insurance system. This can be thought of as a rationale why Anglo-Saxon countries (in particular the USA) have less evolved public social security systems.

According to the *Second Welfare Theorem*, the questions of efficiency and equity can be separated. In particular, the role of the government boils down to influencing the initial endowment of agents that is socially acceptable. Starting with this endowment a Pareto efficient outcome will be reached in the market economy. The endowment in this case is the ability of the agent. As argued in Piketty and Saez (2013b) the labor

[12] Note that a similar argument was put forward in section 2.2 discussing the issue of *Ricardian equivalence*.

income is driven by four components. It can result from (1) abilities given at birth, (2) acquired skills actively accumulated in life-time, (3) heterogeneous tastes, or (4) pure luck. Whereas the first factor and the last factors are beyond individual control, the other factors are endogenous to each agent and given by active decisions (2) or preferences (3). The idea is that the tax system should not only - and as already put forward - provide insurance against idiosyncratic shocks of bad luck (4), but also against bad starting conditions (1). An example for the latter would be being born with a physical or mental disability. Thus, the latter argument is very close to the rationale of the *Rawlsian veil of ignorance*. Furthermore, the government should provide an environment in which one can actively build up human capital (2). More concretely, this requires for a subsidy of education. Education is usually considered a public good. Any form of public goods - such as national defense, infrastructure, or police - are also a standard rationale for taxation. On the other hand, the economic literature emphasizes that heterogeneous tastes (3) should not be accounted for by the tax system. A liberal society (in contrast to a paternalistic) should tolerate individual tastes - in particular a high taste for leisure as opposed to work - however not actively subsidy them. The key problem of the tax system is that it can only imprecisely monitor whether a particular outcome is result of tastes (3) or abilities (1). If heterogeneous earnings are only the result of heterogeneous tastes, a progressive income tax system would not be desirable.[13]

The transfer to private households can occur in various forms. The already mentioned form of public goods can be considered a demogrant (Piketty and Saez, 2013b) defined as as an access to a good regardless of the individual income situation. In fact, and as will be more precisely discussed in section 7.2.2, the presence of a demogrant constitutes a progressive tax system. Another question that emerges is whether the transfers should be conducted in terms of money or as in-kind transfers (i.e. in the forms of goods) (Piketty and Saez, 2013b). A rationale for the latter would be that individuals have a tendency to invest the money in *demerit goods* such as alcohol or cigarettes. The general literature, however, favors transfers in terms of money from a liberal or anti-paternalistic motive.

While the concrete design of transfers will not be subject of this work[14], we discuss the design of the tax system. Taxation can be classified in broad categories. We present these categories in table 7.1 and give concrete values and specifications for the case of the German tax system.[15] Firstly, income can be taxed. In this case, we can further decompose functionally into labor and capital income. The government, however, can also tax the use of income in the form of consumption. While the mentioned categories

[13] We present this in a more formal argument in section A.8.

[14] Note, however, that the transfers are granted in form of money transfers rather than in-kind allowing individuals to independently decide its use.

[15] The data originates from the German Treasury (Bundesfinanzministerium, 2014) . This summary does not include taxes imposed on firms and thereby indirectly on private households as it is unclear whether they will be transfered to consumers by means of higher prices or to workers or rentiers by means of lower wages or rents. In particular, our data does not include the *Gewerbesteuerumlage* and the *Köperschaftssteuer* imposed on firms. We, however, capture 94% of total tax.

tax flow variables, government can also impose a tax on stock (i.e. on accumulated capital). In the following, we discuss each category separately.

We also compute values of average tax rates defined as the ratio of taxes T_i to tax basis for different taxable factors x_i ($\tau_i = \frac{T_i}{x_i}$). All taxable factors can be related to the national income Y by means of a specific multiplier M_i making the average tax rate $\tau_i = \frac{T_i}{M_i Y}$. Labor income is related to national income by means of the labor share $1-\alpha$, whereas consumption is related by means of the consumption ratio $c_y = 1 - s$.[16] The value of capital to national income ($\kappa = \frac{k}{y}$) originates from the study of Piketty and Zucman (2014).

The highest average tax rate is imposed on consumption. This rate, however, is smaller than the standard VAT of 19%, implying that the effect of lower taxes on special goods of 7% outweighs the effect of higher taxes on demerit goods such as tobacco.[17] We also find that the average tax on capital income is (slightly) higher than the average tax on labor income. Yet, the average tax rate on capital income is way below the flat tax level (*Abgeltungssteuer*) of 25%, indicating the presence of strong tax exemptions.[18] The taxation of stock is very modest in Germany.[19] The most important categories are land purchase tax (*Grunderwerbssteuer*) and inheritance tax (*Erbschaftssteuer*), which, however, only occur at time of transaction. Our back of the envelope computation assumes that total capital is transferred annually and thereby underestimates the applied tax computed to be at a very low level of 0.15%. All in all, it is also worth noting that the aggregate average tax rate $\tau = \frac{T}{Y}$ is higher than all individual levels $\tau_i = \frac{T_i}{M_i Y}$ as the total national income is taxed at various times being at its formation as flow income, its employment by means of consumption, as well as its storage as stock of capital.

Taxation of Consumption

In our survey of the taxation forms, we start with the tax on consumption, since in Germany it is the category with the highest revenue. Most of the income stems from the *Value Added Tax* (VAT). From a distributive point of view, a VAT, however, is undesirable since in a scenario with a subsistence consumption motive it is regressive - implying that is mostly imposed on low-income individuals. Consider a Keynesian consumption function with a subsistence level $\bar{c}$ and a marginal propensity to consume out of income $0 < c_y < 1$. The tax rate τ_C is imposed on all goods identically:

$$T_C = \tau_C \cdot C = \tau_C(\bar{c} + c_y Y). \tag{7.1}$$

[16] These values originate from the Federal Statistical Office (*Statistisches Bundesamt*) Statistisches-Bundesamt (2014).

[17] Moreover, the result is also highly sensitive to the savings ratio.

[18] The average tax on a taxable source x_i in a system with a flat marginal tax τ_i and an exemption level $\bar{x}_i$ is: $\frac{T_i}{x_i} = \tau_i \left(1 - \frac{\bar{x}_i}{x_i}\right)$. For the concrete case of taxes on capital income this implies a level of $\frac{\bar{x}_i}{x_i} \approx 44\%$.

[19] In fact, there are negative flows from estate taxes.

		Major Categories	[1,000 Euro]	Share	Multiplier		Average Tax τ_i
Tax Revenue	Consumption	VAT, diverse taxes on specific goods (tobacco, alcohol, coffee,...)	299,991,162	55.61%	1-s	0.90	15.68%
	Wealth	Land purchase tax, inheritance tax,...	3,026,655	2.41%	κ	4.00	0.15%
	Capital Income	Abgeltungssteuer	50,943,903	9.44%	α	0.33	14.14%
	Labor Income	Wage tax	175,457,046	32.53%	$1-\alpha$	0.67	12.39%
	Sum		539,418,766	100.00%			
	National Income	Labor, capital income	2,118,790,000				25.46%

Table 7.1.: Taxes in Germany 2013 (Data source: Bundesfinanzministerium (2014))

The average tax rate ($AT_C = \frac{T_C}{Y}$) increases with income due to the existence of the subsistence level ($\bar{c} \neq 0$):

$$\frac{T_C}{Y} = \frac{\tau_C \bar{c}}{Y} + c_y \tau_c \leftrightarrow \frac{d(T_C/Y)}{dY} = -\frac{\tau_C \bar{c}}{Y^2} < 0, \tag{7.2}$$

implying a regressive system. The problem could eventually be solved by allowing a tax free level of consumption equal to the subsistence level of consumption. Besides the problem of concrete implementation[20], there would be issue of the determination of the level of $\bar{c}$. As intensively discussed in section 4.3, the subsistence consumption is hard to disentangle from relative consumption concerns leading to a potential arm's race situation.

Note that up to this point - and in line with a standard VAT - we assumed a uniform tax rate. Another solution to the problem would be imposing heterogeneous tax rates on heterogeneous goods. This can take two forms: luxury goods could be subject to higher tax rates. The other option would be to impose a lower tax on inferior goods in the microeconomic sense. In Germany a lower VAT applies to goods such as groceries (in stores and not in restaurants) or public transport. This differentiated tax, however, is dismissed by economic theory, arguing that the government should not distort the vector of market prices (e.g. Atkinson and Stiglitz (1976)). Nevertheless, heterogeneous taxation of different goods is common practice in Germany. Beside the general VAT, there are specific taxes on goods considered demerit (such as alcoholic beverages, tobacco, coffee, lotteries and cars due to the pollution aspect). In this case, however, once again the anti-paternalistic rationale applies as a counterargument.

Taxation of Labor Income

The second most important category of tax revenues in Germany is taxes on labor income. Moreover, it is also the most controversially debated component in economic theory. Atkinson and Stiglitz (1976) in particular argue that labor income should be the only source of income subject to taxation.[21] The key idea derives from the Second Welfare Theorem. Labor income is a proxy for abilities. The tax system should impact on the distribution of labor income to generate equal opportunities. All other market prices will adjust, accordingly creating a Pareto efficient outcome.

The seminal model discussing taxation of labor income is Mirrlees (1971). As already presented in section 7.1 from both a utilitarian and Rawlsian perspective equally distributed income would be socially optimal. In a flat tax regime with a minimum income (cf. section 7.2.2), this would imply a flat tax rate of $\tau_L = 100\%$. The other extreme result would emerge for a representative agent model: any form of taxation distorts the optimal market outcome and thereby reduces the utility of the (representative) agent.[22]

[20] A feasible solution would be a sort of *food stamps* that allow individuals to buy a certain amount of goods tax-free.

[21] The model result of Atkinson and Stiglitz (1976), however, relies on specific assumptions. In fact, agents have homothetic preferences, ruling out the case of luxury goods. Moreover, leisure choice l is assumed to be separable from consumption choice c ($\frac{\partial^2 U}{\partial l \partial c} = 0$).

[22] The latter is presented based upon a formal argument in the appendix A.8.

In a heterogeneous agent model, the central problem of a tax on labor is that it has adverse effect on the supply of labor. The key contribution of Mirrlees (1971) is trading off the equity concern against the labor supply - being the efficiency concern - in order to design an optimal tax system.

Note that in our model we do not model labor supply decisions by assuming a constant labor which thereby is totally inelastic. This is a major caveat of our analysis. Thus, theoretically the $\tau_L = 100\%$ result would be socially optimal. In our simulations in section 7.2.4, we, however, put forward another argument against this extreme tax. One might think of as our model as a very long-run model. As emphasized in Piketty and Saez (2012), the long-run labor supply is very inelastic. Despite a large increase in overall income in the last 100 years labor supply only reacted very modestly.[23]

However, we do not want to fully ignore the important issue of labor supply. As it, however, is not central to our argument we chose to treat it separately and based on formal arguments in appendix A.8. The key insights, however, are the following: taxes on labor lead to a substitution effect of labor in favor of leisure. This can be in the form of an extensive margin (a continuous variable measuring the concrete level of labor supply) or in an intensive margin (a binary variable determining whether supply labor at the market is provided at all). The latter effect is of particular importance for low-income individuals deciding on whether to supply labor in the first place. This issue is frequently addressed by special subsidies for low-income workers rather than non-workers (Piketty and Saez, 2013b). The presence of the substitution effect also limits the optimal level of taxation even when the single target is maximization of government revenue (the well-known *Laffer-curve* relation). Meanwhile, a very fat tailed labor income distribution calls for high marginal tax rates for very high-income individuals (Diamond and Saez, 2011). A progressive tax system - more severely discouraging labor supply of high-income agents - leads to a compression of inequality even if government does not redistribute tax revenues to low-income agents due to a pure labor supply effect. It is therefore preferable to a flat tax system that only leads to a labor supply compression as a result of heterogeneous tastes.[24] On the other hand, the progressive income system can have adverse long-run effects on labor supply by disencouraging the aggregation of human capital (Heathcote et al., 2014). The majority of agents refrain from accumulating skills in the form of education, generating a huge wage premium for high skill agents in line with the predictions of the *skill-biased technological change* (Acemoglu and Autor, 2012).

Taxation of Capital Income

A very controversial subject is the taxation of capital income. In fact, the established literature argues for a zero tax on capital income ($\tau_{rK} = 0$). Following the argument laid out in section 4.1 savings are only made in order to provide for future consumption.

[23] This is also subject of a collection discussing the predictions of Keynes (1930) regarding growth and labor supply (Pecchi and Piga, 2008). While Keynes (1930) made a rather good prediction of economic growth, he wrongly predicted a large decrease in labor supply. One key rationale brought forward is the existence of relative consumption effects emphasized in this work.

[24] However, the progressive system also impacts on the taste parameter.

In the rationale of Atkinson and Stiglitz (1976), a tax system that taxed capital income would thereby discriminate between future and current consumption in favor of current consumption. As already presented the section on taxation of consumption, a heterogeneous tax rate on different forms of consumption is not desirable. Moreover, Atkinson and Stiglitz (1976) emphasize that labor income is the ultimate source of income. By also taxing capital income, labor income would be taxed multiple times: once when it is earned (in the form of a tax on labor) and various times when the saved labor income in the form of accumulated capital generates capital income. This multiple treatment is considered unnecessary.

Comparable to the taxation of labor income, the tax on capital income distorts a particular economic decision. For the case of labor this is the decision about labor supply, in the case for tax income it is the savings/consumption decision as usually treated in the Euler-equation (cf. section 4.1).

Assume a tax rate[25] on capital income is imposed ($T_{rK} = \tau_{rK} \cdot r \cdot K$). The welfare costs of tax distortion are given by:[26]

$$\frac{dC(t)/C(t)}{d\tau_{rK}/\tau_{rK}} = -\tau_{rK}\frac{rt}{\gamma} + \frac{\tau_{rK}}{1-\tau_{rK}} + \frac{r(1-\gamma)\tau_{rK}}{\rho - r(1-\tau_{rK})(1-\gamma)}, \tag{7.3}$$

or for the special case of log-utility ($\gamma = 1$):

$$\frac{dC(t)/C(t)}{d\tau_{rK}/\tau_{rK}} = -\tau_{rK}rt + \frac{\tau_{rK}}{1-\tau_{rK}} \tag{7.4}$$

This term gives the percentage change of consumption at a certain time t as a result of a percentage change of the tax rate. First of all - in the very long-run - taxes lead to an infinite loss of consumption ($\lim_{t\to\infty} \frac{dC(t)/C(t)}{d\tau_{rK}/\tau_{rK}} = -\infty$). Furthermore, lower levels of risk aversion γ [27] and higher levels of interest rate r [28] lead to the fact that the taxation of capital income is inefficient. Higher risk aversion is accompanied by a lower Intertemporal Rate of Substitution (IES) implying that individuals exhibit lower elasticity to interest rate changes. As put forward by Guvenen (2006), the IES is lower for individuals with low levels of wealth making them suffer little from taxation. Or put

[25] Note that we label τ_{rK} somewhat imprecisely as the tax rate. For a simple flat rate regime without exemptions this rate equals both the marginal and average tax rate. If we, however, as done in the literature, assume that individual agents have heterogeneous tax rates $\tau_{rK,i}$ depending on their underlying features, they also have heterogeneous (marginal respectively average) tax rates. The latter in particular depends on the underlying tax basis $r_i K_i$.

[26] The result can be derived assuming a concrete consumption function in equation 4.52 in section 4.2 amounting to $C(t) = \frac{\rho - r(1-\tau_{rK})(1-\gamma)}{\gamma}\left(\frac{Y}{r(1-\tau_{Kr})}\right)\exp\left(\frac{r(1-\tau_{rK})-\rho}{\gamma}t\right)$ for the case with capital income without initial endowment ($D_0 = -K_0 = 0$).

[27] This is straightforward for the first term. On the other hand there is a short-run gain due to a substitution effect as captured in the third term for which consumption is preponed for lower effective interest rates. Formally, the partial derivative of this term - given by $\frac{-\rho r \tau_{rK}}{[\rho - r(1-\tau_{rK})(1-\gamma)]^2} < 0$ is always negative.

[28] The first term is obvious and unambiguous. The partial derivative of the third term is given by $\frac{(1-\gamma)\tau\rho}{[\rho - r(1-\tau_{rK})(1-\gamma)]^2}$ which is positive for low levels of risk aversion $0 < \gamma < 1$.

differently, low-income agents with a lower IES (higher γ) can be imposed a higher tax rate τ_{rK}. Hence, this simple idea hints at higher taxes of low-income agents effectively calling for a regressive tax. The result is also straightforward from intuition because high-income agents utilize capital markets in a more intense way to smooth consumption they would experience stronger welfare losses if these markets were subject to an inefficiency resulting from a tax on capital income. Basically, this is the rationale put forward by Chamley (1986) arguing that in the case of infinitely living agents, taxation of capital income leads to a infinite consumption loss in the very long-run. Atkeson et al. (1999) furthermore show that this result also holds in when certain strong assumptions of the original work of Chamley (1986) are relaxed. The result in particular holds in any form of infinitely living agent model. They also show that under certain assumptions the result is also sustained in an OLG-model in which agents have a finite life span.

On the other hand, analyzing equation 7.3 in the short-run yields other valuable insights. In particular there is a time t^* below which the tax actually allows for increasing consumption:

$$t < t^* = \frac{\gamma}{r}\frac{1}{1-\tau_{rK}} + \frac{(1-\gamma)\gamma}{\rho - r(1-\tau_{rK})(1-\gamma)}, \tag{7.5}$$

or for the log-case ($\gamma = 1$):

$$t < t^* = \frac{1}{r(1-\tau_{rK})}. \tag{7.6}$$

The positive short-run effect of the tax on consumption stems from the second term and third term in equation 7.3. The second term captures the effect the tax has on the long-run human capital $\frac{Y}{r(1-\tau_{rK})}$. Eventually an increase of the tax rate lowers the effective discount rate increasing the human capital. In the short-run, this permits for higher borrowing activities and higher consumption. The third term captures a short-run gain for low risk aversion ($0 < \gamma < 1$).[29] This effect comes from the prevailing substitution effect leading to a higher level of consumption in the short-run for a lower level of real interest rate. It is only in the long-run that higher taxes, by lowering the effective interest rate, hinder savings for future consumption and thereby decrease it.

If we further analyze the condition 7.6 for the log-case, it becomes evident that the period t^*, in which taxes eventually increase consumption, itself increases with the tax rate τ_{rK} and decreases with the interest rate r. Low values of r furthermore emphasize the positive effect of taxes as a tax does not distort capital income significantly since the rate of interest is already at a low level.

Another seminal work in this field is Judd (1985) also arguing for a zero long-run tax in an environment with infinitely living agents. He models an economy with capitalists and workers. The key idea is that workers would prefer a tax on capital income as it is imposed on the other group of agents. Nevertheless, as Judd (1985) shows, the tax on capital income depresses investment and output leading to adverse aggregate effects for all agents including workers. In the original paper of Judd (1985), a one time tax rate

[29] Note that formally the condition $\rho > r(1-\tau_{rK})(1-\gamma)$ also has to hold which, however, is always the case for the realistic case of $\rho > r$ prevailing in *Bewley*-type models. It does also hold for the standard assumption $r = \rho$ with $\gamma < 1$.

equal to the maximum $\tau_{rK}(t=0) = 100\%$ [30] and a long-run zero tax rate $\tau_{rK}(t) = 0$ for all $t > 0$ are optimal.

The famous result of a zero tax on capital income - well summarized in the work of Atkeson et al. (1999) - however is challenged by several theoretical papers. Their arguments are, however, frequently fundamentally different and at times even rely on diametrical reasons.

The first major challenge to the zero tax result on capital income was put forward by Aiyagari (1995). He argues in a *Bewley*-type model (cf. section 3.3) in which dynamic inefficiency can be an equilibrium outcome. In this case, due to the presence of uninsurable idiosyncratic risks, the equilibrium rate of interest is below the rate of time preference $r < \rho$ and the economy overaccumulates capital above the golden-rule level. The tax on capital income counteracts this inefficient overaccumulation of capital. On the other hand - as argued by Atkeson et al. (1999) - the problem of market inefficiency (in the market for insurance) is counteracted by implementing another market inefficiency (a tax). A more liberal argument would be to reduce the original inefficiency. This rationale was frequently put forward in favor of deregulating financial markets.

Lansing (1999) discusses a special case of Judd (1985) in which the utility function is of the log-specification (a CRRA utility function with $\gamma = 1$) showing that the result of a zero optimal capital income tax ceases to hold. The log-specification has the effect that income and substitution effect perfectly cancel each other out (also cp. section 4.1), implying that future interest rate changes have no effect on the optimal consumption path of agents being completely determined by the rate of time preference ρ and given by $c = \rho k$. Lansing (1999), moreover, shows that the optimal level of taxation increases with the welfare weight on workers assigned by the social planner, the level of inequality (proxied by the capital share in total income α as an index of the functional distribution), and increases with the time preference rate ρ of capitalists. As he furthermore shows the direct link between the level of capital k and consumption c is broken once government bonds as well consumption taxes on capitalists are introduced implying that in the latter case the zero optimal tax result would be preserved. Or put differently, the welfare enhancing effects of tax can be achieved by other measures such as government debt or consumption taxes.

A very recent critique by Straub and Werning (2014) closely follows the original argumentation of Chamley (1986) respectively Judd (1985) and further generalizes the rationale of Lansing (1999). In particular, they show that the optimal tax rate is not zero in the model of Judd (1985)[31] for the case of an IES smaller than one (respectively a high risk aversion $\gamma > 1$). In this case, the tax rate increase in time to a positive level, as the income effect prevails and a decrease in the effective rate of interest increases savings of capitalists. The optimal level of taxation negatively depends on the level of government consumption indicating a trade-off. For the inverse case with $0 < \gamma < 1$ there is a short-run positive taxation of capital yet in the long-run it is zero. Concerning

[30] The condition $r \geq 0$ equivalent to $\tau_{rK} < 1$ must be satisfied in order to provide capitalists to rent out capital in the first place.

[31] It is important to keep in mind that in the framework of Judd (1985) workers consume in a hand-to-mouth manner leading to the fact that the tax on capital income only applies to capitalists.

the model of Chamley (1986), Straub and Werning (2014) show that the zero tax result implies the absurd result of a zero level of wealth - implying a zero tax base - or a labor tax of zero, already implying the existence of a first-best solution.

Saez (2013) also takes the rationale for a zero capital income tax in an infinitely living agent setup seriously. He, however, argues that a positive tax on capital income can be desirable if it is only imposed for a finite time. In particular, the agents are exempt from the tax once they undercut a certain threshold of capital. Thereby, in the long-run all agents fall below this level and no agents are eventually taxed. The rationale for the tax in his model is an equity concern. The proposed tax - eventually being progressive due to the exemption level - truncates the wealth distribution. In the work of Saez (2013) this equity concern has to be traded-off against the hazard of overconsumption. The trade-off goes in favor of the tax if the condition $\frac{a}{\gamma} < 1$ holds, for which a represents the tail index of the wealth distribution.[32] In effect, the tax can be efficient for highly unequal wealth distributions (low values of a) and low IES (high values of γ) restating an argument put forward in the previous stanza.

A taxation of capital income does little else but reduce the effective rate of interest $r(1-\tau_{rK})$ and thereby is comparable to expansive monetary policy. As already presented in section 4.1 in a standard framework in which the utility function is of the CRRA-type the impact of an interest rate change on the savings propensity crucially depends on the level of γ capturing the risk aversion. The level of the latter has to be quantified by empirical studies. The case of a prevailing substitution effect (tacitly assumed in standard undergraduate textbooks) leads to a standard savings function with a positive slope. A survey of the empirical literature, however, also confirms this effect (Oeffner, 2008). Table 7.2 summarizes the effects and some empirical literature which are also of importance for the case of capital taxation.[33] Note that in our model the partial derivative is given by $-\frac{dC_i}{dr} = \frac{dS_i}{dr_i} = D_i$ [34] and thereby depends on the fact whether individuals are debtors or lenders. If they are debtors ($D_i > 0$), agents have a (standard) positive sloped savings function and vice versa.

Note that in the discussion of the literature we focused on work arguing from a social planner's perspective deriving a social optimum. In contrast to this, the political economic literature argues that taxation of capital income will always prevail. Scheuer and Wolitzky (2014) for instance show that in a direct democracy (i.e. one in which all laws are directly decided by voters by referendum) the tax rate on capital income is u-shaped presenting the lowest burden for middle-income agents at the expense of poor and rich agents confirming *Director's Law* (Stigler, 1970). In contrast to that, in representative

[32] The distribution in this case follows a density function of the type $f(Y) \sim Y^{-(a+1)}$.

[33] Note that this table only summarizes the standard approach detailed in section 4.1. For the *buffer stock theory* an increase in the rate of interest does not per se affect the saving propensity, but only leads to a larger equilibrium level of the buffer stock level of wealth (Carroll and Toche, 2009). If agents were to maintain a constant buffer level of wealth higher interest rates would require lower levels of savings implying a negative slope.

[34] Formally, this results from the disposable income effect, necessary to guarantee a stable level of debt as thoroughly discussed in section 4.2.

Coefficient of risk aversion (CRRA utility)	Definition	$0 < \gamma < 1$ (low)	$\gamma = 1$ (log-utility)	$\gamma >> 1$ (high)
	Effect on savings	$\frac{ds}{dr} > 0$ (substitution$>$ income effect)	$\frac{ds}{dr} > 0$ (substitution $=$ income effect, pure wealth effect)	$\frac{ds}{dr} < 0$ (income$>$ substitution effect)
Empirical evidence	Variation with education (Cagetti, 2003)	Low education		High education
Intertemporal Elasticity of Substitution	Definition ($\frac{1}{\gamma}$)	IES$>$1 (strong reaction to monetary shocks)	IES$=$1	IES$<$1 (little reaction to monetary shocks)
Empirical Evidence	Variation with wealth (and income) (Guvenen, 2006)	Wealthy individuals		Individuals poor in terms of wealth
	Cross-country variation (Havranek et al., 2013)	Developed financial markets		Low financial development
Effect for capital income taxation	Critique of Judd (1985) by Straub and Werning (2014)	$\tau_{rK,\infty} = 0$ $\frac{d\tau_{rK,t}}{dt} < 0$	$\tau_{rK,\infty} > 0$ $\frac{d\tau_{rK,t}}{dt} = 0$ (Lansing, 1999)	$\tau_{rK,\infty} > 0$; $\frac{d\tau_{rK,t}}{dt} > 0$

Table 7.2.: Empirical evidence for the shape of the utility function and theoretical implications

democracies in which politicians have the opportunity of conducting a *surprise* political reform the tax rate will increase with income.

While the previous arguments are highly theoretical, one can also put forward more hands-on arguments in favor of a tax on capital income. As emphasized by Piketty and Saez (2013b), not taxing capital income allows for massive tax avoidance opportunities. In particular, top management can reshift or (even only) relabel their income from labor to capital income being paid out in stock options. As shown in Piketty and Saez (2012) for the case in which both labor and capital income are virtually indistinguishable - which they label as *complete fuzziness* - both should be taxed at the very same rate to counteract tax avoidance.

One can counteract this very hands-on argument with another very practical argument. In contrast to labor that is very immobile - e.g. due to cultural barriers and family ties - capital is very mobile. In the presence of bank secrecy laws, the latter creates a strong tax competition between countries that can eventually even result in a *beggar-thy-neighbor* result in which there is a race to the bottom in the tax rates on capital income. Piketty and Saez (2012) also predict that smaller countries will have lower taxes on capital as their labor force is reuqired to be more mobile due to a narrow local labor market.

We can also compare this result to the result of our model in particular concerning the distribution of income. Consider a simple case in which both capital and labor income are subject to a simple flat tax without exemption ($\tau_y > 0$ and $\tau_{rK} > 0$). As shown in section 5.5.1, in our model capital and labor income are positively correlated. In fact the level of capital K_i for a particular labor income Y_i is given in the following form:

$$K_i = k_y Y_i + k_0, \tag{7.7}$$

with $k_y > 0$. We furthermore showed that under the realistic scenario in which human capital dominates inherited wealth, low-income households eventually are indebted ($k_0 = \frac{\bar{c}}{rs - c_w} < 0$). The total tax is given as follows:

$$T = \tau_y Y + \tau_{rK} rK = \tau_{rK} r k_0 + Y(\tau_y + \tau_{rK} r k_y), \tag{7.8}$$

in which the average tax changes with income in the following manner:

$$\frac{d(T/Y)}{dY} = -\frac{\tau_{rK} r k_0}{Y^2}. \tag{7.9}$$

The latter term is positive - indicating a progressive tax system - for the realistic case of $k_0 < 0$. Thereby, in our case a progressive tax system can simply be implemented by imposing a flat tax on capital income $\tau_{rK} > 0$.[35] The progressivity of this tax increases with level of the flat tax τ_{rK}. It is furthermore worth noticing that the system collapses to a neutral tax system (neither progressive nor regressive) if there is no subsistence level of consumption ($\bar{c} = 0 \rightarrow k_0 = 0$). Thereby, the condition that makes a VAT regressive, is the same condition that makes the tax on capital income progressive. In a scenario in

[35] Note that capital income is subject to a flat tax in Germany.

which inherited wealth dominates labor income - as emphasized by Piketty (2014) - we have $k_0 > 0$.[36] As a result, the tax on capital income would eventually contribute to a regressive tax system ($\frac{d(T/Y)}{dY} < 0$).

The literature labeled *New Dynamic Public Finance* presents arguments in the favor of a positive (yet moderate) tax on capital income. Their reasoning is, however, diametrical to the arguments already put forward. In particular, Golosov et al. (2003) argue that the presence of saving - as a form of insurance against adverse future labor shocks - imposes a moral hazard problem. Agents who posses a high amount of savings are discouraged of providing effort in future periods. Thus, they argue for a tax on capital income as it - in contrast to the tax on labor income - increases (future) labor supply. In a another paper, Golosov et al. (2013) - following the empirical results of Cagetti (2003) - assume that savings is a good preferred by individuals with higher ability. As shown by Cagetti (2003)[37], highly educated agents - being the ones with the highest abilities as well as the higher income - are more patient and thereby save more. As abilities - in contrast to tastes - should be taxed a tax on capital income is desirable. In their calibrated model, Golosov et al. (2013) show that the optimal tax rate is, however, u-shaped and at a very modest level. The high tax rate for low-income individuals is introduced in order to prevent low-income individuals from high savings - i.e. overconsuming the luxury good. Given the empirical evidence that the poor tend to undersave in any case (e.g. Fulford (2012)), this proposal, however, seems highly questionable. Golosov et al. (2013) estimate that for very high-income individuals the rate should only be as high as 4.5%. Once again, the tax rate is very sensitive to the risk aversion γ respectively the IES $1/\gamma$. A low level of risk aversion requires for low taxes.

Taxation of Wealth

As presented in table 7.1, the pure wealth taxes in Germany only represent a very small share of total tax revenues and the taxable base is only treated with a very low tax rate. On the other hand, one should keep in mind that in contrast to all other taxes considered so far this form of taxation is (directly) imposed on a stock quantity rather than a flow quantity. Given the multiplier relationship ($\frac{K}{Y} > 1$) the average tax rates are not directly comparable. As the empirical evidence - also confirmed for other developed countries (Kopczuk, 2013) - shows, the tax on wealth only plays a very minor practical role as there are some very strong objections against it. In a macroeconomic sense, a tax on wealth lowers capital accumulation and thereby the steady state of capital and output.[38]

First of all, the tax imposes - so to speak - an artificial depreciation on capital and therefore the wealth tax is often opposed by arguing that it presents a form of *expro-*

[36] For details of the derivation of the result the reader is referred to section 5.5.1.

[37] And as already put forward in section 4.1.

[38] In their calibrated model for the US economy, Castaneda et al. (2003) test the macroeconomic impact of an abolishment of the estate tax. Due to its very low level, it only increases wealth inequality marginally and has little impact on total output (increase <1%).

priation. In fact, it is very similar to the flow tax on capital income. The two forms of taxation can be related in the following manner:[39]

$$\tau_K = \tau_{rK}\frac{r}{1+r} \leftrightarrow \tau_{rK} = \frac{1+r}{r}\tau_K, \tag{7.10}$$

with r being the return on capital. Thus, to generate the same tax revenue for the government a lower tax on wealth compared to the tax on capital income is needed. In fact, the two forms of taxation can be distinguished by the fact that the wealth tax not only taxes the returns on capital, but also the underlying principal. This, however, is also a major problem with a uniform tax rate on wealth as it subsidizes agents with high returns in favor of agents with low returns. This is easy to show if we consider agents with different returns $r_h > r_l$ (high h and low l) on capital who otherwise, however, are completely identical. Assume the government decides to abolish the tax on capital income in favor for a pure tax on wealth. The design is made according to the previous equation assuming the low rate of interest r_l:

$$\tau_K \equiv \tau_{rK}\frac{r_l}{1+r_l}, \tag{7.11}$$

implying that agents receiving the low rate of interest are equally well-off under both regimes. We can, however, also compare the taxes of agents with high returns $r_h > r_l$ under both regimes:

$$\frac{T_{K,h}}{T_{rk,h}} = \frac{\tau_K(1+r_h)K}{\tau_{rK}r_hK} = \frac{1+r_h}{r_h}\frac{r_l}{1+r_l} < 1, \tag{7.12}$$

being smaller than one since:

$$\frac{1+r_h}{r_h} < \frac{1+r_l}{r_l} \leftrightarrow \frac{1}{r_h} < \frac{1}{r_l} \leftrightarrow r_h > r_l. \tag{7.13}$$

Thus, the tax on wealth deteriorate the position of agents who made a low return on capital. In particular, a wealth tax is even imposed if losses are made ($r < 0$). If this is the result of an idiosyncratic shock, the system would eventually go against its original motive of providing an insurance. Moreover, it is well-known that high net-worth individuals earn higher returns (Yitzhaki, 1987), thus the system eventually has an implied regressivity.

The tax on wealth can at times, however, be desirable to implement extreme policies. Sometimes it can be desirable to achieve a negative real rate of interest. If the standard monetary mechanisms ensuring theses results are hampered (e.g. due to a binding zero lower bound on nominal interest), this can eventually be achieved by the government by means of taxation. If only capital income is taxed, this, however, would require for a tax rate $\tau_{Kr} > 100\%$ which would never be politically feasible. The tax on wealth would only require:

$$\tau_K > \frac{r}{1+r}. \tag{7.14}$$

[39]This result is easy to grasp using the definitions: $T_{t,K} = K_{t-1}\tau_k(1+r) \stackrel{!}{=} T_{rK} = K_{t-1}r\tau_{rK}$.

In particular, if the topic nominal rate of interest r is already low this would be easy to attain. For example for a nominal interest rate of 1% this would only require a tax of $\tau_K > 0.99\%$.

Note that in equation 7.10 we implicitly assumed that the tax on wealth is imposed annually, e.g. in the form of permanent taxes on estate. In fact, the tax on wealth mostly emerges at certain events, in particular when wealth is transferred. The most common form of wealth taxation is the taxation of inheritance. To close the loop hole of escaping the tax by means of inter-vivo *gifts*, frequently large gifts are also subject to taxation. Moreover, the purchase of large assets, in particular real estate, is also taxed in which the tax base is given by the current market value of the transfered good.[40] Consider that the transactions only occur once in the length of a generation $G > 1$. To satisfy equivalence between both forms of taxes the following equation must hold for a constant return on capital r:

$$(1 + (1 - \tau_{rK})r)^G \stackrel{!}{=} (1 + r)^G(1 - \tau_K). \tag{7.15}$$

Using the exponential approximation[41] the following result can be derived:

$$\tau_{rK} = \frac{-\ln(1 - \tau_K)}{rG}. \tag{7.16}$$

This not only - once again - shows that for a given tax on capital income τ_{rK} the equivalent tax rate on wealth τ_K falls with the level of the interest rate r, but also that the same relation holds for longer holding periods G. The latter is straightforward as longer holding times imply that the number of taxable events is reduced. For the value of $\tau_{rK} = 14.14\%$ reported in table 7.1 and a long-run return on capital assumed to be 4.5% (Piketty, 2014) and a length of a generation $G = 30$ (Piketty and Saez, 2012), using the previous equation an equivalent tax on the stock of wealth as high as $\tau_K = 17\% > \tau_{rK}$ can be computed. The key argument for this high value is that the taxable events only happen very infrequently.

Once again returning to the seminal result of Atkinson and Stiglitz (1976) one can state that - if accumulated wealth results from labor income - it does not require for a specific tax. Wealth can be attained in two ways: (1) accumulating wealth in the form of savings out of labor income during life time or (2) inheriting wealth from parent generations. In our model, we take the strong notion that inheritance is perfectly correlated with future labor ($P_0 q_{i,0} = HY_i$). Thereby, following Atkinson and Stiglitz (1976), our model does not provide a rationale for taxing the stock quantity wealth. Piketty and Saez (2013a), however, argue that there are two distinct forms of capital acquisition, thus also two forms of taxes - (1) a tax on labor and (2) a tax on inheritance - are necessary. Their key rationale is that in a non-aristocratic society wealth should not depend on the ability of parents, arguing in a tradition of the *veil of ignorance* requiring similar starting conditions for all agents. If we, however, think that the ability of parents is

[40] Note that in some legislation there are *standard* values for real estate to circumvent the problems of unequal taxation in boom and bust cycles.

[41] This assumes that $(1 + r)^G \approx \exp(rG)$ since $G \ln(1 + r) \approx Gr$ for low values of r.

completely transfered by means of *biological* inheritance, there is also no need to tax *economic* inheritance. A tax on inheritance not only hits those about to inherit but also the parent generation and thereby discourages great inheritances. In their calibrated model, Piketty and Saez (2013a) compute values for the inheritance tax as high as high as 50-60%. Meanwhile, they argue that higher taxes on inheritance can be compensated by lowering the tax rate on labor income. Interestingly, the tax rate decreases for very high net-worth individuals to a negative level eventually presenting a subsidy. This is the case if the social planner puts a high weight on receivers of the bequests.

In this respect, the result of Piketty and Saez (2013a) is identical to the result in the literature of *New Dynamic Public Finance* by Farhi and Werning (2010). The latter argue that bequests are a positive externality to the offspring generation and thereby should be treated with a *Pigouvian subsidy* rather than a tax. Besides providing wealth to the children generation the parent generation also incurs a utility gain if we assume a *Warm Glow* motive. Moreover, it is important to point out that the altruistic *Joy of Giving* motive and the *Love of Wealth* motive[42] are virtually indistinguishable in a theoretical framework despite the fact that from an ethical perspective they should be treated very differently. Empirical evidence, showing that individuals without offspring also leave sizable bequests, points to the fact that bequeathing wealth to children is not the major driver of accumulating wealth (Kopczuk, 2013). Yet, another reason for providing bequests is that parents want to compensate children for providing grandchildren or nursing to their parents. Farhi and Werning (2010) thereby argue with fairness between generations and not within a generation of heterogeneous agents. One might therefore criticize them for supporting an *aristocratic* society (Piketty and Saez, 2013a). It is, however, important to point out that a positive tax on capital can eventually be optimal under some conditions in their model. Due to a general equilibrium effect, the tax on capital lowering the current value of the assets can increase the future return factually subsidizing capital. It is also interesting to compare this result to the seminal result of Stiglitz (1978), who argues in a similar vein. In his framework, there can be unintended consequences to estate taxes that eventually countervail the targeted goal of decreasing wealth inequality. The estate tax lowers capital accumulation and increases return on capital and thereby can finally lead to an increase in the capital share and an increase in wealth inequality.[43] If one follows the argument of Piketty and Saez (2013a) and Piketty (2014) fearing a return of patrimonial capitalism, a tax on inheritance and a subsidy on charity - i.e. on donations outside the family sphere - can lead to welfare enhancing results.

In the following, we summarize other major (more practical) pros and cons regarding a tax on the stock quantity of wealth. The estate tax is frequently refused. However, and in contrast to a simple tax on capital income, this tax is able to capture *unrealized* capital gains. A practical problem of a permanently installed estate tax constantly taxing the stock level of wealth, however, is finding the correct valuation of the assets.

[42] One might also label this a *Scrooge McDuck*-motive for whom the accumulation of wealth is the final goal.

[43] As already discussed in section 6.2.1, this effect prevails in a scenario with a CES production with a dominating price rather than a volume effect ($\rho < 1$).

If a market valuation is chosen, the tax is very sensitive to boom-bust-cycles. As an avoidance of the tax, agents could try to sell assets on the market once the taxation event occurs, leading to negative fire-sale externalities.

Mostly taxes on transfers of wealth, in particular on bequests, are discussed. Comparable to savings, bequests are a luxury good. A tax on inheritance thereby can lead to the desired progressive taxation. The economic literature usually differs between planned and accidental bequests caused by *sudden death*, arguing that the latter should be severely taxed (Kopczuk, 2013). In practice, however, it is difficult to clearly distinguish the two motives. In practice, bequest taxes are subject to a high tax exemption only posing a tax on very high net worth individuals in which the accumulated wealth does not primarily stem from precautionary savings (Carroll, 1998). The exemption level being fixed in nominal terms, however, is subject to a bracket creep. This is especially important as durable assets - as also emphasized in this work - are subject to a strong temporary form of inflation (asset price booms).

There are, however, more practical counterarguments against a tax on wealth. Bequests also play an important role in transferring family business. A tax on inheritance would virtually lead to the carving-up of family-owned firms (being of significant importance in Germany). Moreover, this tax thereby imposes an (implicit) subsidy on other legal forms in particular joint stock companies. The key concern of Piketty (2014) fearing a return of a wealth dominated society is - at least at this point of time - not accounted for by the data. A simple test - proposed by Kopczuk (2013) - is looking at the Forbes list of the wealthiest individuals. If wealth is mostly inherited, one would expect a an approximately equal number of male and female individuals on the list - in line with the distribution amongst the overall population. The list, however, is highly dominated by male individuals propping the hypothesis of self-made billionaires.[44]

Summary

The last sections presented major views about taxation of different sources of income. This section summarizes the key results and states a normative conclusion applied to our model in order to justify our subsequent approach. We focused on the theoretical literature in economics. While most models provide interesting and elegant toolboxes, some of the results are highly quixotic.

Taxes on consumption are problematic as they - in particular the VAT - are regressive. A progressive taxation of labor income is supported by a high number of economists, yet the concrete size is subject to a large debate. The major concern with the progressive taxation of labor income is its adverse effect on labor supply. There is a long-run tradition in the economic literature opposing any tax on wealth. However, in the more recent literature there is a slight consensus of imposing a tax on capital income not least due to the massive possibility of tax avoidance if one major income category remains untaxed. However, the concrete values span from very modest (Golosov et al., 2013) (max. 4,5%)

[44] This simple does however not account for gender differences in inheritance. In particular, the presence of primogeniture (inherting the total wealth to the first born male offspring) should be taken into account.

to strong progressivity (Saez, 2013) (up to 73%). Following the argument of Atkinson and Stiglitz (1976), in the framework we discuss a tax on wealth is never desirable as all inequality stems from labor inequality. For the tax on wealth, there exists no consensus in the academic literature. Yet, there are some very strong arguments against it - in particular its (implicit) regressivity and its adverse effects on family business.

7.2.2. A Tax System with a Flat Tax and a Minimum Income

In this section we present a highly stylized tax system that is aimed at redistributing income making it factually a combined system of taxation and transfers close to the negative income tax system of Friedman and Friedman (1962). In the subsequent section, we contrast this framework with a more realistic progressive tax system.

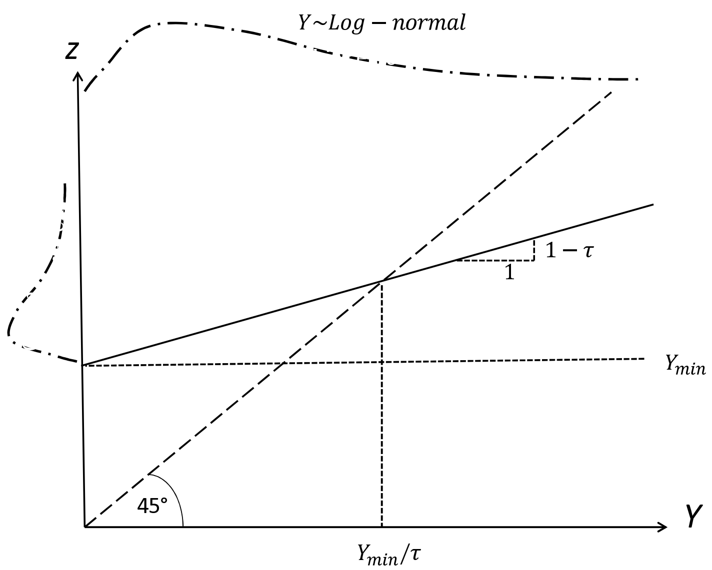


Figure 7.2.: Relation between market and post-tax income

We assume that taxes are only imposed on labor income which we assume to be the exogenous source of income inequality. The relation between market income Y_i and income after taxes and transfers Z_i is given as follows:

$$Z_i = (1 - \tau)Y_i + Y_{min}. \tag{7.17}$$

We assume a flat tax rate τ and a minimum income Y_{min}. In the public economics literature, the level Y_{min} is also referred to as the *demogrant*, being a minimum income level that is guaranteed to all individuals regardless of their respective labor income.

The relation between market and post-tax income is presented in figure 7.2. To ban perverse labor supply incentives ($\frac{\partial Z_i}{\partial Y_i} = 1 - \tau \overset{!}{>} 0$), this system only requires the flat tax to be below $\tau < 100\%$. In fact, a tax of $\tau = 100\%$ would lead to a maximal compression coinciding with a totally egalitarian society ($Y_i = Y_{min}$). One interpretation of this system is that every agent pays a flat tax τ while being guarantee a basic income Y_{min}. Another interpretation would be that tax is only paid on income exceeding a certain threshold that can be thought of as being tax-free ($Y_{TF} \equiv \frac{Y_{min}}{\tau}$):

$$Z_i = (1-\tau)(Y_i - Y_{TF}) + Y_{TF} = (1-\tau)\left(Y_i - \frac{Y_{min}}{\tau}\right) + \frac{Y_{min}}{\tau}. \tag{7.18}$$

While the marginal tax rate is constant ($MT = \frac{\partial(Y_i - Z_i)}{\partial Y_i} = \tau$), the average tax rate ($AT = \frac{Y_i - Z_i}{Y_i} = \tau - \frac{Y_{min}}{Y_i} = \tau\left(1 - \frac{Y_{TF}}{Y_i}\right)$) is positive for income above a certain threshold ($Y_i > \frac{Y_{min}}{\tau} = Y^{TF}$) and negative below this threshold, signifying that these individuals are net transfer receivers. For high income the average tax rate converges to the marginal tax rate ($\lim_{Y_i \to \infty}(\frac{Y_i - Z_i}{Y_i}) = \tau$). By definition a flat tax is not progressive. The existence of a minimum income level ($Y_{min} \neq 0$), however, introduces progressivity in the system. We can also compute the average income-weighted marginal taxes:

$$\sum_{i=1}^{N} MT(Y_i)\frac{Y_i}{\sum_{i=1}^{N} Y_i} = \frac{1}{Y}\sum_{i=1}^{N} \tau Y_i = \tau, \tag{7.19}$$

with total income of the economy being $\sum_{i=1}^{N} Y_i = Y$. As the marginal tax rate is identical for all individuals, the average income-weighted marginal tax rate also equals τ. We will use this result to compare this system with a progressive system discussed in the subsequent section.

We can also quantify the degree of progressivity. A well-known index of progressivity is the elasticity of average taxes to income changes ($\varepsilon_{AT,Y_i} = \frac{dAT/AT}{dY_i/Y_i}$) which yields:

$$\varepsilon_{AT,Y_i} = \frac{Y_{min}}{\tau Y_i - Y_{min}}, \tag{7.20}$$

which is positive for an income above the tax free level $Y_i > Y_{TF} = \frac{Y_{min}}{\tau}$. In fact, the highest progressivity is measured at the inflection point $Y_i = Y_{TF}$ where agents change from being net receivers to being net tax payers. For high incomes progressivity converges to zero ($\lim_{Y_i \to \infty} \varepsilon_{AT,Y_i} = 0$).

If the tax system shall generate an income G for the government in a society with N heterogeneous agents, it must be covered by an aggregate tax revenue $T > 0$:

$$G \stackrel{!}{=} T = \sum_{i=1}^{N}(Y_i - Z_i) = \sum_{i=1}^{N}(\tau Y_i - Y_{min}) = \tau Y - NY_{min} = N(\tau E(Y) - Y_{min}). \tag{7.21}$$

In this case, $Y = \sum_{i=1}^{N} Y_i$ is the total income, whereas $E(Y) = \frac{Y}{N}$ is the mean income. To make the system self-financing the following condition for the flat tax rate τ must hold:[45]

$$\tau = \frac{G + NY_{min}}{NE(Y)} = \frac{G}{Y} + \frac{Y_{min}}{E(Y)}. \tag{7.22}$$

[45] A similar discussion in a flat tax rate regime in which poverty level is a function of mean post-tax income is presented in Thompson (2010).

The financing condition links the tax rate τ and the minimum income Y_{min} and thus only leaves one degree of freedom for shaping the tax system. If there is no government income generating motive ($G = 0$), a pure *Robin Hood tax* (Bilbiie et al., 2013) emerges for which the only motive of the taxing authority is income redistribution, leading to an even simpler condition for the flat tax:

$$\tau = \frac{Y_{min}}{E(Y)}. \tag{7.23}$$

The intuition of these equations is that the financing motive of the government increases the level of the flat tax with the amount of the government expenditure ratio $\frac{G}{Y}$. In the *Robin Hood* case, the tax-free level Y_{TF} furthermore equals the mean income ($Y_{TF} = E(Y)$). If the tax also has an income generating purpose for the government ($G > 0$), the tax-free level falls below the mean income level. A positive level of $G > 0$ can also be interpreted in the sense of Okun (1975) as a *leaky bucket* for which the total amount of transfers is lower than the total taxes. By setting $G = 0$, we implicitly disregard the *leaky bucket* effect. In the following we only consider the *Robin Hood* case, implying that the only motive of the government to install a tax and transfer system is to achieve redistribution between agents and thereby income generation ($G > 0$) or other motives (e.g., influencing the consumption of demerit goods) do not matter.

It is important to note that the given system changes if we assume that the minimum income level - being the demogrant - is set to zero ($Y_{min} = 0$). As all agents have to pay taxes (the tax-free income is zero; $Y_{TF} = \frac{Y_{min}}{\tau} = 0$) the government is net beneficiary of the system ($G = \tau Y > 0$). Moreover, this system loses its progressivity since marginal and average tax rates are identical ($AT = MT = \tau$).

The minimum income Y_{min} is usually motivated in order to prevent poverty. Poverty itself normally is not defined in absolute terms but rather in relative terms making it - similar to consumption in the Duesenberry (1949) sense - a positional *bad*. This implies that poverty depends on the overall income per capita level.[46] If we define the poverty level as a proportion α of the mean income, the following flat tax can lead to redistribution:

$$\tau = \frac{\alpha E(Y)}{E(Y)} = \alpha. \tag{7.24}$$

This implies that the tax rate equals the poverty ratio (defined as a relative value of mean income).

In policy debates, the level of poverty is frequently defined as a proportion of median income. Since the income distribution is right-skewed, the median is lower than the mean income leading to lower poverty levels. Since we assume a log-normal distribution, a closed-form relation between median $\bar{Y}$ and the mean $E(Y)$ is given as follows:

$$E(Y) = \bar{Y} \exp\left(\frac{1}{2}\sigma_y^2\right). \tag{7.25}$$

[46] Similar to the discussion about the subsistence level of consumption presented in section 4.3, poverty depends on the overall level of income. Poverty in a low-income country might be classified as restricted access to food or basic shelter, whereas the lack of a TV set or a computer in a developed country might rank an individual as poor.

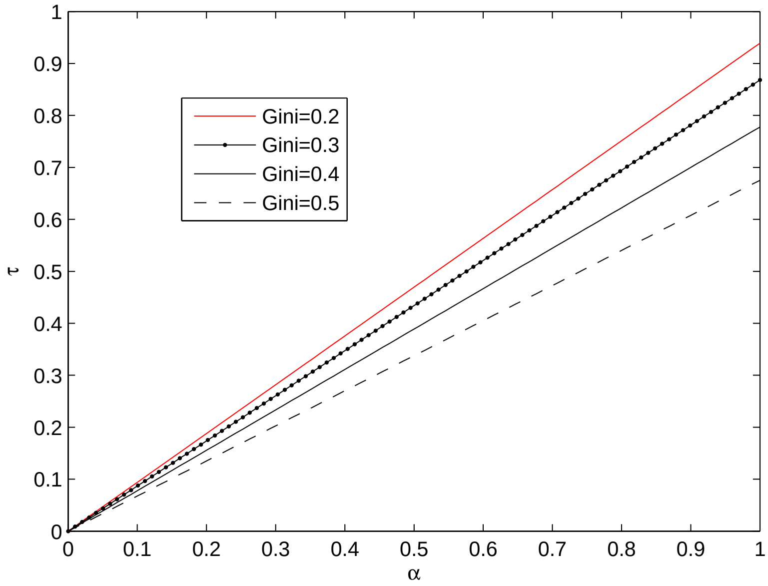


Figure 7.3.: Poverty level α as a function of Gini coefficient and flat tax τ

Inserting this result into the self-financing condition (for $G = 0$, cf. eq. 7.23) yields the following flat tax rate

$$\tau = \frac{\alpha \bar{Y}}{\bar{Y} \exp\left(\frac{1}{2}\sigma_y^2\right)} = \alpha \exp\left(-\frac{1}{2}\sigma_y^2\right). \tag{7.26}$$

For a log-normal distribution (in a first-order approximation) we can derive a relation between the Gini coefficent and the standard deviation of the distribution:[47]

$$Gini(Y) \approx \frac{\sigma_y}{\sqrt{\pi}}, \tag{7.27}$$

leading to the following relation between market income Gini, flat tax rate τ and poverty ratio α:

$$\tau \approx \alpha \exp\left(-\frac{\pi}{2} Gini(Y)^2\right). \tag{7.28}$$

Figure 7.3 plots a relation for realistic parameter values. The latter equation computes a relation between an exogenously given distribution ($Gini(Y)$), a desired poverty level (α), and a resulting policy parameter (τ). Interestingly, societies with higher levels of inequality require lower levels of flat tax rates τ to achieve a certain poverty ratio α, due to some very high-income individuals[48].

[47] The proof for this is presented in appendix A.4.

[48] It is also interesting to relate the linear tax rate to the results of Uhlig and Ljungqvist (2000). In their work, the authors assume a relative consumption effect and compute an optimal tax ratio that is able to outdo the negative externality resulting from the conspicuous consumption effect. The optimal

If we want to evaluate the redistributive effect of the tax, we can relate the market income Gini (GBT) to the post-tax Gini (GAT). Given the first-order approximation of a linear relation between Gini and the standard deviation ($Gini \sim \sigma_y$), we can perform a *back of the envelope calculation* for the redistributive effect:[49]

$$\frac{GAT}{GBT} = \frac{\sigma(Z)}{\sigma(Y)} = \frac{\sigma[(1-\tau)Y + Y^{min}]}{\sigma[Y]} = (1-\tau)\frac{\sigma(Y)}{\sigma(Y)} = (1-\tau). \tag{7.29}$$

This, however, is an approximated calculation.[50] A real closed-form solution can be derived for the coefficient of variation which is also frequently used to measure the effect of inequality:

$$\begin{aligned} \frac{CoV(Z)}{CoV(Y)} = \frac{\sigma(Z)}{E(Z)} \cdot \frac{E(Y)}{\sigma(Y)} &= \frac{(1-\tau)\sigma(Y)}{(1-\tau)E(Y) + Y_{min}} \cdot \frac{E(Y)}{\sigma(Y)} \\ &= \left(1 + \frac{Y_{min}}{(1-\tau)E(Y)}\right)^{-1}. \end{aligned} \tag{7.30}$$

Using, the equation for the self-financing of the *Robin Hood tax* (eq. 7.23), the identical result as for the approximated calculation for the Gini emerges:

$$\frac{CoV(Z)}{CoV(Y)} = \left(1 + \frac{\tau}{1-\tau}\right)^{-1} = 1-\tau. \tag{7.31}$$

The total redistribution via a complete redistributive tax ($\tau = 100\%$) leading to an egalitarian society results in both a Gini and a coefficient of variation of zero. Moreover, we have a simple linear relation between inequality before and after redistribution described by the redistribution parameter τ.

The presented system therefore not only has some appealing analytical properties but, furthermore, as put forward frequently in favor for a simple tax system, the administration costs and the tax evasion opportunities are minimized.

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level is given by $\tau^* = \frac{\bar{c}}{E(C)}$ and thereby increases with the level of relative consumption. This makes the minimum income level $Y_{min} = \tau E(Y) = \bar{c}\frac{E(Y)}{E(C)}$. For an economy that is balanced in the aggregate ($E(C) = E(Y)$) this implies $Y_{min} = \bar{c}$ accompanied by the fact that the minimum consumption level $\bar{c}$ can be sustained by any agent as it equals the level of minimum income. In contrast to our model the minimum consumption level $\bar{c}$ does not change after redistribution. Another important result of Uhlig and Ljungqvist (2000) is that taxes should be procyclical (i.e. high in booms and low in busts). This very *Keynesian* result relies on the notion of the relative consumption problem and thereby follows from a completely different argument than the standard Keynesian literature.

[49] Another way of putting it would be $\log(GBT/GAT) = -\log(1-\tau) \approx \tau$.

[50] Due to the minimum income assumption the post-tax distribution is left-censored at a level of Y_{min} which (in contrast to Fischer (2013) assuming an exogenous variation of the labor income distribution) does not conserve the log-normal distribution assumption. Hence, the approximation $Gini \sim \sigma_y$ (which is a first-order approximation for the log-normal distribution anyhow) does not hold.

7.2.3. A Combined System with Progressive Taxation and Transfers

We contrast this simple linear income tax system with a progressive income system, as implemented in most developed countries. Following the rationale of Bénabou (1996), the disposable income after taxes and transfers is modeled as follows:

$$Z_i = \left(\frac{Y_{TF}}{Y_i}\right)^{\tau} Y_i = Y_i^{1-\tau} Y_{TF}^{\tau}. \tag{7.32}$$

Note that we use the parameter τ once again, even though it has a different meaning as in the previous chapter in which it was flat tax rate. As we will discuss more thoroughly in the following the measures are highly comparable as they imply a similar degree of redistribution. As before, perverse labor supply effects are ruled out:

$$\frac{\partial Z_i}{\partial Y_i} = (1-\tau)\left(\frac{Y_{TF}}{Y_i}\right)^{\tau} \overset{!}{>} 0, \tag{7.33}$$

for all $\tau < 1$. As in the linear system, the special case of $\tau = 1$ implies an egalitarian distribution for which post-tax income is equal for all agents and equals the given minimal income ($Z_i = Y_{TF}$).

In a progressive system, the marginal tax rate is always higher than the average tax rate. The marginal tax rate (MT) is given as follows:

$$MT = \frac{\partial(Y_i - Z_i)}{\partial Y_i} = 1 - \frac{\partial Z_i}{\partial Y_i} = 1 - (1-\tau)\left(\frac{Y_{TF}}{Y_i}\right)^{\tau}, \tag{7.34}$$

whereas the average tax rate (AT) amounts to:

$$AT = \frac{Y_i - Z_i}{Y_i} = 1 - \frac{Z_i}{Y_i} = 1 - \left(\frac{Y_{TF}}{Y_i}\right)^{\tau}. \tag{7.35}$$

The condition of average tax rate being lower than marginal tax rate is satisfied for:[51]

$$\tau > 0, \tag{7.36}$$

imposing the second restriction on the value of τ. For the special case of $\tau = 0$ there is no effective tax system ($Z_i = Y_i$). Moreover, a value of $\tau < 0$ is accompanied by marginal taxes that are higher than average taxes resulting in a regressive system.

For the case of $\tau > 0$, it is furthermore important to point out that both marginal and average tax rates grow with income, yet marginal taxes at a lower pace ($0 < \frac{\partial MT}{\partial Y_i} = \tau(1-\tau)\left(\frac{Y_{TF}}{Y_i}\right)^{\tau}\frac{1}{Y_i} < \frac{\partial AT}{\partial Y_i} = \tau\left(\frac{Y_{TF}}{Y_i}\right)^{\tau}\frac{1}{Y_i}$). Thereby, we can differ this system from the flat tax system with a demogrant in the previous section, in which only the average taxes grew with income. Moreover, both average and marginal tax rate converge to a rate of

[51] This is easy to verify: $AT < MT \leftrightarrow (1-\tau)\left(\frac{Y_{TF}}{Y_i}\right)^{\tau} < \left(\frac{Y_{TF}}{Y_i}\right)^{\tau} \rightarrow 1-\tau < 1$.

one for high incomes, implying that only for the unrealistic case of infinite income, the income after taxes is zero ($\lim_{Y_i \to \infty} MT = \lim_{Y_i \to \infty} AT = 1$).

We also can compute the level of tax-free income Y_{TF} for which pre- and post-tax income are identical:

$$Y_i \equiv Z_i \equiv Y_{TF} = \left(\frac{Y_{TF}}{Y_{TF}}\right)^{\tau} Y_{TF}. \tag{7.37}$$

Below this level the average tax rate is also negative, implying that agents receiving market income below this level are net transfer receiver.[52]

The tax elasticity as an index of progressivity is equal to:

$$\varepsilon_{AT,Y_i} = \tau \left(\frac{Y_{TF}^{\tau}}{Y_i^{\tau} - Y_{TF}^{\tau}}\right). \tag{7.38}$$

As in the flat tax case, this is positive for all incomes above the tax-free level ($\varepsilon_{AT,Y_i} > 0$ for $Y_i > Y_{TF}$) and converges to zero for high incomes ($\lim_{Y_i \to \infty} \varepsilon_{AT,Y_i} = 0$).

In the model, we assumed that income follows a log-normal distribution. This allows us to compute the redistributive effect of the tax. Note that post-tax income is distributed as follows:

$$Z_i \sim Y_i^{1-\tau}, \tag{7.39}$$

yielding the following ratio between standard deviations is proportional to the Gini-values (the formal proof is presented in appendix A.4):

$$\frac{\sigma(Z)}{\sigma(Y)} \equiv \frac{Gini(Z)}{Gini(Y)} = 1 - \tau. \tag{7.40}$$

It is important to point out that this result is identical to the result for the case of a linear tax, for which redistribution solely depends on the variable τ. Therefore, both types of taxation lead to similar redistributive effects.

We still have a degree of freedom, since our tax system has two parameters Y_{TF} and τ. To connect them we once again impose a self-financing condition of the system:

$$\sum_{i=1}^{N} Y_i \stackrel{!}{=} \sum_{i=1}^{N} Z_i = Y_{TF}^{\tau} \sum_{i=1}^{N} Y_i^{1-\tau}, \tag{7.41}$$

leading to:

$$Y_{TF} = \left(\frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} Y_i^{1-\tau}}\right)^{\frac{1}{\tau}}. \tag{7.42}$$

For the special case of the log-normal distribution this yields:[53]

$$Y_{TF} = \exp[\tau \log(\bar{Y}) + 0.5\sigma_y^2(1 - (1-\tau)^2)]^{\frac{1}{\tau}}, \tag{7.43}$$

[52] Note that for the special case of zero income $Y_i = 0$, this situation would not change under the tax system as $Z_i(Y_i = 0) = 0$, implying that under this system no minimum subsistence level is sustained. In contrast to that for the simple linear system we have $Z_i(Y_i = 0) = Y_{min} > 0$.

[53] This result is easy to verify if we keep in mind that $\sum_{i=1}^{N} Y_i = N \cdot E(Y) = N\bar{Y}exp(0.5\sigma_y^2)$ and due to the transformation property of log-normal distributions $\sum_{i=1}^{N} Y_i^{1-\tau} = N \cdot \exp([1-\tau]\log(\bar{Y}) + (1-\tau)^2 0.5\sigma_y^2)$.

which in logs equals:

$$\log(Y_{TF}) = \log(\bar{Y}) + 0.5\sigma_y^2(2 - \tau). \tag{7.44}$$

For the special case of total redistribution ($\tau = 1$) the minimum income equals the mean income; as in the case of the linear tax system. However, in the progressive the minimum income *decreases* with the degree of redistribution τ. To grasp this surprising result, we can compare the two tax systems.

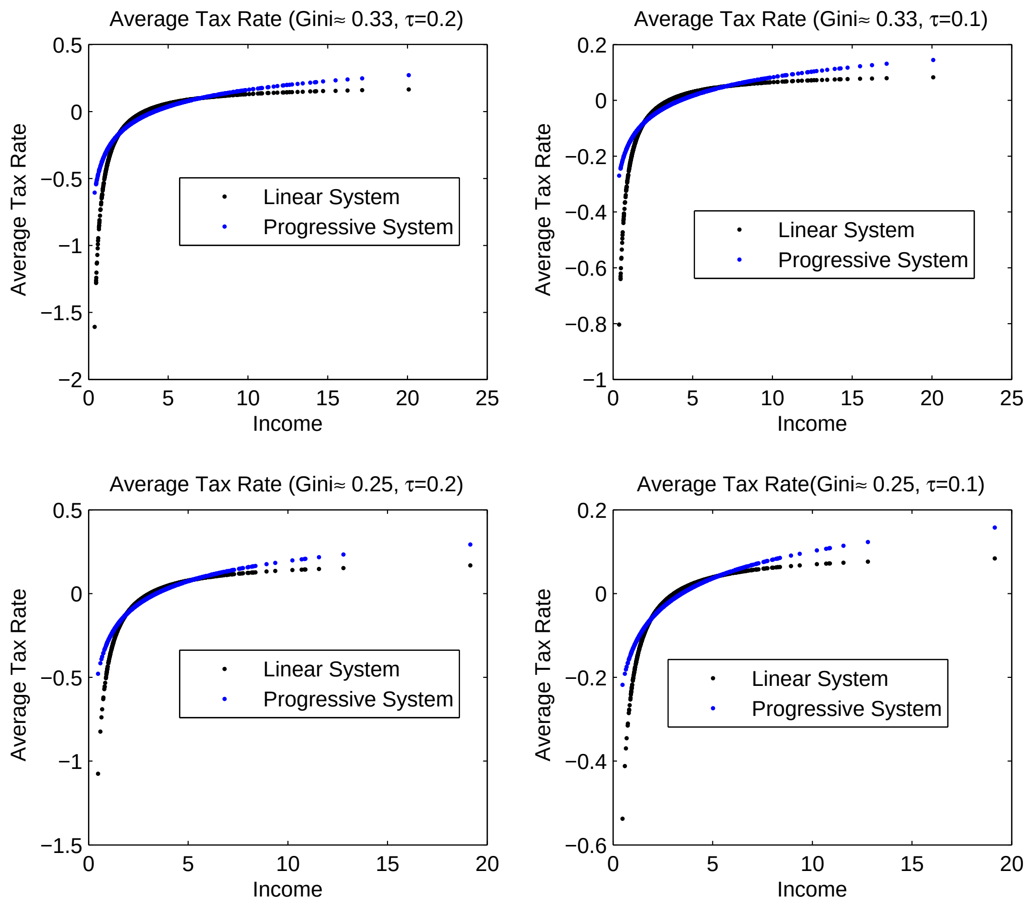

Figure 7.4.: Average tax rates in the linear (black) and progressive (blue) tax system with variation of inequality and redistribution

Once again - following Heathcote et al. (2014) and also using the result for the tax-free income Y_{TF} in a self-financed system - we can compute the average income-weighted marginal tax rate:[54]

$$\begin{gathered}\sum_{i=1}^{N} MT(Y_i)\frac{Y_i}{Y} = \frac{1}{Y}\sum_{i=1}^{N} Y_i - (1-\tau)Y_i\left(\frac{Y_{TF}}{Y_i}\right)^{\tau} = \\ 1-(1-\tau)\sum_{i=1}^{N}\frac{Y_{TF}^{\tau}}{Y}Y_i^{1-\tau} = 1-(1-\tau) = \tau.\end{gathered} \tag{7.45}$$

It is important to acknowledge that in both cases - the progressive system and the linear tax system with a demogrant - both take the same value. Therefore, both system can be easily compared if we keep the same value τ.

In the following we compare both systems by means of numerical simulation. To do so we set an environment of $N = 1,000$ agents in which income is log-normal distributed with $\log(\bar{Y}) = 1$. We vary both the initial inequality and the level of redistribution. We discuss the case of high and low initial inequality ($Gini(Y) \approx 0.35$ respectively $Gini(Y) \approx 0.28$ [55]). In figure 2.4, we presented empirical evidence for redistribution, which we measured as:

$$Redist = \log(GBT) - \log(GAT) = \log\left(\frac{GBT}{GAT}\right). \tag{7.46}$$

This ratio is easy to transfer to our model, for which:

$$\log\left(\frac{GBT}{GAT}\right) = \log\left(\frac{1}{1-\tau}\right) \approx \tau. \tag{7.47}$$

In particular we assume $\tau = 0.2$ - reflecting continental European countries with high redistribution - and $\tau = 0.1$ mimicking the behavior of low redistribution countries (especially the USA). Heathcote et al. (2014) estimate a progressive system in the Bénabou (1996) tradition using PSID-data and derive a value of $\tau = 0.151$ for the USA.

$\log(Gini(Z)) - \log(Gini(Y))$		$\tau = 0.2$		$\tau = 0.1$	
		Linear	Progressive	Linear	Progressive
$Gini(Y)$	0.352	0.223	0.210	0.105	0.099
	0.282	0.223	0.216	0.106	0.102

Table 7.3.: Redistributive effect with different forms of initial inequality and taxation policies

As already put forward and documented in table 7.3 both systems achieve similar means of redistribution. Nevertheless, the two systems are completely different. The

[54]It is important to point out that this result holds without making specific assumptions about the distribution of income.

[55]Both results from log-normal distributions with $\sigma_y = \frac{2}{3}$ respectively $\sigma_y = 0.5$.

$Y_{TF}/E(Y)$		$\tau = 0.2$		$\tau = 0.1$	
		Linear	Progressive	Linear	Progressive
$Gini(Y)$	0.352	1.000	1.180	1.000	1.205
	0.282	1.000	1.110	1.000	1.125

Table 7.4.: Tax-free income as a ratio of mean income with different forms of initial inequality and taxation policies

$Y_{TF}/\bar{Y}$		$\tau = 0.2$		$\tau = 0.1$	
		Linear	Progressive	Linear	Progressive
$Gini(Y)$	0.352	1.234	1.456	1.234	1.487
	0.282	1.144	1.270	1.144	1.287

Table 7.5.: Tax-free income as a ratio of median income with different forms of initial inequality and taxation policies

relevant figure of the average tax rate is visualized in figure 7.4 as a function of underlying income. The average tax rate in fact give the ratio of taxes to income ($AT_i = \frac{T_i}{Y_i} = \frac{Y_i - Z_i}{Y_i}$). If the ratio is negative, agents are net transfer receivers. Not surprisingly, the average tax rate for high-income households increase in a scenario with progressive income taxation. Thus, rich individuals are worse off in a progressive taxation scenario. Moreover, the transfer rates for low-income individuals also increase - i.e. the ratios are less negative and low-income individuals receive a lower level of transfers and thereby are also worse off under this scenario. The higher taxes on high-income individuals and the lower transfers to low-income individuals, however, are not gains for the government, but are redistributed to the middle-income agents, who are the net winners under this scenario. Middle-income agents in particular gain for higher inequality - allowing for a higher tax base of rich individuals - and lower progressivity as measured by τ. In fact, the curvature for the average tax rate is lower for the progressive scenario.

This effect becomes evident if we compare the level of tax-free income as presented in table 7.4 and 7.5. As put forward earlier, for the case of a linear tax the ratio of tax-free to the average income equals one, implying that the household with medium income is exactly at the knife-edge case of zero taxes nor transfers.[56] If we compare it to the median income - as usually done in poverty analysis - this ratio is slightly higher (due to the effect that in an unequal society median income is lower than mean income). This implies that the median household is always a net receiver. This is also important as it is in line with the median voter theorem of political economy.

A different scenario emerges for the progressive tax. The tax-free income is *above* the mean income. Thus, the mean income household is actually a net receiver. We can even compute a closed-form relation for the log-normal case:

$$\frac{Y_{TF}}{E(Y)} = \frac{\exp(\log(\bar{Y}) + 0.5\sigma_y^2(2-\tau))}{\exp(\log(\bar{Y}) + 0.5\sigma_y^2))} = \exp(0.5\sigma_y^2(1-\tau)) > 1, \tag{7.48}$$

[56]It is important to point out that this result does not require a specified distribution of income.

respectively:

$$\frac{Y_{TF}}{\bar{Y}} = \frac{\exp(\log(\bar{Y}) + 0.5\sigma_y^2(2-\tau))}{\exp(\log(\bar{Y}))} = \exp(0.5\sigma_y^2(2-\tau)) > \frac{Y_{TF}}{E(Y)} > 1. \tag{7.49}$$

As becomes evident, this ratio once again increases for higher inequality, but decreases for higher progressivity. In this case, the middle-class profits from high inequality as it is accompanied by a higher base of taxation allowing for more transfers to the middle-class. The explanation for the effect of progressivity τ is as follows: higher values of τ increase the progressivity, which is highest at the point $Y_i = Y_{TF}$. Therefore, in a progressive system the middle-class suffer most from an increase in progressivity as their status can turn from net transfer receiver to net taxpayers. Or put differently, in a progressive tax system compared to a flat tax system middle-class households are better off. However, this position is very fragile. Only slight changes in their income or in the structure of the system can change them from net receivers to net tax payers.

All in all, albeit similar redistributive effects progressive systems are more favorable for middle-income households, whereas high-income and low-income households would prefer a linear tax system. In the following, we assess the impact in our complete model, that focuses on the effect of financial stability.

7.2.4. Effect of Taxation

In this section we present simulation results for the case with a tax on labor income Y. It is important to point out that we only impose the tax on labor income Y rather than total including capital income ($X = Y + rK$). First of all, a general tax on total income X would result in an identical tax on both labor and capital income. In general, capital income is covered with a lower marginal tax rate.[57] In fact, a vast amount of the theoretical literature summarized so far even favors zero taxes on capital income. In our modeling choice, we follow this literature. However, a natural candidate for extension would be to discuss a tax that also envisions stock quantities such as a tax on wealth or capital income.

In this section, we take the benchmark model for the *Austerity*-case as presented in section 6.1 and vary the parameter τ in a *ceteris paribus* fashion.[58] We start by discussing the flat tax regime as presented in section 7.2.2 and then contrast it to the progressive system as discussed in section 7.2.3. For this section, we assume that - rather than being formed on the distribution of the market income Y_i - the subsistence level of consumption depends on the distribution of the post-tax income $Z_{i,t}$:

$$\bar{c} = quantile_j(Z_{i,t}). \tag{7.50}$$

[57] In Germany a flat tax on capital income of 25% is applied. The average tax on capital income (also having a lower average income tax rate than 25%) is below 25% due to exemption levels. For very high-income individuals, the tax exemption does not matter in a significant manner, yet the capital tax rate of 25% is way below the maximal marginal tax rates for total income ranging up to 45%.

[58] To ensure exactly similar results, we once again assume the same noise trading vector as well as the exact same pick of the log-normal distribution for the wage income distribution.

This assumption is essential for the results. As the post-tax income determines the available income for all agents, it also the underlying variable to derive the subsistence level of consumption. As shown in figure 7.5, an increase of the tax level τ increases the income level of the low-income households also leading to a higher subsistence level of consumption. This is one of the central problems in redistributing flow income in the presence of relative consumption. The relative position of individuals does not change, while the minimum consumption level even increases. In fact, as also shown in figure 7.5, the minimum consumption level is always above the minimum income[59] provided by the tax system. Even though the gap between minimum income and minimum consumption narrows, it still persists (only converging to zero for an egalitarian society) requiring debt financing of consumption for some individuals.

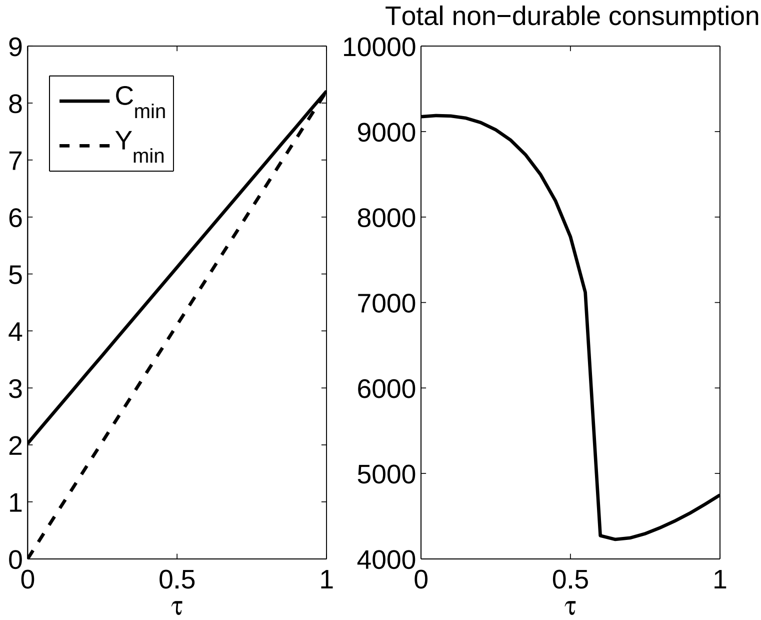

Figure 7.5.: Subsistence consumption $\bar{c} \equiv c_{min}$ and minimum income Y_{min} as well as total non-durable consumption as a function of tax rate τ

Interestingly, the total level of non-durable consumption $\left(\sum_{i=1}^{N} C_i\right)$ also falls compared to a society without redistribution (see figure 7.5) at odds with the Post-Keynesian argument that redistribution leads to higher overall consumption. As, however, presented in appendix A.9, this is only the case for a concave consumption function. As shown in section 6, in our model with debt and positive interest rates the consumption function is convex, leading to a lower overall consumption, albeit a higher level of minimum consumption level. As formally proved in appendix A.9 the convexity of the consumption

[59]Formally, the minimum income level is given by: $Y_{min} = \tau \cdot \bar{Y} \cdot exp\left(0.5\sigma_y^2\right)$. For a given level of income and initial inequality determined by $\bar{Y}$ and σ_y, the minimum income level increases with the flat tax rate τ in a linear manner. The closed-form value of the minimum consumption level is more complicated, yet has near-linear properties.

function stems from the presence of the collateral constraint. Or - as already emphasized in section 6.2.1 - the decrease in the number of middle-class households presented in figure 7.6 contributes to the decline of overall consumption.[60] [61]

It is also interesting to compare this result with the work of Foellmi and Oechslin (2008). The authors assume a standard concave production function with negative scale effects, implying that aggregate output will increase with redistribution. As the latter, however, also increases the demand for credit, the interest rate increases, making the credit constraint more binding for poor households. Thus, in result the effect of redistribution is ambiguous. In our model, redistribution also increases the demand for credit and the interest rate making the agents class-wise worse off. As a result, we have a similar channel. Note that we, however, do not have a production function but discuss a consumer-only economy. Moreover, in our case consumption C is a convex function out of labor income Y resulting in a decreasing aggregate demand for redistribution (also refer to appendix A.9).

As shown in figure 7.6, the *Robin Hood tax* reduces the number of high-income households by taking from the rich. The social function of high-income households in the model context is to provide debt for lower income households. As the high-class households vanish, the supply of debt decreases, accompanied by a current account deficit and high interest rates.[62] The high interest rate furthermore lowers the disposable income of lower income households leading to an actual increase of low-class households. Therefore, and somehow surprisingly, in a society with relative consumption, an income tax not only transforms high-class into medium-class households, but also leads to the fact that medium-class households turn into low-class households. In result, the situation with taxes *class-wise* is Pareto inferior to a situation without taxes.

As presented in figure 7.6, moderate taxes reduce price volatility in durable markets and decrease prices for durables. This can be attributed to the effect that mean net worth decreases with convergence to a current account deficit leaving less budget for financial speculation and promoting a flight to debt.

[60] Another way of rationalizing is that with higher flat tax rates the economy transforms into a deficit economy lowering the foreign income that can be reemployed for consumptive purposes.

[61] Athreya et al. (2014) discuss the effect of redistribution on aggregate consumption within a calibrated *Bewley*-type model and emphasize the role of labor supply not considered in our model. While they put forward that due to the lower MPC of high-income households a redistribution from rich to poor increases aggregate consumption, they argue that it also contributes to an aggregate decrease in working hours in favor of leisure (as well as investment). This effect outdoes the positive effect of higher consumption as the authors assume that leisure is a luxury good whose demand increases with income. The latter, however, is in stark contrast to the findings presented in Pecchi and Piga (2008) arguing that we live in a time with (over)worked rich. Athreya et al. (2014), however, also put forward that due to the more productive working hours of high income increasing the effective working hours contract less than the actual hours slightly dampening the negative effect of redistribution.

[62] This relation is not presented in figure 7.6. However, as shown in section 5.4.1 we know that $r_{t+1} = r_0 \exp(\mu_r D_t)$, implying a clear positive relation between the level of accumulated total debt and the level of the interest rate. The maximum level of debt is jointly determined by the level of assets and the equity requirement and given by $D_{max} = (1-m)H \cdot N \cdot E(Y) \approx 1.31 \cdot 10^5$ which for the given values implies an interest rate of $r_{t=T}(\tau = 1) \approx 3.87\%$.

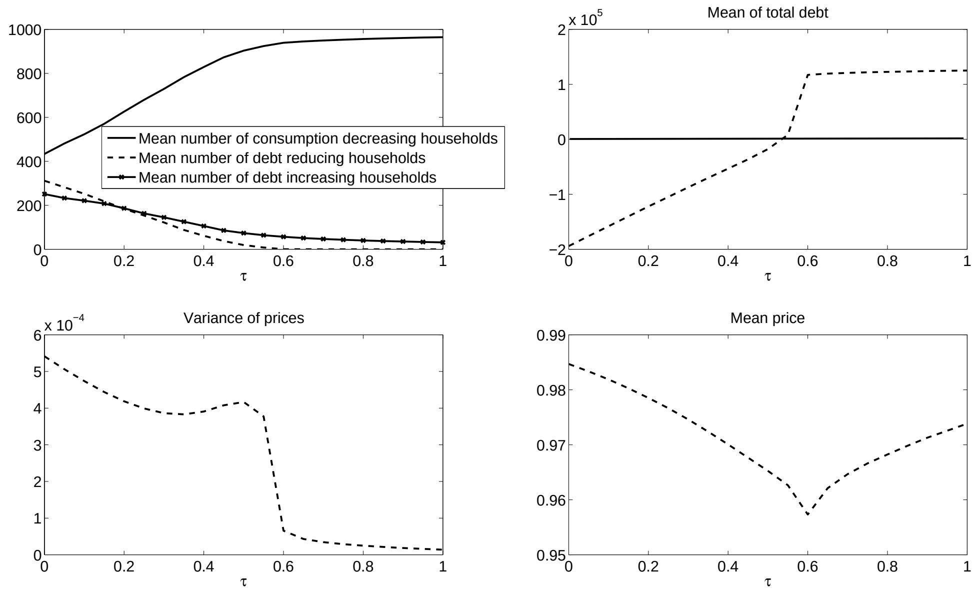

Figure 7.6.: Agent composition and market conditions in the market for durables and debt with variation of the flat tax rate τ

A special region can be identified for tax values of approximately $\tau > 60\%$. In this region, domestic savers disappear resulting in a current account deficit. While some agents, due to their inherited initial endowment, are able to lever up in the short-run making them *middle-class*, in the long-run all agents fall into the class of consumption-decreasing and low-class agents. This can be attributed to the relation between the distribution of post-tax income and the chosen consumption function.[63] The increased prices and lower financial volatility for the region $\tau > 60\%$ should therefore not be misinterpreted, but are rather a result of a breakdown in trading for the durable market accompanied by a convergence of price to its fundamental and initially assumed value ($P_0 = F = 1$).

Finally, we can make a statement about different forms of inequality in the tax-regime case (see figure 7.7). As formally presented in the previous sections, the flat tax rate and labor income inequality can be related in a simple linear manner. Inequality of consumption, however, is above labor income inequality for a reasonable level of the flat tax rate ($\tau < 60\%$). In this region, the gap between these two forms of inequality increases due to consumption out of net worth (stock) as well as the total income inequality stemming from capital income. This effect can be attributed to the higher net worth (stock) inequality (documented in the right panel of figure 7.7). While the tax is

[63] As shown in the robustness tests (see section 6.2), factors that increase consumption (higher values of c_y, H, c_w, and $\bar{c}$ as well as lower values of ε) contribute to a current account deficit. The same holds true for a lower level of inequality as the number of high-income households holding capital (rather than debt) decreases. Future work should try to find closed-form solutions in order not only to identify but also to quantify the underlying factors leading to the current account deficit.

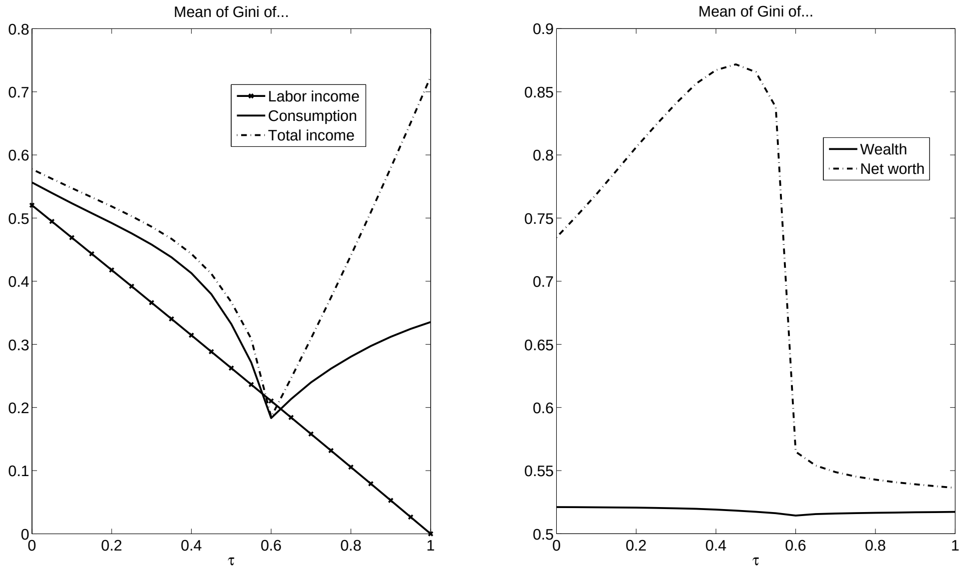

Figure 7.7.: Different Gini coefficients (mean in time) for variation of flat tax rate τ

able to address the effects of flow inequality, the resulting increase in mutual debt/credit positions eventually increases the stock value of net worth inequality.

Once again, we have to note the different behavior for the current account deficit situation ($\tau > 60\%$). As all agents in this case end up being low-class households, they are all levered up to their maximum $D_{max_i} = (1-m)HY_i$. Thereby, the distribution of net worth is completely determined by the distribution of wealth which is exogenously assumed to be distributed in the exact manner as labor income. In result for the case, both distributions - net worth and wealth - converge leading to the surprising result that net worth inequality decreases for higher flat tax rates in a current account deficit scenario. Moreover, the inequality of total income and consumption also increases in this region. To comprehend this surprising result it is important to point out that the tax impacts only on the flow quantity of labor income. In result, agents with a high labor income Y_i have a very high ratio of wealth (given by $W_i = HY_i$) to post-tax income (given by $Z_i = (1-\tau)Y_i + Y_{min}$) and vice versa for agents with a low level of labor income. This fact allows high income agents to accumulate a higher level of debt (also relative to post-tax income Z_i). In effect the total income $X_i = Z_i - rD_i$ unambiguously *decreases* with labor income Y_i respectively post-tax income Z_i. For very high income total income is negative ($X_i < 0$) implying that they are in a Ponzi trap, for which interest on debt exceeds current available income. As modeled in equation 6.3, there is even a sub-category within the lower-class households. Agents with negative total income are assumed to have zero consumption, whereas the other agents act in a hand-to-mouth manner also leading to the fact the effect of net worth on consumption ceases to exist.[64] In result, the absolute level of consumption decreases with the level of post-tax income. Moreover and as a result of the latter finding, for this region the redistribution eventually (slightly) increases aggregate consumption as shown in figure 7.5. The key of the analysis, however, lies in the domain of a current account surplus ($\tau < 60\%$) for which the economy is not governed by strange *topsy-turvy* dynamics.

We can also compare the effect of the flat tax rate - being a policy parameter - to other parameters presented in the section for which we varied the *exogenous* parameters, which are beyond the control of the policy institution (cf. section 6.2).

First of all, the increase of the flat tax rate bears close resemblance to an increase in the minimum consumption level as documented in section 6.2.1. This is straightforward, since - as presented in figure 7.5 - the decreased inequality increases the level of minimum consumption. In both cases, the increased consumption desire, however, is financed by a higher level of domestic debt which furthermore increases the rate of interest (also cf. 6.12). As a result for both - an increase in minimum consumption c_{min} or the flat tax rate τ - all agents are worse off class-wise. A pure increase in the minimum consumption level increases all forms of inequality as the increased level of debt increases net worth inequality, which increases the inequality of total income, also feeding back into inequality of consumption (cf. figure 6.13). The taxation case - by construction - lowers the level of labor income inequality also lowering total income inequality as well as consumption inequality. The level of net worth inequality, however, also increases in this case due to the extended use of debt.

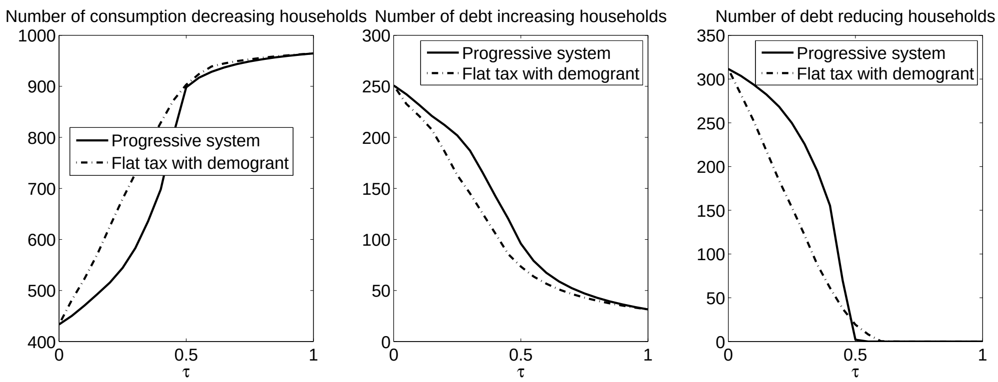

Figure 7.8.: Class composition of a progressive opposed to a linear tax system with demogrant

Another natural candidate for comparison would be contrasting the effect of the flat tax rate to an exogenously assumed variation of the inequality as captured by the standard deviation of the income distribution σ_y in our model and discussed in section 6.2.2. For the exogenous variation we not only lower the level of income inequality, but also the level of wealth inequality, which we assume to be perfectly correlated. Once again, all agents are worse of in terms of class for lower inequality (lower values of σ_y), yet

[64]Note that we do not illustrate these results in graphs due to space constraints. They are, however, available on request.

the variables of inequality decrease. For an exogenous decrease in inequality, the level of net worth inequality also decreases (cf. figure 6.26). This is because the decrease in wealth inequality outdoes the increase in debt inequality leading to an overall decrease in net worth inequality. For a lower level of σ_y, the level of debt and the rate of interest increase, whereas the volatility in asset markets decreases (cf. figure 6.25). Note that the lower level of overall non-durable consumption for an exogenous variation of inequality σ_y stems from a pure mathematical identity as we assume the median income to be fixed, resulting in the fact that mean as well as the aggregate income increases with inequality.[65]

In section 7.2.3, we introduced a *real* progressive tax system, for which - in contrast to the linear tax system with demogrant (introduced in section 7.2.2) - the progressivity not only resulted from the presence of the demogrant. Qualitatively, the progressive tax system has the same properties as the linear tax system. Namely, all agents are worse off class-wise (cf. figure 7.8), debt and interest rates increase (cf figure 7.9). The demise of the middle-class contributes to the decline of aggregate consumption (cf. figure 7.10). Meanwhile, the volatility in durable markets is reduced (cf. figure 7.9). While the inequality of consumption decreases, inequality of net worth increases (cf. figure 7.9).

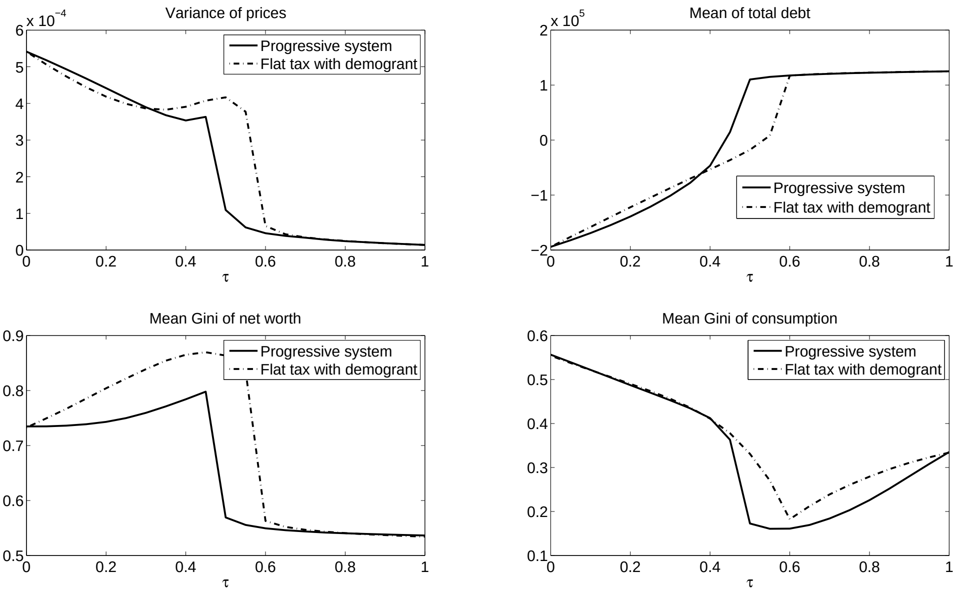

Figure 7.9.: Variance of prices, total debt, Gini of net worth, as well as Gini of consumption in the progressive and the linear tax system

If we, however, make a quantitative assessment, the two type of taxes have different outcomes. As already presented in section 7.2.3, both type of tax systems have the same impact on the aggregate distribution and therefore are easy to compare by means of the variable τ. Their macroeconomic impact, however, is different as they imply different

[65]Formally, this is because of $E(Y) = \bar{Y}exp(0.5\sigma_y^2)$.

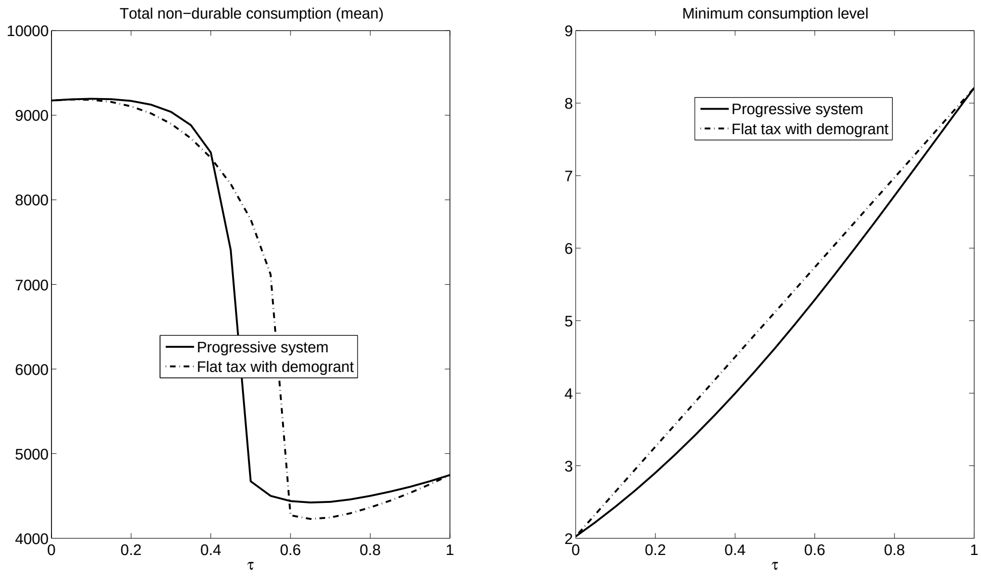

Figure 7.10.: Aggregate non-durable consumption and minimum consumption in the progressive and the linear tax system

individual average tax rates (respectively transfer rates) for different agents (also cf. figure 7.4).

First of all, the point for which the economy turns from a surplus into a deficit economy is reached for a lower level of redistribution τ in the case of the progressive tax system. The following analysis, yet, focuses on reasonable and moderate levels of taxation with domestic supply of credit ($\tau < 40\%$) for which a current account surplus economy prevails.

As depicted in figure 7.8, for a given level of redistribution given by τ, there are more middle and high-class agents as well as fewer low-class agents under a progressive tax system as opposed to a linear system with demogrant. As already put forward in section 7.2.3, the middle-class profits from a progressive tax system as compared to a flat tax system. Hence, the aggregate consumption level - being closely related to the number of middle-class households - is also higher for a progressive tax system (cf. figure 7.10). The key rationale for the different behavior of the linear tax system as compared to the progressive system lies in the difference in the minimum consumption level as depicted in the right panel of figure 7.10. For the linear case, the minimum consumption level is always slightly above the level witnessed in the progressive case. The fact that agents are better off class-wise is also reflected in a lower level of debt, furthermore resulting in lower net worth inequality (cf. figure 7.9). On the other hand, for a given level of redistribution τ the volatility of asset markets is higher under the progressive system owing to the higher number of high-class agents.

In effect, the progressive tax system can be considered a *light*-version of the linear tax system. Both the negative effects (more debt, higher net worth inequality, lower consumption) as well as the positive effects (less volatility in durable markets) emerge, yet are diminished. Regarding the fact, that besides increasing stability in durable markets, taxation has negative net effects, one might say that the progressive tax system wins the *horse race* between the two systems.

All in all, the redistributive tax is successful in lowering income inequality and thereby also reducing the number of high-income individuals that engage in potentially destabilizing speculation in the markets for durables. On the other side, the tax can transform a surplus economy in the presence of strong conspicuous consumption into a deficit economy for which debt is a society-wide phenomenon. Moreover, lower income (flow) inequality is accompanied by higher net worth (stock) inequality and therefore is accompanied by unintended consequences.

8. Conclusion

Turn the light out,
say goodnight
No thinking for a little while
Let's not try to figure out
everything at once.

The National - Fake Empire (2007)

This dissertation presents a theoretical model in order to shed light on certain aspects of the recent financial crisis. In particular, the model links the issues of financial (in)stability and inequality for developed countries.

The dissertation relies on the novel method of Agent-Based Modeling. This class of models has been very successful in explaining boom-bust cycles in asset markets.[1] As these models emphasize the aspect of *heterogeneous* agents, it is also straightforward to address issues of inequality within this setting. The latter issue, however, is only covered sparsely in the existing literature.[2] In section 3 we compare this modeling paradigm with other competing approaches and highlight advantages of the novel method but also emphasize potential shortcomings. As with any sophisticated model that relies on numerical methods the underlying mechanisms are hard to understand. Moreover, the question of the robustness of the (numerical) results arises. We try to account for these problems in section 5 by presenting closed-form solutions and in section 6 by performing numerical robustness checks of the parameters. Moreover, this class of models usually assumes behavioral decision rules that are frequently criticized for being of an *ad-hoc* nature. We address this issue in section 4 by showing that the key decision in the model (the consumption decision) can be micro-founded.

As discussed in section 4, a key component to explain the events surrounding the financial crisis is to argue on the basis of the theory of relative consumption (Duesenberry, 1949), for which the level of consumption is determined in a process of social interaction. In particular, low-income individuals try to imitate the consumption of high-income individuals. The latter effect is fortified in times of high income inequality as witnessed in recent years and documented in section 2. In the simulations of our model (section 6), we show that higher consumption of low-income individuals (temporarily) decreases consumption inequality to a level lower than labor income inequality. The increased

[1] For excellent surveys the reader is referred to Hommes and Wagener (2009), Chiarella et al. (2009), and Lux (2009).

[2] A notable exception is Dosi et al. (2013), focusing however, on the functional rather than the personal distribution of income.

consumption was financed by means of credit both in the model and in the years leading to the crisis. The latter was provided by high income domestic individuals as well as foreigners (especially for the USA and the European periphery) as reflected in current account data (cf. section 2). As a result, in the short-run the consumption inequality was able to decrease relative to labor income inequality. This, however, was accompanied by higher net worth inequality (stock) and a disparity of current accounts. As a result, the short-run decrease in consumption inequality also promoted an increase in net worth inequality (stock) and disparity of current accounts. Moreover, the credit expansion fueled a real estate bubble. The burst of the bubble for this *popular* asset led to severe adverse consequences especially resulting from the problem of deleveraging. We are able to reproduce these facts in the model (cf. section 6). Moreover, we show that higher income inequality can promote financial instability. We decompose financial instability into asset price bubbles (mostly driven by high-income individuals employing a large share of their labor income for financial speculation) and Ponzi-schemes in the debt market (driven by low-income agents that are indebted) and discuss common factors. Furthermore, the model is also able to explain the decline of the real interest rate also discussed under the aspect of the *global savings glut hypothesis* as a result of higher inequality.

While the major economies still struggle with the consequences of the crisis, the public debate about financial stability slightly abated. The discussion about economic inequality, however, gained a strong momentum after the publication of the book of Piketty (2014).[3] Similar to our work, Piketty (2014) emphasizes the distribution of stock rather than flow quantities. This dissertation also critically discusses some of the major theoretical claims of Piketty (2014). First of all, the role of debt is only sparsely covered in his work. As emphasized in our theoretical work it is a major determinant for explaining net worth inequality.[4] Piketty (2014) emphasizes that a dynamic efficient economy (in which the interest on capital r exceeds the total growth rate g) goes along with an ever-increasing wealth inequality. As clarified in section 5.5.2, this only holds for the strong and unrealistic assumption of agents saving their total flow income and not consuming out of their stock level of wealth. In contrast, we emphasize other key determinants for wealth inequality. Wealth inequality is mainly driven by saving ratios and rates of returns that grow with the level of wealth.[5] We also offer some critical thoughts on the prediction of Piketty (2014) of a society dominated by inherited wealth.

As a policy conclusion Piketty (2014) advocates a global wealth tax in order to halt the growth of wealth inequality also revitalizing the public debate about redistribution by means of a system of taxes and subsidies. We discuss this issue in section 7 and show that

[3]It is interesting to point out that the book was already published in its original French version one year earlier without attracting major public reactions. The English translation (also referred to in this work) was able to get a massive press attraction in the USA as well as worldwide.

[4]A very recent empirical investigation by Saez and Zucman (2014) moreover affirms that the higher participation rates in real estate were more than offset by the increase of debt leading to a decline of middle-class wealth contributing to the increase of net worth inequality.

[5]As moreover discussed in the section 5.5.2 the latter factors are also documented by the empirical literature.

redistribution can have unintended consequences. In particular stronger redistribution may have an *arms' race* property[6] by raising the desired relative consumption level. In effect, the use of debt increases, also promoting financial instability in the form of Ponzi schemes.

The model presented in this work can be modified and extended in several directions. First of all, the model does neither feature labor market decisions (important for labor income inequality) nor markets for investment goods. The latter extension would allow to discuss the role of productive *capital* in the sense of Piketty (2014). Furthermore, there are important policy questions that could be addressed in extensions of the model. So far we only introduced taxation of the flow level labor income. As however, argued in section 7.2.1 taxes that affect stock levels are at the center of the academic and public debate. This not only includes pure wealth taxes but also taxes on the flow of capital income. Finally, there has been an increased interest in the effects of monetary policy on inequality[7] - which are far from being well understood. The model developed in this work may be a starting point to further investigate this question.

Both issues - inequality and financial instability - are a *bad* and call for actions of the policy maker, yet are also necessary byproducts of a capitalist society that has promoted growth in the standard of living in the last 100 years. This in particular becomes obvious when imagining the complete opposite case sometimes depicted as a social utopia in popular comments. A society that collects all individual income and redistributes it in an egalitarian manner - addressing inequality in the most radical manner - provides no incentive to display effort in the first place. Meanwhile, a society that would completely be absent of risk-taking financial institutions - also not being subject to financial instability - would also completely lack visionary entrepreneurial activities (Schumpeter, 1931). A balanced view trades off the opposing poles. As a result a market economy requires clear rules (e.g. banking regulation) and instruments (e.g. taxes and transfer systems) that are able to control financial stability and inequality without restricting the activity of visionary entrepreneurs. The role of economists is to provide (quantitative) answers to design these rules for the given trade-off. This not only involves testing measures ex-post in empirical frameworks, but also deriving ex-ante counter-factual predictions by means of theoretical models.

[6] The later issue is also brought up in more narrative work emphasizing the problems prevailing in positional race (Grolleau et al., 2012) posing the question of an satiation level in consumption (Skidelsky and Skidelsky, 2012).

[7] Recent leading contributions include Gorodnichenko et al. (2012) and Brunnermeier and Sannikov (2012).

A. Additional Proofs

A.1. Derivation of the Optimality Condition for Consumption

The optimal sequence of consumption is derived by maximizing the following target function:

$$\max_{C_t} \sum_{t=1}^{\infty} \beta^t U(C_t), \tag{A.1}$$

subject to the intertemporal budget constraint:

$$\sum_{t=1}^{\infty} R^t C_t = W_0 + \sum_{t=1}^{\infty} R^t Y_t. \tag{A.2}$$

The Lagrangian reads as follows:

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^t U(C_t) - \lambda \left(\sum_{t=1}^{\infty} R^t C_t - W_0 - \sum_{t=1}^{\infty} R^t Y_t \right), \tag{A.3}$$

with the following general first-order condition:

$$\frac{\partial \mathcal{L}}{\partial C_i} = \beta^i U'(C_i) - \lambda R^i \overset{!}{=} 0 \rightarrow \lambda = \frac{\beta^i}{R^i} U'(C_i). \tag{A.4}$$

Consider a period t and a later period $t+T$. From the equivalence of the shadow price λ, we have:

$$\frac{\beta^t}{R^t} U'(C_t) = \frac{\beta^{t+T}}{R^{t+T}} U'(C_{t+T}), \tag{A.5}$$

resulting in the standard optimality condition (the *Euler*-equation):

$$\frac{U'(C_t)}{U'(C_{t+T})} = \left(\frac{\beta}{R} \right)^T. \tag{A.6}$$

For the special case of two subsequent periods t and $t+1$ (implying $T = 1$) this results in:

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{\beta}{R} \tag{A.7}$$

A.2. Conditions for the *Fallacy of Composition* not to Hold

As already presented in the text, we assume an economy with n agents where a specific quantity (e.g. income Y or asset holding q) follows a given distribution $f(Y)$ with a minimum value Y_{min}, a maximum value Y_{max} [1] and a mean value $E(Y)$. In fact, the value of $f(Y)$ gives the number of agents within the population, who have a specific income level Y. Therefore, for the whole sample the following results holds:

$$\int_{Y_{min}}^{Y_{max}} df(Y) = \int_{Y_{min}}^{Y_{max}} f(Y)dY = [F(Y)]_{Y_{min}}^{Y_{max}} = n, \tag{B.8}$$

with $F(Y)$ being the compounded density function (CDF). The value $F(Y)$ informs about the number of agents receiving income not higher than a level Y. The latter result is also depicted in figure A.1. Moreover, the integral of the CDF - depicted as a grey area in figure A.1 - yields:

$$\int_{Y_{min}}^{Y_{max}} dF(Y) = \int_{Y_{min}}^{Y_{max}} F(Y)dY = \int_{Y_{min}}^{Y_{max}} \int_{Y_{min}}^{Y_{max}} f(Y)dYdY = E(Y)n, \tag{B.9}$$

indicating the total income or GDP of the economy as being the product of the mean income $E(Y)$ with the number of agents n. Note that the figure shows the special case of a distribution without skewness. In particular, for this symmetrical distribution the mean ($E(Y)$) equals the median ($\bar{Y}$) income ($E(Y) = \bar{Y}$). This, however, is not the case for distribution of income exhibiting negative skewness with a fat right tail. Nevertheless, by definition the integral - indicated as the grey area in figure A.1 - always equals the product of mean income $E(Y)$ and the number of agents n.

Other economic variables (e.g. consumption C or savings S) are functional relations of income. Note that it is usually assumed - and also the case in our model - that consumption of all agents follows the same functional form, such that the heterogeneous outcomes are solely owed to the different input variable, implying that there is no heterogeneity in preferences. If we e.g. want to compute the total consumption we have to solve the following integral:

$$C_{total} = \int_{Y_{min}}^{Y_{max}} C(Y)dF(Y) = \int_{Y_{min}}^{Y_{max}} C[F(Y)]dY. \tag{B.10}$$

Only for the case of a linear function:

$$C(Y) = c_y Y, \tag{B.11}$$

we are not subject to the *fallacy of composition*, implying:

$$C_{total} = \int_{Y_{min}}^{Y_{max}} C[F(Y)]dY = c_y \int_{Y_{min}}^{Y_{max}} F(Y)dy = c_y \cdot E(Y) \cdot N. \tag{B.12}$$

[1] To be more general and given the positive value nature of income, the upper and lower bounds can be replaced by 0 and ∞.

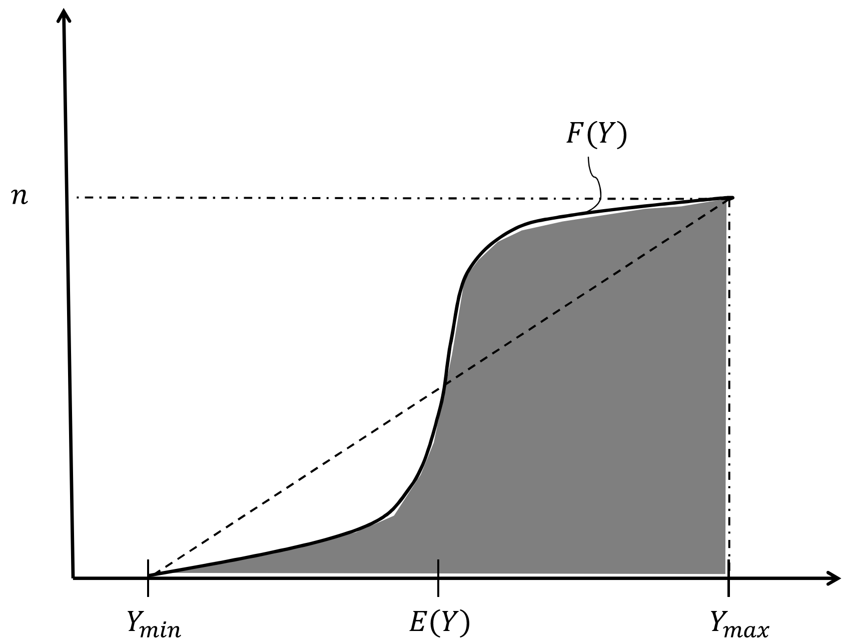

Figure A.1.: Graphical solution of the integral of the CDF

More generally, in this case, the aggregation operator (the integral) and the functional operator (in this case represented by square brackets) can be switched without affecting the results:

$$C\left[\int_{Y_{min}}^{Y_{max}} F(Y)dY\right] = \int_{Y_{min}}^{Y_{max}} C[F(Y)]dY = C_{total}. \tag{B.13}$$

Or put differently, for the linear case to describe the aggregate behavior it is sufficient to rely on the first statistical moment being the expectation operator ($E(\cdot)$). In particular, this is the case for an underlying homothetic utility function such as the CRRA function (Chipman, 1974).

It is important that this result is independent of the concrete distribution $f(Y)$ allowing us to derive aggregate results from aggregate variables - e.g. aggregate consumption from GDP.

Up to this point we chose a continuous formulation which is frequently used in theoretical papers aiming at deriving closed-form solutions. However, this is not very practical since when using data we deal with discrete observations. Therefore, for the rest of this section we use a discrete formulation. The last equation can be rewritten as follows in a discrete scenario:

$$C\left[\sum_{i=1}^{N} Y_i\right] = \sum_{i=1}^{N} C[Y_i]. \tag{B.14}$$

This also implies that the behavior of a single agent with mean income is representative for the whole economy. In fact, the aggregate behavior can be derived by multiplying the behavior of the representative agent with the total number of agents in the system:

$$\begin{aligned} C_{total} = N \cdot C[E(y)] = N \cdot C\left[\frac{1}{n}\sum_{i=1}^{N} Y_i\right] &= N \cdot \frac{c_y}{N}\sum_{i=1}^{N} Y_i \\ &= c_y Y \equiv \sum_{i=1}^{N} c_y Y_i = \sum_{i=1}^{N} C[Y_i]. \end{aligned} \tag{B.15}$$

Now let us assume a slight modification of the consumption function. While the consumption is still linear in its nature, we also allow for an offset:

$$C = \bar{c} + c_y Y. \tag{B.16}$$

As extensively argued in section 4.3, this can be thought of as reflecting a relative consumption motive or a subsistence level. Once again, the aggregate consumption can be deduced from the behavior of the representative agent multiplied by the number of agents N with $Y = \sum_{i=1}^{N} Y_i$:

$$C_{total} = N \cdot C\left[E(Y)\right] = N \cdot C\left[\frac{1}{N}\sum_{i=1}^{N} Y_i\right] = N\left(\frac{c_y}{N}Y + \bar{c}\right) = c_y Y + N \cdot \bar{c}. \tag{B.17}$$

However, in this case the derivation of total consumption from total income is not possible:

$$C[Y] = C\left[\sum_{i=1}^{N} Y_i\right] = c_y Y + \bar{c} < C_{total} = c_y Y + N \cdot \bar{c}, \tag{B.18}$$

as the conspicuous consumption effect emerges for all agents. Thus, this derivation underestimates total consumption.

A.3. The Transversality Condition in the Gordon-Growth Asset Pricing Model

In section 4.2 we presented a model with optimal consumption and showed that for any relation between the growth rate of consumption $r - \rho$ [2] and the growth rate of income

[2]Even more generally for a general CRRA utility function the following growth rate is reported $\frac{\dot{C}}{C} = \frac{r-\rho}{\gamma}$. We focus on the isoelastic case ($\gamma = 1$).

g a stable outcome emerges. For $r - \rho < g$ we have consumers who accumulate debt, whereas agents with $r - \rho > g$ accumulated assets or capital indefinitely without the transversality condition being violated. For the margin-case $r - \rho = g$, agents consume all disposable income in a *hand to mouth* manner and accumulate assets (for the case of inherited capital $-D_0 > 0$ and vice versa for inherited debt) growing at the same rate as the income $r - \rho = g$. In this section we show that this *borderline* case is the only feasible case for asset pricing. Any non-zero value of $r - \rho - g$ would lead to asset bubbles.

We consider an asset that provides an annual cash flow (i.e. a dividend d_t). Furthermore, we assume that the dividend-growth follows an exponential process $d_{t+n} = (1+g)^n d_t$. As usual, the future cash flow has to be discounted by means of an adequate interest rate r, implying the following long-run valuation equation:

$$p_t = \sum_{i=t}^{t+n} \frac{d_t(1+g)^i}{(1+r)^i}. \tag{C.19}$$

Following from the standard geometric series logic the long-run price is given by:

$$p_t = \sum_{i=t}^{t+\infty} \frac{d_t(1+g)^i}{(1+r)^i} = d_t \frac{1+g}{1+r} \frac{1 - \left(\frac{1+g}{1+r}\right)^\infty}{1 - \frac{1+g}{1+r}}, \tag{C.20}$$

which only converges for $r > g$ leading to:

$$p_t = \frac{d_t(1+g)}{r-g}, \tag{C.21}$$

which for small values of r and g equals:[3]

$$p_t = \frac{d_t}{r-g}. \tag{C.22}$$

Following Becker (2008), we can also write the total return of the asset ρ as the sum of the dividend yield and the capital gain:

$$\rho = \frac{p_{t+1} - p_t}{p_t} + \frac{d_t}{p_t}, \tag{C.23}$$

equivalent to:

$$p_{t+1} = (1+\rho)p_t - d_t. \tag{C.24}$$

Using the derivation of the previous stanza we know that the correct discount ratio is $(1+\rho) = \frac{1+r}{1+g} \leftrightarrow \rho = \frac{r-g}{1+g}$. For the subsequent period we therefore have:

$$p_{t+2} = \left(1 + \frac{r-g}{1+g}\right) p_{t+1} - d_t. \tag{C.25}$$

[3] Note that $p_t = \sum_{i=t+1}^{t+\infty} \frac{d_t(1+g)^i}{(1+r)^i} = \frac{d_t(1+r)}{r-g}$

We can reinsert the previous result, leading to:

$$p_{t+2} = \left(1 + \frac{r-g}{1+g}\right)^2 p_t - d_t \left[1 + \frac{r-g}{1+g}\right]. \tag{C.26}$$

With recursion, we have:

$$p_{t+n} = \left(1 + \frac{r-g}{1+g}\right)^n p_t - d_t \sum_{i=0}^{n-1} \left[1 + \frac{r-g}{1+g}\right]^i = \left(\frac{1+r}{1+g}\right)^{n-1} p_t - d_t \sum_{i=0}^{n} \left(\frac{1+r}{1+g}\right)^i, \tag{C.27}$$

which - using the rules for geometric series - can also be written as:

$$p_{t+n} = \underbrace{\left(\frac{1+r}{1+g}\right)^n \left[p_t - \frac{d_t(1+g)}{r-g}\right]}_{\text{bubble component}} + \underbrace{\frac{d_t(1+g)}{r-g}}_{\text{fundamental component}}. \tag{C.28}$$

In fact, the latter equation can be decomposed into two components: the *bubble* component and the *fundamental* component. Any (even small) initial mispricing ($p_t \neq \frac{d_t(1+g)}{r-g}$) will perpetuate itself and is accompanied by a *rational* bubble. In particular for the case of $p_t > \frac{d_t(1+g)}{r-g} \leftrightarrow \rho < r - g$ we will have a positive bubble (for all $n > 0$ $p_{t+n} > \frac{d_t(1+g)}{r-g}$). For the inverse case ($\rho > r - g$) a negative bubble will emerge (cf. figure A.2). The growth rate of the bubble is given by the correct discount factor ($1 + \rho = \frac{1+r}{1+g}$).

In fact, the transversality condition for the asset pricing model reads as follows:

$$\lim_{n\to\infty} p_{t+n} \left(\frac{1}{1+\rho}\right)^{n-1} \stackrel{!}{=} 0. \tag{C.29}$$

We can insert the result of the previous equation, leading to:

$$\lim_{n\to\infty} \left\{ \frac{1+r}{1+g} \left[p_t - \frac{d_t(1+g)}{r-g}\right] + \frac{d_t(1+g)}{r-g} \left(\frac{1+g}{1+r}\right)^{n-1} \right\} \stackrel{!}{=} 0. \tag{C.30}$$

The second term converges if the condition $r > g$ - already emphasized - is satisfied. The first term is zero only if there is no initial mispricing $p_t = \frac{d_t(1+g)}{r-g}$.

To summarize, the necessary conditions for a correct asset pricing are $r > g$ (implying dynamic efficiency) and the borderline case $\rho = r - g > 0$.

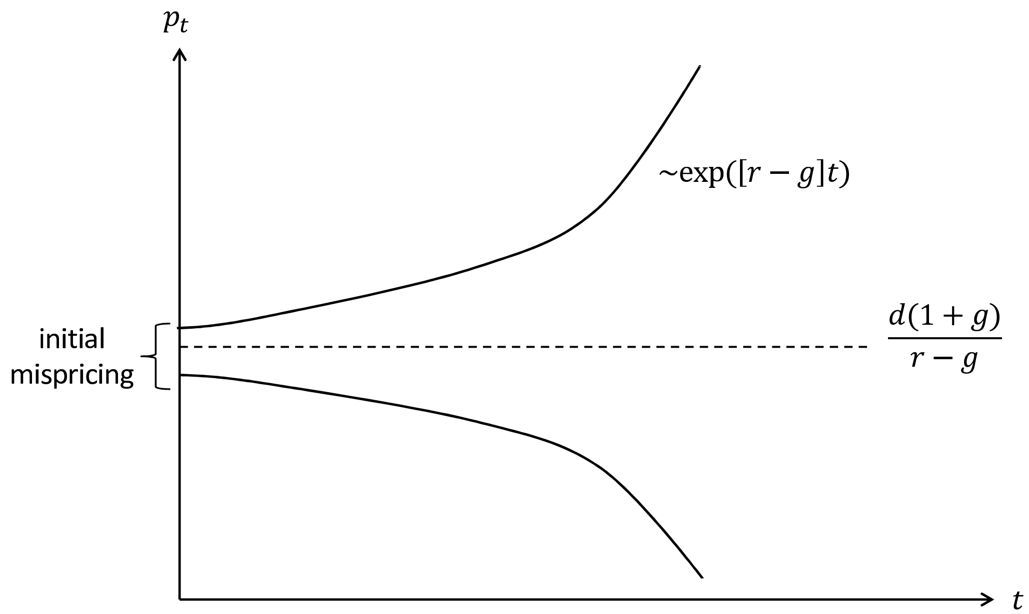

Figure A.2.: The evolution of a rational bubble out of an initial mispricing

A.4. Proof of the Relation between the Log-normal Distribution and the Gini-ratio

As presented in Aitchison and Brown (1957) the log-normal function and the Gini-ratio can be related as follows:

$$Gini = 2\Phi\left(\frac{\sigma_y}{\sqrt{2}}\right) - 1. \tag{D.31}$$

In this case Φ represents the Cumulative Probability Density Function (CPDF) of the normal distribution. This function can be related to the error function ($erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt$) as follows:

$$\Phi(x) = 0.5 + 0.5 \cdot erf(x/\sqrt{2}). \tag{D.32}$$

Inserting this result and using the first-order Taylor approximation for the error function ($erf(z) = \frac{2}{\sqrt{\pi}}(z + O(z^3))$) the following result can be derived:

$$Gini = 2[0.5(1 + erf(0.5\sigma_y))] - 1 = erf(0.5\sigma_y) = \frac{\sigma_y}{\sqrt{\pi}} \approx 0.5642 \cdot \sigma_y. \tag{D.33}$$

As a rule of thumb we can therefore say that for sufficiently low values of σ_y the Gini-ratio and the distribution parameter can be related with the factor 0.5.

A.5. Proof for the relation between Gini-ratio and the Growth Rate of Inequality

We assume that in a society with growing inequality in a dynamic scenario incomes are given as follows:

$$Y_{t+1} = (1 + g_{ineq} \log(Y_t))Y_t. \tag{E.34}$$

Log-linearizing the result using the first-order Taylor approximation for the log operator ($log(1 + z) \approx z$), the following result can be derived:

$$\log(Y_{t+1}) - \log(Y_t) \approx g_{ineq} \log(Y_t) \Rightarrow \frac{\log(Y_{t+1})}{\log(Y_t)} = 1 + g_{ineq}. \tag{E.35}$$

If a function is log-normally distributed ($Y \sim L$) its logs are normally distributed $\log(Y) \sim N$ (Evans et al., 2000) and the given ratio can be related to the ratio of two normally distributed function:

$$\frac{\log(Y_{t+1})}{\log(Y_t)} = 1 + g_{ineq} = \frac{N(\mu_{t+1}, \sigma_{y,t+1})}{N(\mu_t, \sigma_{y,t})}. \tag{E.36}$$

More specifically, the following equation relates normal and log-normal distributions (Evans et al., 2000):

$$\log(L : \log(\mu), \sigma_y) \sim (N : \mu, \sigma_y) \sim \mu + \sigma_y \cdot N(0, 1). \tag{E.37}$$

This helps us calculate the second moment:

$$\sqrt{Var(\log(Y_{t+1}))} = (1 + g_{ineq})\sqrt{Var(\log(Y_t))}. \tag{E.38}$$

As already presented in section 5.2, in a first-order approximation there is a proportional relationship $Gini \sim \sigma_y$, leading to the following result:

$$\frac{\sqrt{Var(\log(Y_{t+1}))}}{\sqrt{Var(\log(Y_t))}} = \frac{Gini_{t+1}}{Gini_t} = (1 + g_{ineq}). \tag{E.39}$$

Furthermore, for a multi period approach the following result holds:

$$\frac{\sqrt{Var(\log(Y_{t+T}))}}{\sqrt{Var(\log(Y_t))}} = \frac{Gini_{t+T}}{Gini_t} = (1 + g_{ineq})^T. \tag{E.40}$$

Log-transforming leads to the following equation:

$$\log(Gini_{t+T}) = \log(Gini_t) + T \cdot \log(1 + g_{ineq}). \tag{E.41}$$

For small values of g_{ineq} the following approximation holds:

$$Gini_{t+T} = \exp(g_{ineq} \cdot T) \cdot Gini_t. \tag{E.42}$$

For the even stronger assumption that the product $0 < g_{ineq} \cdot T << 1$ is small the following linear relation can be derived:

$$Gini_{t+T} = g_{ineq} \cdot T \cdot Gini_t. \tag{E.43}$$

A.6. Inequality of Wealth

In our model, wealth is defined as the current asset holding q_t evaluated at current market price P_t. For the sake of readability we refer to this in this proof as $A_{i,t} \equiv P_t q_{i,t}$. In general holdings of assets evolve as follows:

$$A_{i,t} = P_t(q_{i,t-1} + d_{i,t}), \tag{F.44}$$

where the demand for new assets is defined in equation 5.14. This leads to the following equation:

$$A_{i,t} = P_t(q_{i,t} + MPCD_t W_{i,t}). \tag{F.45}$$

For the sake of illustration, we firstly assume that demand for new assets does not depend on net worth $W_{i,t}$ but rather on total assets $A_{i,t}$ leading to the following recursive equation for total asset holdings:

$$\begin{aligned} A_{i,t} = P_t(q_{i,t} + MPCD_t A_{i,t-1}) &= P_t \left(\frac{A_{i,t-1}}{P_{t-1}} + MPCD_t A_{i,t-1} \right) \\ &= A_{i,t-1} \left(\frac{P_t}{P_{t-1}} + P_t MPCD_t \right). \end{aligned} \tag{F.46}$$

The first term in brackets captures the change in wealth due to reevaluation effects and the latter due to active buying or selling (for $MPCD_t > 0$ respectively $MPCD_t < 0$). The equation can be reformulated as follows:

$$A_{i,t} = A_{i,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau MPCD_\tau. \tag{F.47}$$

The evolution of assets captured in the product term is independent of any idiosyncratic effects. This can be easily verified if we compute the ratio of assets between two heterogeneous agents $i \neq j$ for a specific time t:

$$\frac{A_{i,t}}{A_{j,t}} = \frac{A_{i,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau MPCD_\tau}{A_{j,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau MPCD_\tau} = \frac{A_{i,0}}{A_{j,0}}. \tag{F.48}$$

The distribution of assets thereby only depends on the initial distribution. By assumption ($A_{i,0} = HY_{i,0}$) this is totally determined by the inequality in income (flow):

$$\frac{A_{i,t}}{A_{j,t}} = \frac{A_{i,0}}{A_{j,0}} = \frac{HY_{i,0}}{HY_{j,0}} = \frac{Y_{i,0}}{Y_{j,0}}. \tag{F.49}$$

Therefore, the asset distribution takes the same value as the distribution of income ($Gini(A) = Gini(Y)$). The speculative market, moreover, has the long-run property $E(P_t) = F = 1$. This is also accompanied by the effect that mean demand ($E(MPCD_t) = 0$) as well as long-run returns are zero ($E(1 + r_t) = E\left(\frac{P_t}{P_{t-1}}\right) = 1$). Therefore, long-run wealth is identical to initial wealth:

$$A_{i,t} = A_{i,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau MPCD_\tau = A_{i,0} \prod_{\tau=1}^{t} (1 + 0) = A_{i,0}. \tag{F.50}$$

However, up to this point we disregarded the effect of demand being a function of net worth and therefore also depending on individual debt $D_{i,t}$. Equation F.46 therefore changes as follows:

$$A_{i,t} = P_t(q_{i,t} + MPCD_t W_{i,t-1}) = P_t \left(\frac{A_{i,t-1}}{P_{t-1}} + MPCD_t (A_{i,t-1} - D_{i,t}) \right). \tag{F.51}$$

Now, we have to distinguish between two cases. In the long-run low and medium-income households only satisfy minimum equity requirements as given by the collateral constraint (equation 5.17). This implies $W_{i,t} = A_{i,t} - D_{i,t} = mA_{i,t}$, resulting in the following long-run equation:

$$A_{i,t} = A_{i,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau m MPCD_\tau. \tag{F.52}$$

Meanwhile, high-income households are net lenders, implying that $D_{i,t} < 0$ and $W_{i,t} > A_{i,t}$. As suggested in figure 6.7 the ration between claims and income as well as the ratio of income and net worth are constant. Without further specifying we assume that $0 < \frac{A_{i,t}}{W_{i,t}} = \frac{W_{i,t} + D_{i,t}}{W_{i,t}} = k < 1$.[4] The evolution of their wealth therefore is described as follows:

$$A_{i,t} = A_{i,0} \prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + \frac{P_\tau}{k} MPCD_\tau. \tag{F.53}$$

This helps us relate the wealth of low-income (index (l) to a high-income households (index h):

$$\frac{A_{h,t}}{A_{l,t}} = \frac{A_{h,0}}{A_{l,0}} \frac{\prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + \frac{P_\tau}{k} MPCD_\tau}{\prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau m MPCD_\tau} = \frac{Y_{h,0}}{Y_{l,0}} \frac{\prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + \frac{P_\tau}{k} MPCD_\tau}{\prod_{\tau=1}^{t} \frac{P_\tau}{P_{\tau-1}} + P_\tau m MPCD_\tau}. \tag{F.54}$$

[4] Formally - and as shown in section 5.5.1 - the value is given by $\frac{1}{k} = 1 + \frac{K}{HY} = 1 + \frac{k_y}{H} + \frac{k_0}{HY}$. For the given calibration and the a long-run rate of interest of $r \approx 0.5\%$, it shows little reaction to changes in labor income Y. Thus, the value is given by $k \approx 0.66$.

For no demand $MPCD_t = 0$ the ratio between low and high-income households once again is entirely given by the income distribution ($Gini(A) = Gini(Y)$). This is also the case in the long-run, since - as already shown - there we have $E(MPCD_t) = 0$. Let us now go to the short-run. Let us assume that $\frac{A_{h,t-1}}{A_{l,t-1}} = \frac{Y_{l,0}}{Y_{h,0}}$:

$$\frac{A_{h,t}}{A_{l,t}} = \frac{A_{h,t-1}}{A_{l,t-1}} \frac{\frac{P_t}{P_{t-1}} + \frac{P_t}{k} MPCD_t}{\frac{P_t}{P_{t-1}} + P_t m MPCD_t} = \frac{Y_{l,0}}{Y_{h,0}} \frac{\frac{P_t}{P_{t-1}} + \frac{P_t}{k} MPCD_t}{\frac{P_t}{P_{t-1}} + P_t m MPCD_t}. \tag{F.55}$$

If trading rationale implies going long $MPCD_t > 0$ inequality increases, as $\frac{1}{k} > 1 > m$. Going long also implies increasing prices. Therefore, in upswings inequality increases. The reverse argument holds for short positions and asset price decreases. Thus, inequality of wealth and asset prices are positively correlated as depicted in figure 6.3.

A.7. Optimal Distribution of Economic Resources

This section shows a utilitarian approach for determining the *optimal* distribution of resources. The social planner's problem reads as follows:

$$\begin{aligned} \max_{x_1, x_2, \cdots, x_N} \sum_{i=1}^{N} U(x_i) = \max_{x_1, x_2, \cdots, x_N} \sum_{i=1}^{N} \left[\frac{1}{1-\gamma} x_i^{1-\gamma} \right] \\ s.t. \quad x = \sum_{i=1}^{N} x_i. \end{aligned} \tag{G.56}$$

Social welfare (the objective function) is defined as the sum of individual utility. The utility function itself is assumed to be of the CRRA type. The social planner faces the constraint of a finite amount of resources x.

The problem can be solved using the following Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{1-\gamma} x_{i,t}^{1-\gamma} - \lambda \left(x_t - \sum_{i=1}^{N} x_{i,t} \right), \tag{G.57}$$

with the following general first-order condition:

$$\frac{\partial \mathcal{L}}{\partial x_i} = x_i^{-\gamma} - \lambda = 0 \Rightarrow \lambda = x_i^{-\gamma}. \tag{G.58}$$

Using the shadow price identity the following general condition for $i \neq j$ holds:

$$x_i^{-\gamma} = x_j^{-\gamma}. \tag{G.59}$$

For any γ, this condition only holds for:

$$x_i = x_j, \tag{G.60}$$

implying total equality. We also have to control the second-order condition to ensure a local maximum:

$$\frac{\partial^2 \mathcal{L}}{\partial x_i^2} = -\gamma x_i^{-\gamma-1} \overset{!}{<} 0, \tag{G.61}$$

which only holds for $\gamma > 0$ since $x_i > 0$. Therefore, for risk-averse agents ($\gamma > 0$), the optimal distribution is total equality. For the inverse case ($\gamma < 0$), the first-order condition constitutes a local minimum. A maximum, however, is reached when giving one agent i all resources ($x_i = x$) and all other agents nothing ($j \neq i \; x_j = 0$), thus implying total inequality. For the general case, the social planner is indifferent to which agent to distribute all resources.

The results can be confirmed by inserting the optimal result into the target function. For risk aversion ($\gamma > 0$, index RA) the target function for the optimal distribution ($x_i = \frac{x}{N}$) yields:

$$U_{\vec{x}^*} = \frac{1}{1-\gamma} \sum_{i=1}^{N} \left(\frac{x}{N}\right)^{1-\gamma} = \frac{N}{1-\gamma} \frac{x^{1-\gamma}}{N^{1-\gamma}} \equiv U_{RA}. \tag{G.62}$$

For the case of risk loving individuals $\gamma < 0$ for the optimal case of total inequality the utility function takes the following value:

$$U_{\vec{x}^*} = \frac{1}{1-\gamma} x^{1-\gamma} \equiv U_{RL}. \tag{G.63}$$

The ratio between the two is given as follows:

$$\frac{U_{RA}}{U_{RL}} = \frac{N}{N^{1-\gamma}} = N^{\gamma}. \tag{G.64}$$

For all $N > 1$ and risk averse agents ($\gamma > 0$) this ratio is larger than one ($\frac{U_{RA}}{U_{RL}} > 1$), implying that $U_{RA} > U_{RL}$ and vice versa for risk loving individuals ($\gamma < 0$).

For the special case of a linear utility function with risk-neutrality ($\gamma = 0$), the total objective function reads as follows:

$$U_{tot,t} = \sum_{i=1}^{N} U_i(x_{i,t}) = \sum_{i=1}^{N} x_{i,t} = x_t. \tag{G.65}$$

In this case, distribution does not matter. Total utility can only be increased by increasing the total resources available x_t, thus implying a society with growing resources. The case of risk neutrality also implies $U_{RA} = U_{RL}$.

We, however, take a standard economic approach in summing up all individuals - therefore trading-off the lower outcome of one individual against the higher outcome of another. In this case, an egalitarian society generally is not welfare optimal. This, however, is the case due to the *leaky bucket problem* (Okun, 1975). Rather than designing a society from scratch, redistribution involves an inefficient government system where total subsidies are lower than total taxes. It is feasible to only consider the two agents at the edges of the distribution - namely the very poorest agent (index p) and the very richest agent (index r) - with a market income Y_i.[5] The government system is described by a leakiness $0 < \Phi < 1$, where low values indicate higher leakage. In fact, the total income after taxes and subsidies Z_i is lower than the sum of the market income:

$$\sum_{i=1}^{2} Z_i = Z_r + Z_p = Y_r - t + Y_p + \Phi \cdot t = \sum_{i=1}^{2} Y_i + (\Phi - 1)t < \sum_{i=1}^{2} Y_i, \tag{G.66}$$

where t represents the taxes. The total loss is higher for low values of Φ indicating stronger leakage. Let us optimize utility of total after tax income:

$$max_t \sum_{i=1}^{2} U(Z_i), \tag{G.67}$$

with a standard utility function of the Constant Relative Risk Aversion (CRRA) type ($U(\bullet) = \frac{1}{1-\gamma}(\bullet)^{1-\gamma}$). The first order condition with respect to t yields:

$$\frac{(Y_r - t)^{-\gamma}}{(Y_p + \Phi t)^{-\gamma}} = \Phi = \frac{(Z_r)^{-\gamma}}{(Z_p)^{-\gamma}} = \left(\frac{Z_p}{Z_r}\right)^{\gamma} \leftrightarrow \Phi^{\frac{1}{\gamma}} = \frac{Z_p}{Z_r}. \tag{G.68}$$

For the case of no leakage ($\Phi = 1$), the result breaks down to the very result presented in the beginning of this section, where total equality is optimal. Hence, the only obstacle hindering a egalitarian society being welfare optimal is the inefficiency of the government system aimed at conducting the redistribution. As furthermore shown in Arrow (1973), for extreme risk aversion ($\gamma \rightarrow \infty$) total equality is optimal - therefore mirroring the Rawlsian case - for any Φ, implying an egalitarian distribution. On the other hand, it should also not be forgotten that any redistribution is Pareto inefficient since at least one agent (the *rich* one) ends up worse off.

A.8. The Tax Effect on Labor Supply

The presence of adverse labor supply effects is the key argument put forward against taxation of labor income. This is not considered in our model, as we assume a constant and thereby inelastic supply of labor. In the design of our tax system, we only address the problem of - as we call it - *perverse* labor supply effects - implying that higher pre-tax income should not result in lower post-tax income.

Private Labor Supply Decision

Usually, the private labor supply decision is presented as the following simple optimization problem of private households.[6] Households try to maximize a utility function that

[5] A more general approach with N agents is e.g. presented in Stark et al. (2014).

[6] Note that we only discuss the case of the intensive margin with h being a continuous variable. For the extensive case (h being a binary), which is very important for low-income households, the agents decide whether to work at all.

increases with the level of consumption c and decreases with the amount of hours worked h:

$$U = u(c) + v(h), \tag{H.69}$$

with the properties $v_h < 0$ and $v_{hh} < 0$.[7] It is feasible to assume the following specification:

$$U = \frac{c^{1-\gamma}}{1-\gamma} - \alpha \frac{h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, \tag{H.70}$$

subject to the following budget constraint:

$$z(h) = c, \tag{H.71}$$

where z is the disposable income as a function of hours worked. In the simple framework with an hourly rate of w and linear tax system without demogrant this can be rewritten as:

$$z(h) = w(1-\tau)h = c. \tag{H.72}$$

The solution of the problem leads to the following first order condition (FOC):

$$c^{-\gamma} w(1-\tau) = \alpha h^{\frac{1}{\nu}}. \tag{H.73}$$

In fact, the left side of the equation captures the marginal benefit of working whereas the right side summarizes the marginal disutility out of work resulting from a preference for leisure. In a flat tax system (with or without demogrant) the marginal tax rate is constant. In a progressive tax-system with increasing marginal tax rates - as discussed in section 7.2.3 - marginal taxes increase with income and with hours worked, implying that the marginal benefits of working decrease with hours worked. The previous equation also allows us to derive a closed-form solution of the labor supply function h:

$$h = \left(\frac{w(1-\tau)}{\alpha c^{\gamma}}\right)^{\nu}. \tag{H.74}$$

The variable ν is commonly referred to as the Frisch-elasticity of labor supply[8] (cf. e.g. Heer and Maussner (2005)). The elasticity quantifies the substitution effect of labor in favor of leisure as a response to a tax change:

$$\frac{dh/h}{d(1-\tau)/(1-\tau)} = \nu. \tag{H.75}$$

A more economic interpretation of ν is that it captures the amount of disutility stemming from a higher amount of work, where low values of ν signify that agents strongly despise work. In equilibrium, agents with lower values of ν therefore exhibit lower values of labor supply.[9] In our model, we implicitly assume $\nu = 0$. Our model could be considered a

[7]The utility of consumption function has the usual properties $u_c > 0$ and $u_{cc} < 0$ and is assumed to be of the CRRA type.

[8]The name comes from Ragnar Frisch, the laureate of the very first Nobel prize in economics.

[9]The same result also applies to higher values of α.

very long-run model, since as empirically confirmed, the long-run labor supply is very inelastic. The latter could be explained by the relative consumption effect, where agents try to keep up with the Joneses and thereby do not decrease labor-supply despite higher standards of living (Stiglitz, 2008).

Using the First Order Condition (FOC) and the budget constraint yields the following optimal level of consumption c^* and hours h^* :

$$h^* = \alpha^{-\frac{\nu}{\gamma\nu+1}} (w[1-\tau])^{\frac{\nu(1-\gamma)}{1+\gamma\nu}}, \tag{H.76}$$

and

$$c^* = \alpha^{-\frac{\nu}{\gamma\nu+1}} (w[1-\tau])^{\frac{\nu+1}{1+\gamma\nu}}. \tag{H.77}$$

For the sake of simplicity, we consider the isoelastic case ($\gamma = 1$) yielding:

$$h^* = \alpha^{-\frac{\nu}{\nu+1}}, \tag{H.78}$$

and

$$c^* = \alpha^{-\frac{\nu}{\nu+1}} w(1-\tau), \tag{H.79}$$

For this case, the following utility prevails:

$$U^*(c^*, h^*) = \ln\left((1-\tau)w\alpha^{-\frac{\nu}{\nu+1}}\right) - \frac{1}{1+\frac{1}{\nu}}. \tag{H.80}$$

In this case, any tax increase decreases the utility of the representative individual:

$$\frac{\partial U^*}{\partial \tau} = \frac{1}{\tau - 1} < 0, \tag{H.81}$$

for all $\tau < 1$. This is the case, since higher taxes reduce the disposable income available for consumption purposes, thereby lowering the utility. However, in a heterogeneous agent framework the social welfare function taking into account an equity concern increases.

Tax Revenues and the Optimal Tax Rate

An argument frequently put forward in favor of higher taxes is that they increase the revenue of the government G available for redistribution. The latter, however, is also limited by the labor supply. In fact, revenue is an inverse u-shaped function of the tax rate, also often referred to as the Laffer-curve (see e.g. Piketty and Saez (2013b)):

$$G = \tau h(\tau). \tag{H.82}$$

The local maximum of tax revenue using the Frisch-elasticity (cf. equation H.75) can be derived as follows:

$$\frac{dG}{d\tau} = h + \tau\frac{dh}{d\tau} = h - \tau\frac{dh}{d(1-\tau)} \stackrel{!}{=} 0 \leftrightarrow \tau = \frac{d(1-\tau)}{dh}h = \frac{1-\tau}{\nu} \leftrightarrow \tau^* = \frac{1}{1+\nu}. \tag{H.83}$$

This implies that the maximum feasible tax rate increases for low levels of the Frisch-elasticity. For the special case of $\nu = 0$ it is equal to $\tau^* = 100\%$.

Note that this simple analysis does not take into account the effect of tax avoidance. In particular, agents shift within income categories (from labor to capital income), or even migrate to a different jurisdiction. Piketty and Saez (2013b) therefore predict that the encouragement of labor mobility in the European Union will lead to tax competition, resulting in lower taxes on labor. It is furthermore important to point out that especially very high-income agents have a high mobility, constraining the government in setting their top income tax rates.

Piketty and Saez (2013b) derive the optimal tax level also taking into account the Social Welfare Function (SWF). In this case, the optimal tax rate τ^* is given as follows:

$$\tau^* = \frac{1-\bar{g}}{1-\bar{g}+\nu}, \tag{H.84}$$

with $\bar{g}$ being the average welfare weight of the agents. For sake of illustration, we can consider the two most extreme cases. In the *Libertarian* case[10], all agents have the same welfare weight making $g_i = \bar{g} = 1$, leading to an optimal tax rate of zero $\tau^* = 0$. In the other extreme case of a *Rawlsian* case following the *maxi-min-principle*, the social planner only adds a weight to the agent with the very lowest earnings and zero weight to the other agents, making $\bar{g} = 0$. In this case, the optimal tax rate is equal to the tax rate maximizing the income of the government: $\tau = \frac{1}{1+\nu}$.

As put forward in Piketty and Saez (2013b), if tax rates are decided on in a democratic political process the welfare weight $\bar{g}$ is determined by the median voter. In particular it is given by the ratio of median income $\bar{Y}$ to average income $E(Y)$:

$$\tau^* = \frac{1-\bar{Y}/E(Y)}{1-\bar{Y}/E(Y)+\nu}. \tag{H.85}$$

Note that if income is given by a log-normal distribution we can eventually provide a closed-form solution:

$$\tau^* = \frac{1-\bar{Y}/E(Y)}{1-\bar{Y}/E(Y)+\nu} = \frac{1-\exp(-0.5\sigma_y^2)}{1-\exp(-0.5\sigma_y^2)+\nu}. \tag{H.86}$$

In this case the tax rate τ^* unambiguously increases with the level of inequality controlled by the parameter σ_y. Assume a standard value for the Frisch-elasticity $\nu = 0.25$ leading to the following concrete values for two types of economies:

$$\tau^*\left(Gini \approx 0.35, \sigma_y = \frac{2}{3}\right) = 0.44 > \tau^*\left(Gini \approx 0.28, \sigma_y = 0.5\right) = 0.32. \tag{H.87}$$

Note that both values are way above average tax rates in existing tax systems as discussed in section 7.1.

[10] This implicitly assumes that the marginal marginal utility of consumption is not decreasing $u_{cc} = 0$.

For the case of inelastic labor supply ($\nu = 0$), the welfare weights do not matter and the optimal tax rate is always maximum $\tau^* = \frac{1-\bar{g}}{1-\bar{g}} = 100\%$, also implying total equality. As we implicitly assume $\nu = 0$ we can simply disregard the social welfare function.

Diamond and Saez (2011) provide an extension that takes into account income heterogeneity. In particular, they consider the fat tail of top incomes. If this fat tail follows a Pareto distribution with a density function of the type $f(Y) \sim Y^{-(a+1)}$ with $a > 1$, the following optimal tax formula can be derived:

$$\tau^* = \frac{1-\bar{g}}{1-\bar{g}+a\cdot\nu}. \tag{H.88}$$

Fatter tails (i.e. lower values of a) lead to higher optimal tax rates. Diamond and Saez (2011) derive top income tax rates as high as 73% for the very top income. Note that standard distribution functions - e.g. the log-normal distribution of income assumed in our work - are characterized by a finite tail, implying $a = \infty$. In this case the top income tax rate would eventually be zero.

Labor Supply under Different Tax Systems and Form of Heterogeneity

In the following, we compare the labor supply under a flat and a progressive tax system. We further distinguish between heterogeneity in tastes and abilities. We start with the simple case of flat tax rate system. We argue in the general framework provided at the beginning of this section. Following Piketty and Saez (2013b), we assume that substitution effects of consumption prevail the income effects by setting $\gamma = 0$, also simplifying the analysis. In this case, tax increases also increase the amount of working hours supplied.

For the linear tax system the marginal benefit of unit of labor (MB) is independent of the number of hours provided:

$$MB = \frac{\partial z}{\partial h} = w_i(1-\tau). \tag{H.89}$$

Consider two agents with either a low or a high ability (index l respectively h). Higher ability implies that the marginal product of one unit of labor for a high ability agent exceeds the the marginal product of the low ability worker. In a competitive setting, the firm pays workers according to their marginal productivity, implying $w_h > w_l$. The ratio of post-tax income - being the product of supplied hours of work and individual wage - is therefore given as follows:

$$\frac{z_h}{z_l} = \frac{w_h(1-\tau)h_h}{w_l(1-\tau)h_l} = \frac{w_h}{w_l}\frac{\alpha^{-\nu}([1-\tau]w_h)^{\nu}}{\alpha^{-\nu}([1-\tau]w_l)^{\nu}} = \left(\frac{w_h}{w_l}\right)^{1+\nu} > 1. \tag{H.90}$$

In this case, the (post-tax) income inequality is higher than the wage inequality ($\frac{z_h}{z_l} > \frac{w_h}{w_l}$). This effect is furthermore emphasized the more elastic the labor supply is (high values of ν). For the extreme case of an inelastic labor supply ($\nu = 0$) income wage inequality is identical to wage inequality as all agents supply the same amount of labor.

We can compare this case to the case where agents - rather than having different fundamental abilities - only have different tastes for leisure. In particular, one agent assigns a higher disutility to labor than the other agent ($\alpha_h > \alpha_l$).[11] Or put more boldly, the agent with the higher value of α is lazier. The distribution of post-tax income is given as follows:

$$\frac{z_h}{z_l} = \frac{w(1-\tau)h_h}{w(1-\tau)h_l} = \frac{w\alpha_h^{-\nu}([1-\tau]w)^\nu}{w\alpha_l^{-\nu}([1-\tau]w)^\nu} = \left(\frac{\alpha_l}{\alpha_h}\right)^\nu < 1. \tag{H.91}$$

First of all, we have the inverse relationship as the worker with the higher value of $\alpha_h > \alpha_l$ works less and therefore also has a lower income due to identical hourly wages. It is also notable that wage inequality reacts less to preference as opposed to heterogeneity of ability. However, there is a reaction, implying that the tax system also impacts on preferences rather than only on abilities, as it supposed to do (cf. the discussion in section 7.2.1). The most important result of these calculations, however, is that the tax system itself - being a flat tax without demogrant - does not impact on wage inequality, as the results are in both cases - heterogeneous abilities and tastes - independent of the level of τ.

We can now compare this result with the result of a progressive tax system as modeled in section 7.2.3. As the tax system is given by:

$$z = (w_i h_i)^{1-\tau} Y_{TF}^\tau, \tag{H.92}$$

the marginal benefit of one hour of work is given by:

$$MB = \frac{\partial z}{\partial h} = w_i (w_i h_i)^{-\tau} (1-\tau) Y_{TF}^\tau, \tag{H.93}$$

this time also depending on the hours worked. In particular, the marginal benefit of a unit of labor decreases with the provided hours:

$$\frac{\partial MB}{\partial h} = \frac{\partial^2 z}{\partial h^2} = w_i^2 (w_i h_i)^{-\tau-1} (1-\tau)(-\tau) Y_{TF}^\tau < 0. \tag{H.94}$$

The general labor supply function in this case is given as:

$$h_i = \left(\frac{w_i^{1-\tau}(1-\tau)Y_{TF}^\tau}{\alpha}\right)^{\frac{\nu}{\tau\nu+1}}. \tag{H.95}$$

Using this result, we can compute the ratio of labor income for agents with heterogeneous abilities ($w_h > w_l$):

$$\frac{z_h}{z_l} = \frac{(w_h h_h)^{1-\tau}}{(w_h h_h)^{1-\tau}} = \left(\frac{w_h}{w_l}\frac{w_h^{(1-\tau)\frac{\nu}{\tau\nu+1}}}{w_h^{(1-\tau)\frac{\nu}{\tau\nu+1}}}\right)^{1-\tau} = \left(\frac{w_h}{w_l}\right)^{(1-\tau)\frac{1+\nu}{\tau\nu+1}}. \tag{H.96}$$

[11] This analysis can also be redone for different values of labor elasticity $\nu_h > \nu_l$, where agents with higher elasticity supply less labor and thereby have a lower labor income. The results, however, are qualitatively identical to the results for the case of heterogeneous α_i.

First of all, and in contrast to the result for the linear income tax system presented in equation H.90, the inequality is smaller. This is the case since:

$$(1-\tau)\frac{1+\nu}{1+\tau\nu} < 1+\nu \rightarrow 1-\tau < 1+\tau\nu \rightarrow \tau(1+\nu) > 0, \tag{H.97}$$

for all positive tax rates $\tau > 0$. Moreover, the inequality decreases with the level of progressivity τ. For the extreme case of $\tau = 1$, all agents are equal.

We can once again compute the case for heterogeneous tastes ($\alpha_h > \alpha_l$), leading to:

$$\frac{z_h}{z_l} = \left(\frac{w_h h_h}{w_h h_h}\right)^{1-\tau} = \left(\frac{\alpha_l}{\alpha_h}\right)^{\frac{\nu}{\nu\tau+1}(1-\tau)} = \left(\frac{\alpha_l}{\alpha_h}\right)^{\frac{\nu-\tau\nu}{\tau\nu+1}} < 1. \tag{H.98}$$

We compare this result to the case of the linear tax presented in equation H.91. In particular, we compare the coefficients:

$$\nu > \frac{\nu-\tau\nu}{1+\tau\nu} \rightarrow 1+\tau\nu > 1-\tau \rightarrow \tau(\nu+1) > 0, \tag{H.99}$$

showing that the inequality in the progressive system is once again lower than in the linear system. This holds for any progressive system $\tau > 0$. Hence, once again the progressive tax system leads to a compression of inequality. Also note that the system reacts less to inequality in tastes as compared to abilities. This becomes clear when comparing the coefficients:

$$(1-\tau)\frac{1+\nu}{1+\tau\nu} > \frac{\nu-\tau\nu}{1+\tau\nu} \rightarrow 1+\nu > \nu. \tag{H.100}$$

The result can also be presented graphically (cf. figure A.3). As the progressive system is accompanied by a negatively slopped curve of marginal benefits (in contrast to the zero slope for the case of a linear tax system), the inequality in hours is compressed. Graphically, the total income can be presented by the rectangle given the point of origin of the coordinate system and the intersection of the labor supply and the marginal benefit curve. This result in particular is notable since it does not rely on the government redistributing income between agents, but results solely from the effect on labor supply. As a progressive strongly punishes strong labor supply, there is a compression of labor supply and thereby of income inequality between agents. It is important to point out that this is the case where the leaky bucket - as elaborated in appendix A.7 - is at its maximum $\Phi = 0$ implying that all tax revenues are lost within an inefficient government system. It is, however, also important to note that this is only a short-run analysis.

As put forward by Heathcote et al. (2014) the long-run effect can eventually be diametrical. First of all there is a general equilibrium effect of the supply of labor of heterogeneous underlying ability. Since the supply of high-skilled labor decreases whereas the supply of low skilled labor increases, firms can react by increasing the skill premium. This is the case of *endogenous wages*. Moreover, in the very long-run, the progressive tax system also discourages the accumulation of human capital. This leads to a fall in wages for agents with little human capital and generates a large wage premium for the

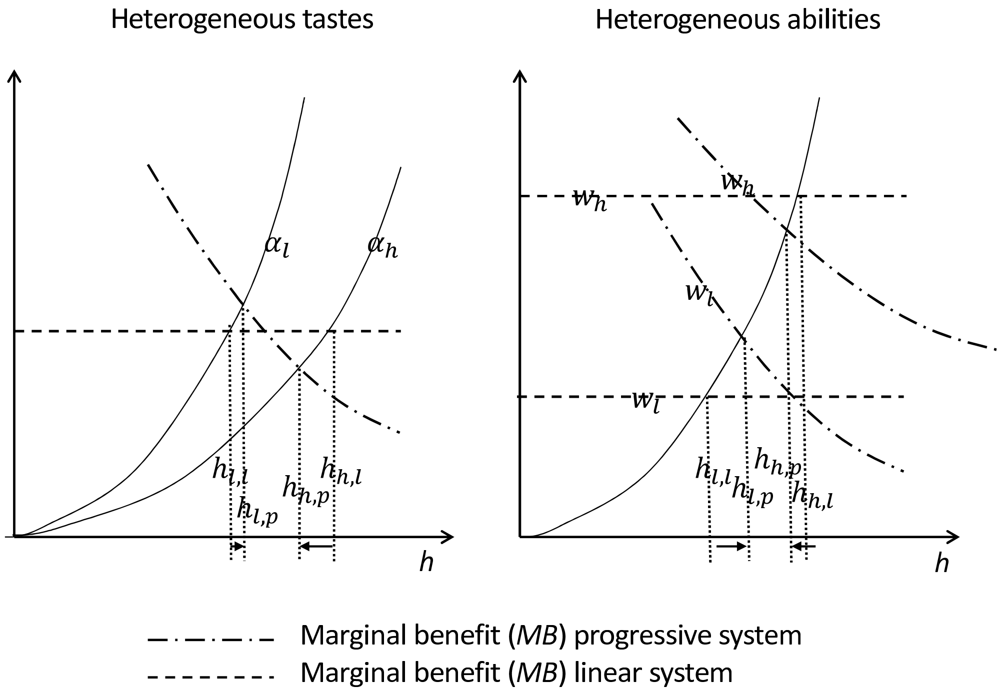

Figure A.3.: The effect of progressive taxation in contrast to flat taxes on inequality resulting out of heterogeneous tastes or abilities

scarce part of the population that holds high human capital contributing to increased inequality in the long-run. This is the case of *endogenous human capital*. Heathcote et al. (2014) eventually compute an optimum level of taxation and show that it is sensitive to the planning horizon of the social planner. If the social planner has a short horizon it favors more redistribution in order to combat consumption inequality, whereas a planner with a long horizon chooses lower redistribution, accounting for the long-run negative effects of redistribution on investment in skills.

A.9. The Aggregate Demand Effect of Redistribution

As presented in section 7.2.4, we identify a Post-Keynesian effect of aggregate demand, implying that a redistribution of labor income from high-income to low-income households boosts aggregate demand. This, however, is only the case if the consumption as a function of labor income is concave ($C''(Y) < 0$). The opposite, lower aggregate demand for for a redistribution from rich to poor, holds true for a convex consumption function ($C''(Y) > 0$). Furthermore, for a linear consumption function ($C''(Y) = 0$) redistributive policy does not impact the overall consumption at all.[12]

The statement can be proofed very easily following a similar rationale as for the case where the optimal distribution of resources from a utility-maximizing view presented in section A.7. Consider a simple consumption function of the type:

$$C_i(Y_i) = Y_i^\theta, \tag{I.101}$$

where for the case of $\theta > 1$ we have a convex consumption function and for $0 < \theta < 1$ we have a concave consumption function. We maximize aggregate consumption defined as:

$$C(Y) = \sum_{i=1}^{N} Y_i^\theta. \tag{I.102}$$

The constraint is the total amount of labor income available for redistribution:

$$Y = \sum_{i=1}^{N} Y_i. \tag{I.103}$$

The first order condition requires:

$$Y_i^{\theta-1} = Y_j^{\theta-1}, \tag{I.104}$$

for any $i \neq j$, which only holds for:

$$Y_i = Y_j, \tag{I.105}$$

indicating a total egalitarian society. We, however, once again have to control for the second order condition requiring:

$$\theta(\theta-1)Y_i^{\theta-2} \overset{!}{<} 0, \tag{I.106}$$

for a local maximum. Since we only have positive levels of income $Y_i > 0$ this condition is identical to:

$$\theta < 1, \tag{I.107}$$

requiring a concave consumption function. Thereby, for a concave consumption function the optimal distribution of labor income to ensure maximum consumption would be total equality. For total equality the target function takes the following value:

$$C_{eq}(Y) = \sum_{i=1}^{N} Y_i^\theta = \sum_{i=1}^{N} \left(\frac{Y}{N}\right)^\theta = N^{1-\theta} Y^\theta. \tag{I.108}$$

[12] It is also interesting to point that the curvature of the consumption function also implies a relation between inequality of consumption and inequality of labor income being: $Ineq(C) > Ineq(Y)$ ($C''(Y) > 0$), $Ineq(C) = Ineq(Y)$ ($C'' = 0$), and $Ineq(C) < Ineq(Y)$ ($C'' < 0$).

The optimal distribution for a convex consumption function would be total inequality giving all income to a single agent, implying a value of the target function of:

$$C_{ineq}(Y) = \sum_{i=1}^{N} Y_i^{\theta} = Y^{\theta}. \tag{I.109}$$

Similar to section A.7, we can compute a ratio between the two target functions:

$$\frac{C_{eq}}{C_{ineq}} = N^{1-\theta}. \tag{I.110}$$

For $0 < \theta < 1$ going along with concave consumption function, this ratio exceeds 1 implying that total equality is optimal. The converse holds true for $\theta > 1$, implying a convex consumption function. Note that for a simple consumption linear consumption function ($\theta = 1$) distribution does not matter for aggregate consumption.

So far we assumed a very abstract consumption function of the type $C_i(Y_i) = Y_i^{\theta}$. Now we classify the function prevailing the one prevailing in our model, in the category of convex or concave. In our model, we assume a special version of the consumption function of the linear type ($\varepsilon = 1$).[13] The linearity in general would lead to neither a positive nor a negative redistributive effect on aggregate consumption. As, however, already suggested in figure 6.7 the consumption exhibits a convex shape. More formally, the consumption function can be written as follows:

$$C = \bar{c} + c_y(Y - rD) + c_w(HY - D). \tag{I.111}$$

The first partial derivative with respect to Y is given as follows:

$$\frac{\partial C}{\partial Y} = c_y - \frac{\partial D}{\partial Y} \cdot c_y r + c_w \left(H - \frac{\partial D}{\partial Y} \right). \tag{I.112}$$

For a concave function, the second-order partial derivative must satisfy the following condition:

$$\frac{\partial^2 C}{\partial Y^2} = -\frac{\partial^2 D}{\partial Y^2}(c_y r + c_w) = C''(Y) \overset{!}{<} 0. \tag{I.113}$$

In section 5.5.1, we derived a simple linear relation between capital $K = -D$ and labor income Y which would imply that $-\frac{\partial^2 D}{\partial Y^2} = 0$ and thereby $C''(Y) = 0$. The simple closed-form computation in section 5.5.1, however, disregarded the role of collateral constraints.

As presented in figure 6.7 the relation between debt D and labor income Y is an inverse u-shape, making the condition $\frac{\partial^2 D}{\partial Y^2} < 0$ hold. In this case the concavity presumption only holds for:

$$c_y r + c_w < 0 \leftrightarrow r < -\frac{c_w}{c_y} < 0. \tag{I.114}$$

[13]We already showed that a higher degree of concavity of the consumption function - given by higher values of $\varepsilon > 1$ - leads to lower aggregate consumption (cf. section 6.2.1.

This implies that the Post-Keynesian case only emerges in a scenario with negative real interest rates below a certain threshold given by the MPC. One might also insert the values of the optimal control derivation of the Keynesian consumption function as presented in section 4.2, implying $c_y = \frac{\rho}{r}$ and $c_w = 0$. In this case the condition would read:

$$c_y r + c_w = \frac{\rho}{r} r + 0 = \rho < 0, \tag{I.115}$$

going along with the unrealistic assumption of a negative rate of time preference, implying that individuals prefer future opposed to current consumption.

All in all - in our model - redistribution of labor income has a negative impact on aggregate demand. On the other hand, the inequality of consumption is lowered, which might be considered an improvement from a welfare point of view. This result can be directly attributed to the collateral constraint. Meanwhile, in a society without financing constraints, redistribution would not matter at all. Therefore, even though we assume a linear consumption function, redistribution matters for aggregate consumption due to the distribution of debt.

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Affidavit

I hereby declare that the dissertation entitled

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is my own work. I have used only the sources indicated and have not made unauthorized use of services of a third party. Where the work of others has been quoted or reproduced, the source is always given. I have not presented this thesis or parts thereof to a university as part of an examination or degree.

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